# The Effects of Nonuniformities on Natural Convection in Annular Receiver Geometries 

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Prepared by Sandia Laboratories. Albuquerque, New Mexico 87115
and Livermore, Califorria Q4550 for the United States Department
of Eneryy under Centrael AT $(20-1) 760$
Printed December 1977


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Printed in the United States of America
Available from
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Price: Printed Copy $\$ 4.00$; Microfiche $\$ 3.00$

SAND77-1641
Unlimited Release

# THE EFFECTS OF NONUNIFORMITIES ON NATURAL CONVECTION IN ANNULAR RECEIVER GEOMETRTES 

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## ABSTRACT

The effects of nonuniform temperature distribution and eccentricity on natural convection heat transfer in a horizontal annulus were assessed through use of a finite element computer program. Calculated results are compared with previously published experimental results. The numerical results indicate that highly nonuniform temperature distributions and large eccentricities are necessary in order to appreciably affect the natural convection process.

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| $\mathrm{C}_{\mathrm{p}}$ | - Specific heat |
| :---: | :---: |
| g | - Acceleration of gravity |
| $\ell$ | - Gap size |
| $\mathrm{N}_{\mathrm{Ra}}$ | - Rayleigh number |
| p | - Pressure |
| Q/L | - Heat transfer per unit length |
| $r_{i}$ | - Outer radius of inner cylinder |
| $r_{0}$ | - Inner radius of outer cylinder |
| T | - Temperature |
| $T^{\prime}$ | - Temperature perturbation |
| $\mathrm{T}_{\mathrm{m}}$ | - Mean temperature |
| $\Delta T$ | - Temp erature difference between cylinders |
| ( $u, \mathrm{v})$ | - Velocity components |
| $(\mathrm{x}, \mathrm{y})$ ) | - Rectangular cartesian coordinates |
| GREEK |  |
| $\beta$ | - Coefficient of volumetric thermal expansion |
| $\varepsilon$ | - Eccentricity |
| $\theta$ | - Angular position |
| K | - Thermal conductivity |
| $\mu$ | - Dynamic viscosity |
| $\rho$ | - Density |

THE EFFECTS OF NONUNIFORMITIES ON NATURAL CONVECTION IN ANNULAR RECEIVER GEOMETRIES

## I. Introduction

An effective device for the collection of solar energy which has received widespread attention is the so called parabolic-cylindrical solar collector. In this device a circular receiver tube, with a suitable selective coating, is enclosed by a concentric glass envelope and situated along the focal line of a parabolic trough reflector. The heat transfer processes which occur in the annular space between the receiver tube and the glass envelope are important in determining the overall heat loss from the receiver tube. In typical high temperature receiver tube designs the rate of energy loss by combined thermal conduction and natural convection is of the same order of magnitude as that due to thermal radiation, and can amount to approximately $6 \%$ of the total rate at which energy is absorbed by the solar collector. The elimination of conduction and natural convection losses can thus significantly improve the performance of a large collector field.

It is well known that natural convection effects have a negligible influence on heat transfer in a uniformly heated annulus when the Rayleigh number, based on gap size, is less than approximately 1000. Below this value, thermal conduction continues to be an important mode of heat transfer. The magnitude of conduction losses can be reduced if the pressure in the annulus is sufficiently reduced. A detailed study of the effects of reduced pressure on conduction heat loss in annular solar receivers is reported in Reference 1.

When the pressure is not maintained at a level low enough to effectively reduce conduction losses, the energy loss, for a uniformly heated annulus, can be minimized by sizing the annulus so as to maintain the Rayleigh number
near 1000 over the expected range of operating conditions. In order to assess the relative importance of nonuniform temperature distribution and eccentricity, a numerical study of the natural convection process in an annulus was performed.

The receiver configuration upon which this study is based is typical of that used in the Solar Total Energy System at Sandia Laboratories. The receiver tube has a "black chrome" selective coating and is 2.54 cm in outside diameter. The inside diameter of the glass envelope is approximately 4.4 cm . Typical operating temperatures of the receiver tube and glass envelope are approximately $300^{\circ} \mathrm{C}$ and $100^{\circ} \mathrm{C}$, respectively.

## II. Analysis

## A. Previous Work

The classical experimental results of Kraussold [2] for natural convection heat transfer between horizontal, concentric, circular cylinders have been used frequently in the thermal analysis of annular solar receivers. Many subsequent studies, dealing with the same topic, have been reported in the literature. Among the more notable are the experimental results of Liu, Mueller, and Landis [3], the experimental and theoretical studies of Kuehn and Goldstein [4] and Powe, Carley, and Carruth [5], and the review by Buchberg, Catton, and Edwards [6].

Recently, Kuehn and Goldstein [7], have compiled a comprehensive review of the available experimental results for natural convection heat transfer between circular cylinders and proposed correlating equations using a conduction boundary-layer model. It is evident from this review that almost all the results available to date are, strictly speaking, valid only for horizontal, concentric, circular cylinders with uniform temperatures. Since the temperature distribution on a typical receiver tube is not uniform, it is important to determine the effect of this variation on the overall heat transfer process. Furthermore, it is also of importance to assess the effect of eccentricity since it is difficult to maintain precise alignment between the receiver tube and envelope due to deflections caused by gravity and differential thermal expansion.

Because of the scarcity of results for nonuniform temperature distributions and eccentric cylinders, a numerical analysis of the natural convection heat transfer process was
performed in order to provide a better understanding of the effects of non-ideal situations. Fundamental aspects of the analysis are outlined in the remainder of this section and the results discussed in the following section.

## B. Numerical Analysis

At this point we wish to summarize the basic theory upon which the numerical analysis is based. The appropriate model is that of steady, planar, laminar thermal convection of a fluid due to nonuniformities of density in a gravitational field. In accordance with the usual Boussinesq approximation, density changes are neglected except insofar as they affect the body force term in the equations of motion. It is, of course, also assumed that the fluid is Newtonian

Subject to the stated assumptions, conservation of mass is expressed by

$$
\begin{equation*}
\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}
\end{equation*}
$$

where ( $x, y$ ) are the rectangular cartesian coordinates and ( $u, v$ ) are the corresponding velocity components. Fluid motion is described by the Navier-Stokes equations

$$
\begin{align*}
& u \frac{\partial u}{\partial x}+v \frac{\partial u}{\partial y}=-\frac{1}{\rho_{O}} \frac{\partial p}{\partial x}-g_{x}\left[1-\beta\left(T-T_{o}\right)\right]+\frac{\mu}{\rho_{O}}\left(\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}\right),  \tag{2}\\
& u \frac{\partial v}{\partial x}+v \frac{\partial v}{\partial y}=-\frac{1}{\rho_{O}} \frac{\partial p}{\partial y}-g_{y}\left[1-\beta\left(T-T_{o}\right)\right]+\frac{\mu}{\rho_{O}}\left(\frac{\partial^{2} v}{\partial x^{2}}+\frac{\partial^{2} v}{\partial y^{2}}\right), \tag{3}
\end{align*}
$$

where $p$ is the pressure, $\left(g_{x}, g_{y}\right)$ are the components of the gravitational acceleration vector, $\rho_{0}$ is the density evaluated at $T_{o}, \mu$ is the dynamic viscosity, $\beta$ is the coefficient of volumetric thermal expansion, $T$ is the temperature, and $T_{0}$ is a reference temperature. The transport of thermal energy is described by

$$
\begin{equation*}
\rho_{o} c_{p}\left(u \frac{\partial T}{\partial x}+v \frac{\partial T}{\partial Y}\right)=\frac{\partial}{\partial x}\left(\kappa \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial Y}\left(\kappa \frac{\partial T}{\partial Y}\right) \tag{4}
\end{equation*}
$$

where $c_{p}$ is the specific heat, $k$ is the thermal conductivity and viscous dissipation effects have been neglected.

For purposes of the present analysis, the appropriate boundary conditions are those of no-slip at a solid surface

$$
\begin{equation*}
u=0=v \text { on } S, \tag{5}
\end{equation*}
$$

and specified temperature

$$
\begin{equation*}
T=f(x, y) \quad \text { on } S \tag{6}
\end{equation*}
$$

Equations (1) through (4) together with the boundary conditions (5) and (6) constitute a complete set for the determination of the velocity, pressure, and temperature fields associated with the natural convection process. It should be noted that the velocity and temperature fields are coupled through the body force term in the equations of motion. It is this effect which provides the driving force for the flow.

The partial differential equations which describe the natural convection process are represented discretely through use of the Galerkin form of the finite element method; a technique which has been described in detail by several authors, e.g. Zienkiewicz [8], and will not be elaborated on here. The resulting methodology has been incorporated, by Gartling [9], into a user-oriented, finite element, computer program called NACHOS. This program makes use of an isoparametric mesh generator to allow complex boundaries to be modeled easily and accurately. The element library consists of an eight-node isoparametric quadrilateral and a six-node isoparametric triangle. Within each element, the velocity and temperature are approximated quadratically and the pressure approximated linearly.

The program is quite versatile and can be used for the analysis of both free and forced convection heat transfer as well as for isothermal flows. Both steady state and transient analyses can be performed. The flows must, however, be incompressible. Temperature dependent fluid properties can be easily incorporated into the analysis at the option of the user. This latter option was used in all the calculations discussed in the remainder of this report. Specific equations for the properties of air as functions of temperature are summarized in the Appendix.

## III. Results of the Analysis

A. Concentric Cylinders - Uniform Temperature Distribution

In order to demonstrate that the results of the numerical analysis are compatible with existing experimental results, an initial series of calculations was performed for horizontal, concentric, circular cylinders with uniform temperatures. The temperatures of the inner and outer cylinders were held constant at 1050 R (583.3K) and 600 R (333.3K), respectively. These values were selected to correspond to the average operating conditions expected for the original Sandia Laboratories Solar Total Energy collector field. The radius of the inner cylinder was held constant at 0.5 in ( 1.27 cm ) and the radius of the outer cylinder varied, producing radius ratios in the range 1.4 to 3.4. Rayleigh numbers ranging from approximately 300 to 97,000 were thus obtained with the Rayleigh number defined in the usual way by

where $\ell$ and $\Delta T$ are the appropriately chosen reference length and temperature difference. The basic parameters and global results for all cases considered are summarized in Table $I$, where cases 1 through 6 apply to the situation currently under consideration.

Following the accepted standard, results of all the analyses are presented in Figure 1 as a plot of the heat loss ratio versus the Rayleigh number, where the heat loss ratio is defined to be the ratio of the energy loss per unit length due to natural convection to that due to thermal conduction acting alone. In Figure 1 , the cross-hatched area indicates the region occupied by the experimental data as compiled by Kuehn and Goldstein [7]. Results of the

Table 1. Tabulated Data for All Casos Considered

| Case No. | $\begin{gathered} r_{i} \\ (f t) \end{gathered}$ | $\begin{gathered} r_{0} \\ (f, t) \end{gathered}$ | $\begin{gathered} T_{i} \\ (f t) \end{gathered}$ | $\mathrm{T}_{\mathrm{o}}$ <br> (R) | $\mathrm{N}_{\mathrm{Ra} a}$ | ( $Q / L$ ) cond. (BTL:/ft-sec) | (Q/I.) conv. (BTU/ft-sec) | ( $\mathrm{e}^{\text {fr }}$ ) | Geometry |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | . 0417 | . 0567 | 1050 | 600 | 331 | . 0557 | . 0553 | 0.993 | concentric cylinders |
| 2 | . 0417 | . 0651 | 1050 | 600 | 1258 | . 0384 | . 0397 | 1.034 | concentric cylinders |
| 3 | . 0417 | . 0736 | 1050 | 600 | 3162 | . 0301 | . 0375 | 1.246 | concentric cylinders |
| 4 | . 0417 | . 0917 | 1050 | 600 | 12142 | . 0217 | . 0435 | 2.005 | concentric cylinders |
| 5 | . 0417 | . 1167 | 1050 | 600 | 40981 | . 0166 | . 0489 | 2.946 | concentric cylinders |
| 6 | . 0417 | . 1417 | 1050 | 600 | 97140 | . 0140 | . 0502 | 3.586 | concentric cylinders |
| 7 | . 0417 | . 0736 | 1050* | 600 | 3162 | . 0301 | . 0346 | 1.150 | concentric cylinders inner cylinder hot on bottom |
| 8 | . 0417 | . 0917 | 1050* | 600 | 12142 | . 0217 | . 0434 | 2.000 | concentric cylinders inner cylinder hot on bottom |
| 9 | . 0417 | . 1417 | 1050* | 600 | 97140 | . 0140 | . 0477 | 3.407 | concentric cylinders inner cylinder hot on bottom |
| 10 | . 0417 | . 0736 | 1050* | 600 | 3162 | . 0301 | . 0455 | 1.512 | concentric cylinders inner cylinder hot on top |
| 11 | . 0417 | . 0917 | 1050* | 600 | 12142 | . 0217 | . 0556 | 2.562 | concentric cylinders inner cylinder hot on top |
| 12 | . 0417 | . 1417 | 1050* | 600 | 97140 | . 0140 | . 0620 | 4.429 | concentric sylinders inner Eylinder hot on top |
| 13 | . 0417 | . 0917 | 1233 | 417 | 21831 | . $0452^{\text {\# }}$ | . 0990 | 2.190 | eccentric cylinders <br> inner cylinder <br> displaced downward $0.025 \mathrm{ft}$ |
| 14 | . 0417 | . 0917 | 1050 | 600 | 12142 | . $024 \mathrm{~g}^{\text {\# }}$ | . 0472 | 2.896 | eccentric cylinders <br> inner cylinder <br> displaced downward <br> 0.025 ft |

[^0]

Figure 1. Natural Convection Heat Transfer in an Annulus Comparison of Computed and Experimental Results
numerical analysis for horizontal, concentric, isothermal cylinders are indicated by the open circles. Computed streamlines and isotherms for each situation considered are presented in Figures 2 through 7. In these and all subsequent streamline plots, a positive value for the stream function indicates a counterclockwise circulation and vice-versa.

It is evident that, in general, there is rather good agreement between the computed and experimentally determined values of natural convection heat transfer rates. As expected, the heat loss is seen to initially deviate from that due to thermal conduction at a Rayleigh number of approximately 1000. The numerical method employed is apparently of sufficient accuracy to warrant its application to the study of the non-ideal situations mentioned previously which have not, as yet, been extensively investigated.

## B. Concentric Cylinders - Nonuniform Temperature Distribution

Two situations involving nonuniform temperature distributions on horizontal, concentric, circular cylinders were selected for study. In both cases the temperature of the outer cylinder was held constant at 600 R (333.3K) and the temperature of the inner cylinder allowed to vary according to

$$
\begin{equation*}
T=T_{m} \pm T^{\prime} \cos \theta \tag{8}
\end{equation*}
$$

where $T_{m}$ is the mean temperature, 1050 R ( 583.3 K ), $\mathrm{T}^{\prime}$ is the perturbation to the mean, and $\theta$ is the angle measured from the bottom of the cylinder. A perturbation of 250 R (138.9K) was used in all calculations, yielding a total variation of $500 \mathrm{R}(277.8 \mathrm{~K})$ around the inner cylinder. The two cases studied are distinguished by the choice of sign in Equation (8); the positive sign corresponding to the occurrence of


Figure 2. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Uniform Temperature, $\mathrm{N}_{\text {Ra }}=331$ (Case 1)


Figure 3. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Uniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=1258$ (Case 2)


Figure 4. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Uniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=3162$ (Case 3)


Figure 5. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Uniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=12,142$ (Case 4)


Figure 6. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Uniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=40,981$ (Case 5)


Figure 7. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Uniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=97,140$ (Case 6)
greatest temperature on the lower surface of the inner cylinder and vice versa. A variation of 500 R ( 277.8 K ) is far in excess of that encountered in conventional receiver tubes but, as will subsequently be shown, even this amount of nonuniformity had only a small effect on the natural convection process. In all calculations, the radius of the inner cylinder was held constant at 0.50 in. ( 1.27 cm ) and the radius of the outer cylinder varied in order to vary the Rayleigh number. It should be noted that the average temperature upon which the Rayleigh number was based had the same value ( $825 \mathrm{R}, 458.3 \mathrm{~K}$ ) as that associated with the uniform temperature case.

When the highest temperature occurred on the lower surface of the inner cylinder the calculated results for the overall heat transfer were virtually indistinguishable from those obtained with uniform wall temperatures, as evidenced by cases 7, 8, and 9 in Table 1 and the plot of Figure 1. The associated streamlines and isotherms are plotted in Figures 8 through lo. It is interesting to note that when the lower surface was at a higher temperature flow patterns were produced which, for the higher Rayleigh numbers of cases 8 and 9, differed significantly from those associated with a uniform temperature distribution. For Rayleigh numbers of approximately 12000 and 97000 (Figures 9 and 10), a two-cell, counter-rotating flow pattern with one cell above the other and each spanning the entire gap between cylinders was obtained. The altered flow pattern, however, produced no appreciable change in the overall heat transfer characteristics.

When the higher temperature occurred on the upper surface of the inner cylinder (cases 10,11, and 12 ) the overall heat transfer rate was enhanced over that obtained with a uniform temperature as shown in Figure 1. Departure from the thermal conduction curve apparently occurred at a Rayleigh number of


Figure 8. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Nonuniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=3162$ (Case 7)


Figure 9. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Nonuniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=12,142$ (Case 8)


Figure 10. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Nonuniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=97,140$ (Case 9)
approximately 1000. Calculated streamlines and isotherms for Rayleigh numbers of approximately 3000, 12000, and 97000 are presented in Figures 11 through 13.
C. Eccentric Cylinders - Uniform Temperature Distribution

The geometry chosen for the study of eccentric cylinders consisted of inner and outer cylinders of radii 0.50 in . $(1.27 \mathrm{~cm})$ and 1.10 in . $(2.79 \mathrm{~cm})$, with the inner cylinder displaced downward a distance equal to one-half the gap width yielding an eccentricity of 0.30 in ( 0.76 cm ) . Each cylinder was held at a uniform temperature. Although other geometries could be analyzed without difficulty, the selection of the case chosen for study was influenced by certain practical considerations. First, a downward displacement of the inner cylinder enhances the convection process more than an upward displacement. Secondly, an eccentricity of one-half the gap width is greater than that encountered in practical receiver designs. Finally, it was convenient to maintain symmetry about a vertical plane in order to simplify the numerical computations.

For proper interpretation of subsequent results, it should be recalled that the heat loss per unit length by thermal conduction between eccentric, circular cylinders with uniform wall temperatures is given by

$$
\begin{equation*}
Q / L=2 \pi \kappa \Delta T / \ln \left(x+\sqrt{x^{2}-1}\right) \tag{9}
\end{equation*}
$$

where

$$
x=\left(r_{0}^{2}+r_{i}^{2}-\varepsilon^{2}\right) / 2 r_{o} r_{i}
$$



Figure ll. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Nonuniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=3162$ (Case 10)


Figure 12. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Nonuniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=12,142$ (Case 11)


Figure 13. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Nonuniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=97,140$ (Case 12)
$r_{i}, r_{o}$ are the inner and outer radii, respectively, and $\varepsilon$ is the eccentricity. Equation (9) is used as the basis of comparison for the heat transfer results calculated for natural convection between eccentric cylinders.

The overall heat transfer results obtained for two different Rayleigh numbers are plotted in Figure 1. The associated streamlines and isotherms are presented in Figures 14 and 15. If the Rayleigh number is based on the mean gap size of 0.60 in. ( 1.52 cm ) and the cylinders held constant at the values previously used in the analysis of concentric cylinders, a Rayleigh number of approximately 12000 results (see case 14 in Table I). For the remaining situation (case 13 in Table I), the temperature of the inner cylinder was increased and that of the outer cylinder decreased, by equal amounts, in order to produce a Rayleigh number of approximately 22000 while simultaneously maintaining the mean temperature constant at 825 R (458.3K). This latter calculation was performed in order to tentatively assess the effect of Rayleigh number on the overall heat transfer process.

It is apparent from Figure 1 that the results for eccentric cylinders coincide with those for concentric cylinders when the appropriate conduction solution is used as a reference and the Rayleigh number is based on the mean gap size. However, it should be noted that the overall heat transfer is slightly enhanced because of the increased effect of conduction for eccentric cylinders. For the variables used, the heat transferred by conduction is $14.75 \%$ greater for eccentric cylinders than for concentric cylinders.


Figure 14. Streamlines and Isotherms for Natural Convection Between Eccentric, Circular Cylinders, Uniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=21,831$ (Case 13)


Figure 15. Streamlines and Isotherms for Natural Convection Between Concentric, Circular Cylinders, Uniform Temperature, $\mathrm{N}_{\mathrm{Ra}}=12,142$ (Case 14)

## Discussion

In the previous section the effects of nonuniform temperature distributions and eccentricity on natural convection heat transfer in an annulus were assessed through use of a finite element computer program. This study was performed in order to obtain a better understanding of the roll of natural convection in the overall heat loss from cylindrical solar receivers typical of those used in conjunction with parabolic trough reflectors. Based on the numerical results, several interesting qualitative and quantitative observations can be made.

The calculated results for horizontal, concentric, isothermal cylinders are in excellent agreement with published experimental results. It is this agreement which lends confidence to the computed results for situations which have not been studied experimentally.

From the results of Section III B, it is evident that highly nonuniform temperature distributions are required in order to appreciably affect the natural convection process between horizontal, concentric cylinders. Since the nonuniformity of temperature in typical receiver geometries is substantially less than that used in the current study, it is anticipated that the effects of nonuniform temperature can be neglected in most instances. Departure from a thermal conduction dominated behavior still occurs at a Rayleigh number of approximately 1000. Hence, this condition can still be used in order to determine the proper sizing for receiver geometries.

The single most important observation regarding the effect of eccentricity is that, for uniform cylinder wall temperatures, a.rather large eccentricity causes only a slight increase in overall heat transfer. This increase is due, in large part, to the enhancement of thermal conduction by
the eccentric geometry. Departure from conduction dominated behavior again seems to occur at a Rayleigh number of approximately 1000 , based on the mean gap size.

In order to fully understand the effec:s of the parameters chosen for investigation it would be necessary to perform a rather lengthy series of calculations. The general trends and initial results obtained from the current study should, nevertheless, be useful for the evaluation of cylindrical receiver designs. It is also worth noting at this point that the numerical technique used can be readily adapted to the analysis of more complex receiver geometries.

## APPENDIX

## PROPERTIES OF AIR

The equations* used for the properties of air as functions of temperature are listed below. In all equations, the temperature $T$ must be expressed in Rankine degrees. In the analysis, the variation of the specific heat $c_{p}$ with temperature was neglected.

$$
\begin{array}{ll}
\rho=39.68 / T & \left(1 \mathrm{bm} / \mathrm{ft}^{3}\right) \\
\beta=1 / T & (\mathrm{l} / \mathrm{R})  \tag{1/R}\\
\mu=\left(7.3094 \times 10^{-7}\right) \frac{\mathrm{T}^{3 / 2}}{\mathrm{~T}+198.7} & (\mathrm{lbm} / \mathrm{ft}-\mathrm{sec}) \\
c_{\mathrm{p}}=0.2238+\left(2.533 \times 10^{-5}\right) \mathrm{T} & (\text { Btu/lbm-R) } \\
\kappa=\frac{\left(1.14 \times 10^{-3}\right) \mathrm{T}^{1 / 2}}{1+\frac{441.7}{T} \cdot 10^{(-21.6 / T)}} & \text { (Btu/hr-ft-R) }
\end{array}
$$

* $\rho, \beta$ : computed from ideal gas law
$\mu, k, c_{p}$ : recommended equations in Handbook of Supersonic Aerodynamics, Vol. 5, Bureau of Ordnance, Dept. of the Navy, Washington, D.C., 1953.


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9515 J. T. Hillman
9570 R. S. Wilson
9572 E. R. Frye
8266 E. A. Aas
3141 C. A. Pepmueller (Actg.)
3151 W. L. Garner (3)
For DOE /TIC (Unlimited Release)
3172-3 R. D. Campbell (25) for DOE/TIC


[^0]:    * denotes mean value
    \# based on eccentric conduction formula

