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A Methodology for Determining the Economic Feasibility of Residential or Commercial Solar Energy Systems

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A METHODOLOGY FOR DETERMINING THE ECONOMIC FEASIBILITY OF RESIDENTIAL OR COMMERCIAL SOLAR ENERGY SYSTEMS

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ABSTRACT

This paper presents a methodology for analyzing the economic feasibility of solar energy systems based upon a life-cycle costing criterion. Both residential and commercial solar energy systems may be handled. The analysis provides for a comparison between solar and conventional energy systems. This methodology has been adapted for use in Sandia Laboratories' systems analysis work in solar-mechanical energy storage, solar photovoltaics, solar irrigation, and solar total energy.

ACKNOWLEDGMENT

Much of the work presented in this paper evolved from interactions with members of the Solar Heating and Cooling Branch's Working Group on Systems Simulation and Economic Analysis. The methodology presented here is a documentation of the techniques currently in use by many members of this group.

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A METHODOLOGY FOR DETERMINING THE ECONOMIC FEASIBILITY OF RESIDENTIAL OR COMMERCIAL SOLAR ENERGY SYSTEMS

Introduction

There are many criteria by which the economic feasibility of a solar energy system (SES) can be assessed. This paper presents a methodology for analyzing the economic feasibility of a residential or commercial solar energy system based upon a life-cycle costing criterion. The methodology takes into account both the initial costs of purchasing a system and the recurring costs associated with operating the system throughout its lifetime. Financial parameters reflecting the state of the economy in general and the economic position of the system's owner in particular are also necessary. The output of the model is the life-cycle cost (LCC) of the solar energy system, which may then be compared to the LCC of a conventional energy system in order to determine economic feasibility.

Life-cycle costing is a convenient method for treating expenditures and receipts that occur at different periods of time. Due to the existence of an interest rate, a dollar received today and invested at the interest rate is worth more than a dollar received at a later date. Therefore, it is not possible to sum the various costs of an SES occurring through time to determine its total LCC. Instead, it is necessary to bring expenditures and receipts occurring at different periods of time to an equivalent basis. The techniques and formulas for moving sums of money back and forth through time may be found in any engineering economics text.*

An LCC may be expressed in two different ways, either as a present value (PV) or as an annual cost (AC). The present value may be thought of as the amount of money necessary at the beginning of system operation which, if invested at the discount rate (defined below), would pay for all costs of purchasing and operating the system throughout its lifetime. The annual cost is equivalent to the PV, but instead represents the yearly amount that must be invested at the discount rate throughout the system's lifetime to pay for all system costs. Both these quantities may be calculated using this methodology.

The methodology is designed to handle both residential and commercial applications. These ownership categories must be treated differently, due to differences in the tax laws. A commercial enterprise is allowed to take tax deductions for operating and maintenance costs, energy costs, property taxes, interest on loan payments and depreciation. A homeowner is allowed to deduct only property taxes and interest on loan payments. The equations for both cases reflect this difference.

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*For instance, Principles of Engineering Economy, Grant, Ireson, Leavenworth.

Although the methodology is designed primarily to determine the LCC of an SES, the LCC of alternative conventional energy systems must also be found in order to determine the SES's economic feasibility. The same methodology may be used to determine the LCC of conventional systems, provided inputs reflecting conventional system costs are used. Once the LCCs of both the SES and a conventional alternative are found, a quantity called solar savings is calculated:

$$\text{Solar Savings} = LCC_{conventional} - LCC_{solar}$$

The solar savings term represents the total life-cycle savings of the SES over its conventional counterpart. If the solar savings term is positive, the SES is competitive and its economic feasibility has been demonstrated on a life-cycle costing basis. However, if the solar savings term is negative, the conventional system is still the less expensive alternative over the period under analysis.*

LCC Methodology

A number of inputs are necessary for calculating the LCC of solar energy systems. These inputs are listed in Table I and are described below. Suggested nominal values for many of these inputs are presented in Appendix A.

TABLE I

Inputs for LCC Analysis

d	Discount rate	N_D	Accounting lifetime
D	Percent down payment	N_M	Borrowing period
e	Energy escalation rate	OM	Miscellaneous costs
F	Backup energy cost	P	Property taxes
g	General inflation rate	SV	Salvage value
i	Interest rate	t	Income tax rate
I	Investment tax credit	y_o	Year of operation
IC_p	Initial system cost	y_p	Price year
N	Period of analysis		

The methodology uses these inputs for calculating the present value of several quantities necessary to determine LCC. A flow chart of the methodology is shown in Figure 1.

*It must be emphasized that although this methodology can be used to determine economic feasibility, the results arrived at are necessarily limited by the accuracy of the inputs used. Many of these input values will be difficult to estimate. Hence, the results will be based on a number of assumptions, and should be treated as approximations to what the true situation would be if all input values were known with certainty.

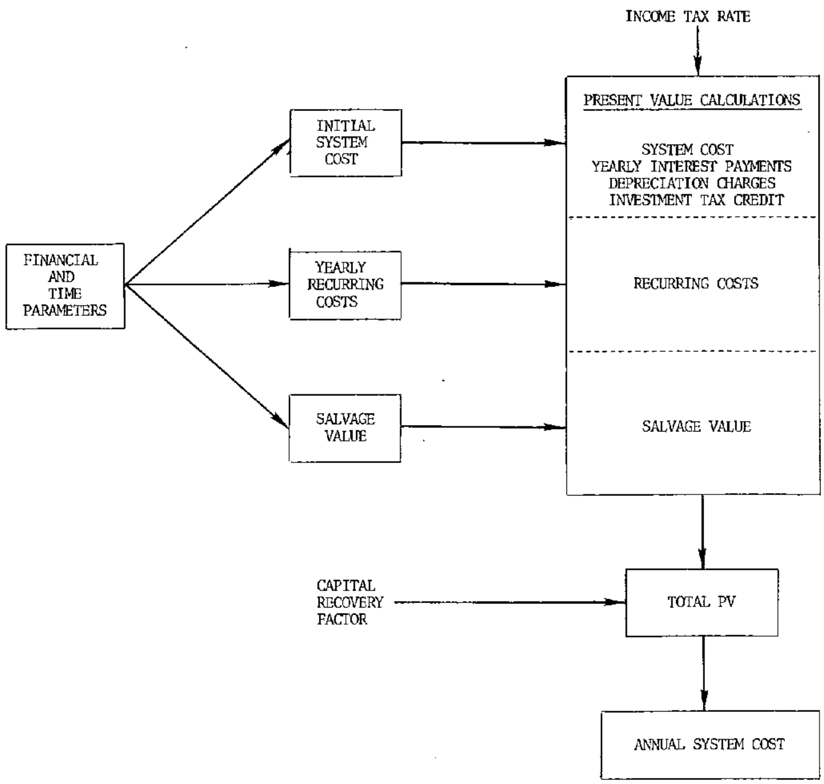

Figure 1. LCC Methodology

Description of Inputs

The inputs necessary for calculating LCC can roughly be divided into three categories; system costs, financial parameters, and time parameters.

System Costs -- System costs include all costs of purchasing, operating and maintaining the SES. All system costs should be expressed in price year dollars.

1. Initial cost of SES. The total initial cost of purchasing an SES for a home or a business. This should include the cost of a backup unit, if necessary, for use when solar energy is insufficient to satisfy the load demand.

2. Yearly miscellaneous costs. The yearly cost of operation, maintenance and insurance of the SES, including the cost of minor repairs and parts. Any other yearly costs may also be included here.*

*Major replacements of system components are sometimes necessary when all system components do not have the same lifetimes. These replacement costs are not rigorously handled in this methodology. However, a factor to account for replacement costs may be included in yearly miscellaneous costs. One possible way of doing this is to allocate some fixed percentage of the initial cost of the SES yearly to account for replacement costs. This yearly cost would then be added to the other miscellaneous costs, and would be available to pay for major replacements when they occurred.

3. Yearly property taxes. The annual amount of property taxes paid on the SES.

4. Auxiliary or backup energy costs. The yearly cost of the energy that must be purchased to supply the load when solar energy is insufficient. This includes either off-site sources (e.g., utilities) or on-site backup (e.g., diesel fuel).

5. Salvage value of SES. The residual value of any SES components at the end of the period of analysis.

Financial Parameters -- The financial parameters include both general economic conditions and the SES owner's particular economic situation. The inputs needed are as follows:

1. Nominal discount rate. The rate of return desired by the homeowner or businessman on his investment in the solar energy system. This is often taken to be the opportunity cost of money--what one could receive by investing in his next best alternative. The nominal discount rate should include, as a minimum, a component for inflation and a component that reflects the real rate of return on money--the rate that would prevail if there were no inflation. It may also be desirable to include a component for risk in the discount rate.

2. Interest or borrowing rate. The cost of borrowing money to the homeowner or business. For the homeowner, this may be his mortgage rate, if the SES is an installation on a new residence. Interest rates that prevail in the current market should be used. These include a component that reflects the real cost of money and a component for inflation, as in the discount rate.

3. Percent down payment. The fraction of the initial cost of the solar system that is paid by the purchaser; i.e., not borrowed.

4. Effective income tax rate. Should include both federal and state taxes. Since state taxes are deductible on the federal return:

$$t = \text{state} + \text{federal} - (\text{state} \times \text{federal})$$

Both the federal and state rates should be the marginal tax rate on the next dollar of additional income.

5. General inflation rate. The yearly rate of inflation in the economy, which is assumed constant throughout the period of analysis. Although the inflation rate is difficult to estimate, its particular value is not crucial as long as an equivalent inflation component is included in both the interest and discount rates. A discussion of several methods of treating inflation in this type of analysis is found in Appendix D.

6. Energy escalation rate. The constant yearly rate of increase in the price of conventional forms of energy. May be different for different energy forms. The energy escalation rate should be derived from the expected real increase in the cost of energy and the general inflation rate as follows:

$$e = \left[(1 + \text{inflation}) \cdot (1 + \text{real})\right] - 1$$

Time Parameters -- The time parameters deal both with system life and with date of system operation as follows:

1. Price year for SES costs. It is necessary to choose a reference year for expressing the costs of the SES. In other words, all costs for the SES should be expressed in dollars of one particular year, referred to in this methodology as the price year. Included are the initial cost, yearly miscellaneous costs, yearly backup energy costs, yearly property taxes, and salvage value.

2. Year of operation of the SES. Year in which the SES begins operation.

3. Period of analysis. Number of years over which the economic analysis will be performed. This is usually just the lifetime of the SES, but may be less if a salvage value for the system is assumed.

4. Borrowing period. Number of years available to pay back the loan taken out on the SES.

5. Accounting lifetime. Number of years over which depreciation is to be carried out. May be less than the period of analysis due to the availability of accelerated depreciation methods.

Other Inputs --

1. Investment tax credit. The amount of money allowed as a one-time tax credit by federal and state governments in the first year of operation. Since this is not expected to vary with inflation, it should be expressed in dollars of the year of operation.

2. Depreciation method. The federal tax laws allow several different methods of depreciation. A choice of straight-line or sum-of-the-years-digits depreciation is given in this methodology.

Calculation of Present Values (PVs)

Given the inputs stated above, it is necessary to calculate a number of present values in order to derive the LCC of an SES. (PVs are determined as of the year of first operation.) This is most easily done by use of a present value function, PVF. This function can be used to determine the PV of a yearly expenditure which escalates at some fixed percentage each year and then is discounted over a number of years.

Let

$$PVF(A,B,C) = \frac{1}{A - B} \cdot \left[1 - \left(\frac{1 + B}{1 + A}\right)^C\right] \text{ for } A \neq B$$

$$PVF(A,B,C) = \frac{C}{1 + A} \text{ for } A = B$$

where

A = discount rate, B = escalation rate, C = number of years.

A derivation of this function and some special uses of it are given in Appendix B.

PV of Initial System Cost -- The PV of initial system cost is made up of two components; the initial downpayment plus the PV of yearly payments necessary to pay back the loan taken out to purchase the SES. Before these can be calculated, inflation in the cost of the SES between the price year and the first year of operation must be accounted for. To do this, the initial cost of the SES in dollars of the first year of operation must be derived:

$$IC = IC_p \cdot (1 + g)^{y_o - y_p} \tag{1}$$

The PV of yearly loan payments is:

$$PVLOAN = (1 - D) \cdot IC \cdot \frac{PVF(d,0,N_M)}{PVF(i,0,N_M)} \text{ if } N \geq N_M \tag{2}$$

$$PVLOAN = (1 - D) \cdot IC \cdot \left[\frac{PVF(d,0,N)}{PVF(i,0,N_M)} + \frac{(1 + i)^{N_M} - (1 + i)^N}{(1 + d)^N \cdot \left[(1 + i)^{N_M} - 1\right]}\right] \tag{3}$$

if $N < N_M$.

Equation (3) gives the PV when the period of analysis is shorter than the borrowing period. This entails calculating not only the PV of the loan payments but also the PV of the remaining loan balance at the end of the period of analysis (the second term in Eq. (3)). The derivation of the formula for remaining loan balance is shown in Appendix B. When the analysis period is longer than the borrowing period, as in (2), it is necessary to calculate only the PV of the loan payments.

The PV of total initial system costs then becomes

$$PVSYS = D \cdot IC + PVLOAN \tag{4}$$

PV of Yearly Interest Payments -- Because interest payments are tax-deductible, it is necessary to calculate the PV of the interest portion of each yearly loan payment. This can be done in closed form as follows:

$$PVINT = (1 - D) \cdot IC \cdot \left[PVF(d,i,N_1) \cdot \left(i - \frac{1}{PVF(i,0,N_M)}\right) + \frac{PVF(d,0,N_1)}{PVF(i,0,N_M)}\right] \tag{5}$$

where

$$N_1 = \min(N_M, N)$$

A derivation of this equation can be found in Appendix B.

PV of Yearly Depreciation Charges -- Two possible methods of depreciation are given in this methodology. Straight-line depreciation provides for equal yearly depreciation charges over the accounting lifetime. The yearly depreciation charge, D_j, is

$$D_j = \frac{IC - SV}{N_D} \quad \text{where } j = 1, \ldots, N_D$$

Sum-of-the-years-digits (SOYD) depreciation provides for acceleration of yearly depreciation charges. The yearly charges decline over the accounting lifetime, with a larger fraction of the initial cost depreciated in the earlier years of system life as compared to the straight-line method. The yearly depreciation charge in this case is

$$D_j = (IC - SV) \cdot \frac{N_D - j + 1}{\frac{N_D \cdot (N_D + 1)}{2}} \quad \text{where } j = 1, \ldots, N_D$$

The equations for the PV of depreciation charges are

Straight-line:

$$PVDEPD = (IC - SV) \cdot \frac{1}{N_D} \cdot PVF(d,0,N_D) \tag{6}$$

Sum-of-the-years-digits:

$$PVDEPD = (IC - SV) \cdot \frac{2 \cdot \left[N_D - PVF(d,0,N_D)\right]}{N_D \cdot (N_D + 1) \cdot d} \tag{7}$$

A derivation of (7) is found in Appendix B.

PV of Investment Tax Credit -- It is assumed that the investment tax credit is received at the end of the first year of operation. Its PV is therefore

$$PVITC = \frac{I}{1 + d} \tag{8}$$

PV of Recurring Costs -- Recurring costs include yearly miscellaneous costs, yearly property taxes, and yearly backup energy costs. It is assumed that both miscellaneous costs and property taxes will escalate yearly at the rate of general inflation. However, any escalation rate may be used in place of the general inflation rate in the following formulas. Backup energy costs will rise at the energy escalation rate. The equations are

$$PVMISC = OM \cdot PVF(d,g,N) \cdot (1 + g)^{y_o - y_p} \tag{9}$$

$$PVPROP = P \cdot PVF(d,g,N) \cdot (1 + g)^{y_o - y_p} \tag{10}$$

$$PVENER = F \cdot PVF(d,e,N) \cdot (1 + e)^{y_o - y_p} \tag{11}$$

The PV factor escalates from the point of first operation, so an additional factor is necessary to escalate the costs from the price year to the year of operation.

PV of Salvage Value -- Salvage value is a one-time receipt occurring at the end of the period of analysis. It represents the residual value of all SES components; i.e., what they could be sold for. This was originally expressed in price year dollars. Assuming that the value of the salvaged equipment will rise due to inflation,

$$PVSV = \frac{SV \cdot (1 + g)^{y_o + N - y_p}}{(1 + d)^N} \tag{12}$$

LCC of Solar Energy System

Given the above present values, it is now possible to calculate the PV and AC of an SES under both residential and commercial ownership.

The total PV is calculated by summing the PVs of all system costs, minus the PVs of tax-deductible items and receipts, using a given tax rate. The equations are

Residential:

$$TPV_R = PVSYS + PVMISC + PVENER$$
$$+ (1 - t) \cdot PVPROP - t \cdot PVINT - PVITC - PVSV$$

$$AC_R = TPV_R \cdot \frac{1}{PVF(d,0,N)}$$

Commercial:

$$TPV_C = PVSYS + (1 - t) \cdot [PVMISC + PVPROP + PVENER]$$
$$- t \cdot [PVDEPD + PVINT] - PVITC - PVSV$$

$$AC_C = TPV_C \cdot \frac{1}{PVF(d,0,N)}$$

The equations for residential ownership reflect the tax deductibility of property taxes and interest on loan payments. Presently, miscellaneous and energy costs are not deductible for a homeowner. The equations for commercial ownership reflect the tax deductibility of miscellaneous costs, property taxes, energy costs, interest payments and depreciation.

The conversion to AC is performed by dividing by the PVF for a zero escalation rate (i.e., B = 0). The reciprocal of this factor is often referred to as the capital recovery factor (CRF). The CRF is commonly used to determine the payments necessary to amortize a sum of money over some period of time at a specified interest rate. In this situation, it is used to find the annual amount necessary to pay for all SES costs.

LCC of Conventional Energy Systems

The above equations are also applicable for calculating the LCC of any conventional energy system, providing the proper inputs are used. Specifically, the total cost of the conventional system should be input as initial system cost. Yearly miscellaneous costs should include the cost of operating and maintaining the conventional system. The total cost of purchasing energy for the conventional system should be input in place of the backup energy cost. Finally, financial arrangements for purchasing a conventional system may be different from those for an SES, and these should be taken into account when determining the conventional system's LCC.

The situation is simplified in the case where the SES's backup unit is identical to the conventional system that would otherwise be in use. The initial costs of the backup unit and the conventional system may be ignored, and the SES's initial cost becomes the incremental cost of the SES over its conventional alternative. The LCC of the conventional system is determined by looking solely at the LCC of purchasing energy for that system (the LCC of the SES must still include the cost of purchasing backup energy). The comparison essentially becomes one of purchasing a solar energy system vs purchasing energy to run a conventional one.

Economic Feasibility

Economic feasibility of an SES is determined by comparing the LCC (either PV or AC) of the SES to the LCC of any conventional alternative. This may be done in a variety of ways, but a convenient method is to examine the life-cycle savings of the solar system over those of its conventional counterpart:

$$\text{Solar Savings} = LCC_{\text{conventional}} - LCC_{\text{solar}}$$

If the solar savings term is positive, the solar energy system is less expensive than its conventional alternative over the period of analysis. The economic feasibility of this system given a life-cycle costing criterion is proven. However, if the solar savings term is negative, the conventional system is the less expensive one, and the SES is not economically feasible on a life-cycle costing basis. An example demonstrating the determination of economic feasibility is shown in Appendix C.

Conclusion

There are many criteria for determining economic feasibility of a solar energy system. This methodology has presented only one such criterion, life-cycle costing. Although in some instances other criteria such as payback periods or rates of return may be more appropriate, life-cycle costing is still valuable for providing a true indication of the economic worth of a system over its lifetime.

APPENDIX A

Suggested Input Values for Calculating the LCC of an SES

The following input values have been suggested in a recent DOE report:*

	Residential	Commercial
OM	$0.015 \cdot IC_p$	$0.015 \cdot IC_p$
P	0.	0.
d	0.10	0.10
i	0.085	0.08
D	0.20	0.30
t	0.30	0.50
g	0.05	0.05
e (electricity or oil)	0.07	0.07
e (natural gas)	0.10	0.10
N	20.0	20.0
N_M	30.0	25.0
N_D	-	20.0
I	0.	$0.10 \cdot IC$
SV	0.	0.

*An Analysis of the Current Economic Feasibility of Solar Water and Space Heating, Office of the Assistant Secretary for Conservation and Solar Applications, January 1978, DOE/CS-0023.

APPENDIX B

Derivation of Equations Used in LCC Methodology

<u>Present Value Function (PVF)</u>

The PVF determines the PV of an expenditure of $1/yr that is escalated at some fixed percentage each year and then discounted over a number of years.

Let A = discount rate

B = escalation rate

C = number of years

The PV of a $1/yr expenditure escalated at B and discounted at A is

$$PVF(A,B,C) = \sum_{i=1}^{C} \frac{(1+B)^{i-1}}{(1+A)^{i}}$$

This assumes no escalation in the first year. This may be simplified as follows:

$$PVF(A,B,C) = \frac{1}{1+A} + \frac{1+B}{(1+A)^2} + \cdots + \frac{(1+B)^{C-1}}{(1+A)^C}$$

$$= \frac{1}{1+A} \cdot \left[1 + \frac{1+B}{1+A} + \cdots + \frac{(1+B)^{C-1}}{(1+A)^{C-1}}\right]$$

Summing the geometric series in brackets:

$$PVF(A,B,C) = \frac{1}{1+A} \cdot \left[\frac{1 - \left(\frac{1+B}{1+A}\right)^{C}}{1 - \left(\frac{1+B}{1+A}\right)}\right]$$

$$= \frac{1}{A-B} \cdot \left[1 - \left(\frac{1+B}{1+A}\right)^{C}\right] \qquad \text{QED}$$

Any yearly expenditure that is expected to escalate at some rate may be multiplied by PVF(A, B, C) in order to determine the total PV of all the expenditures.

For the case where A = B,

$$PVF(A,B,C) = \sum_{i=1}^{C} \frac{(1 + A)^{i-1}}{(1 + A)^{i}}$$

$$= \sum_{i=1}^{C} \frac{1}{(1 + A)}$$

$$= \frac{C}{1 + A}$$

For the special case B = 0:

$$PVF(A,0,C) = \frac{1}{A} \cdot \left[1 - \frac{1}{(1 + A)^{C}}\right]$$

$$= \frac{(1 + A)^{C} - 1}{A \cdot (1 + A)^{C}}$$

When the escalation rate equals zero, the PVF is used simply to find the PV of a constant yearly expenditure, such as loan payment. The reciprocal of the PVF with B = 0 is commonly referred to as a capital recovery factor (CRF). When multiplied by an initial sum of money, the CRF determines the periodic payment necessary to pay back that sum of money at interest rate A over C periods.

PV of Remaining Loan Balance

The PV of the remaining loan balance must be calculated when the period of analysis is shorter than the borrowing period.

N = period of analysis
N_M = borrowing period
d = discount rate
i = interest rate
B_0 = initial loan amount
B_j = loan balance at end of year j
$i \cdot B_{j-1}$ = interest at end of year j
P = constant yearly payment necessary to pay back loan in N_M years

Therefore:

$$B_1 = B_0 - (P - i \cdot B_0) = (1 + i) \cdot B_0 - P$$

$$B_2 = B_1 - (P - i \cdot B_1) = (1 + i) \cdot B_1 - P$$

$$= (1 + i) \cdot \left[(1 + i) \cdot B_0 - P\right] - P$$

$$= (1 + i)^2 \cdot B_0 - (1 + i) \cdot P - P$$

$$\vdots$$

$$B_j = (1 + i)^j \cdot B_0 - (1 + i)^{j-1} \cdot P$$

$$- (1 + i)^{j-2} \cdot P - \ldots - (1 + i) \cdot P - P$$

Therefore:

$$B_j = (1 + i)^j \cdot B_0 - P \cdot \left[(1 + i)^{j-1} + (1 + i)^{j-2} + \ldots + (1 + i) + 1\right]$$

$$= (1 + i)^j \cdot B_0 - P \cdot \left[\frac{1 - (1 + i)^j}{1 - 1 - i}\right]$$

$$= (1 + i)^j \cdot B_0 + P \cdot \left[\frac{1 - (1 + i)^j}{i}\right]$$

The remaining balance at the end of the period of analysis, N, is

$$B_N = (1 + i)^N \cdot B_0 + P \cdot \left[\frac{1 - (1 + i)^N}{i}\right]$$

The payment, P, can be obtained by multiplying the initial loan balance, B_0, by the capital recovery factor:

$$P = B_0 \cdot \frac{1}{PVF(i,0,N_M)} = B_0 \cdot \left[\frac{i \cdot (1 + i)^{N_M}}{(1 + i)^{N_M} - 1}\right]$$

Therefore:

$$B_N = (1+i)^N \cdot B_0 + B_0 \cdot \frac{i \cdot (1+i)^{N_M}}{(1+i)^{N_M} - 1} \cdot \left[\frac{1-(1+i)^N}{i}\right]$$

$$= (1+i)^N \cdot B_0 + B_0 \cdot \left[\frac{(1+i)^{N_M} - (1+i)^{N+N_M}}{(1+i)^{N_M} - 1}\right]$$

$$= B_0 \cdot \left[\frac{(1+i)^{N+N_M} - (1+i)^N + (1+i)^{N_M} - (1+i)^{N+N_M}}{(1+i)^{N_M} - 1}\right]$$

$$= B_0 \cdot \left[\frac{(1+i)^{N_M} - (1+i)^N}{(1+i)^{N_M} - 1}\right]$$

The PV of the remaining loan balance is then

$$PV = \frac{B_0}{(1+d)^N} \cdot \left[\frac{(1+i)^{N_M} - (1+i)^N}{(1+i)^{N_M} - 1}\right] \qquad \text{QED}$$

PV of Yearly Interest Payments

It is necessary to find the PV of yearly interest payments due to the tax deductibility of these payments. Let

N = period of analysis
N_M = borrowing period
N_1 = min (N, N_M)
i = interest rate
d = discount rate
B_0 = initial loan amount
B_j = balance at end of year j
$I_j = i \cdot B_{j-1}$ = interest at the end of year j
P = constant yearly payment necessary to pay back loan in N_M years

The PV of yearly interest payments is

$$PVINT = \sum_{j=1}^{N_1} \frac{I_j}{(1+d)^j}$$

Now:

$$
\begin{aligned}
I_j &= i \cdot B_{j-1} \\
&= i \cdot \left\{(1 + i)^{j-1} \cdot B_0 + P \cdot \left[\frac{1 - (1 + i)^{j-1}}{i}\right]\right\} \\
&= i \cdot (1 + i)^{j-1} \cdot B_0 + \frac{i \cdot B_0}{PVF(i,0,N_M)} \cdot \left[\frac{1 - (1 + i)^{j-1}}{i}\right] \\
&= i \cdot (1 + i)^{j-1} \cdot B_0 + \frac{B_0}{PVF(i,0,N_M)} - B_0 \cdot \frac{(1 + i)^{j-1}}{PVF(i,0,N_M)}
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
PVINT &= \sum_{j=1}^{N_1} \frac{I_j}{(1 + d)^j} \\
&= \sum_{j=1}^{N_1} i \cdot \frac{(1 + i)^{j-1} \cdot B_0}{(1 + d)^j} + \sum_{j=1}^{N_1} \frac{B_0}{PVF(i,0,N_M) \cdot (1 + d)^j} - \sum_{j=1}^{N_1} \frac{(1 + i)^{j-1} \cdot B_0}{PVF(i,0,N_M) \cdot (1 + d)^j} \\
&= i \cdot B_0 \cdot \sum_{j=1}^{N_1} \frac{(1 + i)^{j-1}}{(1 + d)^j} + \frac{B_0}{PVF(i,0,N_M)} \cdot \sum_{j=1}^{N_1} \frac{1}{(1 + d)^j} - \frac{B_0}{PVF(i,0,N_M)} \cdot \sum_{j=1}^{N_1} \frac{(1 + i)^{j-1}}{(1 + d)^j} \\
&= i \cdot B_0 \cdot PVF(d,i,N_1) + \frac{B_0}{PVF(i,0,N_M)} \cdot PVF(d,0,N_1) - \frac{B_0}{PVF(i,0,N_M)} \cdot PVF(d,i,N_1) \\
&= B_0 \cdot \left[PVF(d,i,N_1) \cdot \left(i - \frac{1}{PVF(i,0,N_M)}\right) + \frac{PVF(d,0,N_1)}{PVF(i,0,N_M)}\right] \qquad \text{QED}
\end{aligned}
$$

PV of Sum-of-the-Years-Digits (SOYD) Depreciation Charges

A closed form for the PV of yearly depreciation charges using the SOYD depreciation method may be derived. SOYD provides for acceleration of depreciation charges in the early years of system life.

Let
- D_j = depreciation charge at end of year j
- d = discount rate
- N_D = accounting lifetime
- IC = initial system cost
- SV = salvage value

The yearly depreciation charge is

$$D_j = (IC - SV) \cdot \frac{N_D - j + 1}{\frac{N_D \cdot (N_D + 1)}{2}} = \frac{2 \cdot (IC - SV)}{N_D \cdot (N_D + 1)} \cdot (N_D - j + 1)$$

Therefore:

$$PVDEPD = \sum_{j=1}^{N_D} \frac{D_j}{(1 + d)^j}$$

$$= \frac{2 \cdot (IC - SV)}{N_D \cdot (N_D + 1)} \cdot \sum_{j=1}^{N_D} \frac{N_D - j + 1}{(1 + d)^j}$$

$$= \frac{2 \cdot (IC - SV)}{N_D \cdot (N_D + 1)} \cdot \left[\sum_{j=1}^{N_D} \frac{N_D}{(1 + d)^j} - \sum_{j=1}^{N_D} \frac{j}{(1 + d)^j} + \sum_{j=1}^{N_D} \frac{1}{(1 + d)^j} \right]$$

$$= \frac{2 \cdot (IC - SV)}{N_D \cdot (N_D + 1)} \cdot \left[N_D \cdot PVF(d,0,N_D) - \sum_{j=1}^{N_D} \frac{j}{(1 + d)^j} + PVF(d,0,N_D) \right]$$

A closed form for $\sum_{j=1}^{N_D} \frac{j}{(1 + d)^j}$ may be determined as follows:

$$S = \sum_{j=1}^{N_D} \frac{j}{(1 + d)^j} = \frac{1}{1 + d} + \frac{2}{(1 + d)^2} + \ldots + \frac{N_D}{(1 + d)^{N_D}}$$

$$\frac{S}{1 + d} = \frac{1}{(1 + d)^2} + \frac{2}{(1 + d)^3} + \ldots + \frac{N_D - 1}{(1 + d)^{N_D}} + \frac{N_D}{(1 + d)^{N_D+1}}$$

$$S - \frac{S}{1 + d} = \frac{1}{1 + d} + \frac{1}{(1 + d)^2} + \frac{1}{(1 + d)^3} + \ldots + \frac{1}{(1 + d)^{N_D}} - \frac{N_D}{(1 + d)^{N_D+1}}$$

$$= \sum_{j=1}^{N_D} \frac{1}{(1 + d)^j} - \frac{N_D}{(1 + d)^{N_D+1}}$$

$$= PVF(d,0,N_D) - \frac{N_D}{(1 + d)^{N_D+1}}$$

Therefore:

$$S = \frac{1 + d}{d} \cdot PVF(d,0,N_D) - \frac{N_D}{d \cdot (1 + d)^{N_D}}$$

$$= \frac{1 + d}{d} \cdot \frac{(1 + d)^{N_D} - 1}{d \cdot (1 + d)^{N_D}} - \frac{N_D}{d \cdot (1 + d)^{N_D}}$$

$$= \frac{(1 + d)^{N_D+1} - (1 + d) - N_D \cdot d}{d^2 \cdot (1 + d)^{N_D}}$$

Returning to the original equation:

$$PVDEPD = \frac{2 \cdot (IC - SV)}{N_D \cdot (N_D + 1)} \cdot \left[N_D \cdot PVF(d,0,N_D) - \frac{(1 + d)^{N_D+1} - (1 + d) - N_D \cdot d}{d^2 \cdot (1 + d)^{N_D}} + PVF(d,0,N_D) \right]$$

$$= \frac{2 \cdot (IC - SV)}{N_D \cdot (N_D + 1)} \cdot \left[N_D \cdot \frac{(1 + d)^{N_D} - 1}{d \cdot (1 + d)^{N_D}} - \frac{(1 + d)^{N_D+1} - (1 + d) - N_D \cdot d}{d^2 \cdot (1 + d)^{N_D}} + \frac{(1 + d)^{N_D} - 1}{d \cdot (1 + d)^{N_D}} \right]$$

$$= 2 \cdot (IC - SV) \cdot \frac{d \cdot N_D \cdot (1 + d)^{N_D} - d \cdot N_D - (1 + d)^{N_D+1} + 1 + d + N_D \cdot d + d \cdot (1 + d)^{N_D} - d}{d^2 \cdot (1 + d)^{N_D} \cdot N_D \cdot (N_D + 1)}$$

$$= 2 \cdot (IC - SV) \cdot \frac{d \cdot N_D \cdot (1 + d)^{N_D} - (1 + d)^{N_D} \cdot \left[(1 + d) - d\right] + 1}{d^2 \cdot (1 + d)^{N_D} \cdot N_D \cdot (N_D + 1)}$$

$$= 2 \cdot (IC - SV) \cdot \left[\frac{d \cdot N_D \cdot (1 + d)^{N_D}}{d^2 \cdot (1 + d)^{N_D} \cdot N_D \cdot (N_D + 1)} - \frac{(1 + d)^{N_D} - 1}{d^2 \cdot (1 + d)^{N_D} \cdot N_D \cdot (N_D + 1)} \right]$$

$$= 2 \cdot (IC - SV) \cdot \left[\frac{N_D}{d \cdot N_D \cdot (N_D + 1)} - \frac{PVF(d,0,N_D)}{d \cdot N_D \cdot (N_D + 1)} \right]$$

$$= 2 \cdot (IC - SV) \cdot \left[\frac{N_D - PVF(d,0,N_D)}{d \cdot N_D \cdot (N_D + 1)} \right]$$

QED

APPENDIX C

Numerical Illustration

The methodology may best be illustrated by use of a numerical example for both the residential and commercial applications. The examples given below are purely hypothetical and are used strictly to illustrate the methodology. No generalizations about the economic feasibility of solar energy systems should be drawn from these examples.

Residential Application

A homeowner is contemplating purchasing a solar energy system with an electric resistance backup unit for his new home. If he does not purchase the SES, he will instead install a gas furnace. He is interested in looking at the LCCs of these two alternatives. The cost data in 1978 dollars are as follows:

Initial cost of SES: ICp = \$6000 (including backup)

Backup electricity cost: F = \$100/year

Initial cost of gas furnace: ICp = \$1800

Conventional natural gas cost: F = \$250/year

Year of operation: $y_o = 1979$

The values for all the other inputs necessary to do the analysis are found in Appendix A. (The assumption is made that either system will be financed as part of the purchase of the home.) Using these inputs, the homeowner can calculate the LCC of both the solar and conventional systems.

LCC of Solar Energy System --

PV of initial system cost:

$$IC = 6000 \cdot (1.05)^1 = 6300$$

$$PVLOAN = (1 - .2) \cdot 6300 \cdot \left[\frac{PVF(.1, 0, 20)}{PVF(.085, 0, 30)} + \frac{(1.085)^{30} - (1.085)^{20}}{(1.1)^{20} \cdot [(1.085)^{30} - 1]}\right]$$

$$= (1 - .2) \cdot 6300 \cdot \left[\frac{8.51}{10.75} + \frac{11.56 - 5.11}{6.73 \cdot [11.56 - 1]}\right] = 4447$$

$$PVSYS = .2 \cdot 6300 + 4447 = 5707$$

PV of yearly interest payments:

$$PVINT = (1 - .2) \cdot 6300 \cdot \left[PVF(.1, .085, 20) \cdot \left(.085 - \frac{1}{PVF(.085, 0, 30)} \right) + \frac{PVF(.1, 0, 20)}{PVF(.085, 0, 30)} \right]$$

$$= (1 - .2) \cdot 6300 \cdot \left[16.01 \cdot (.085 - .093) + \frac{8.51}{10.75} \right] = 3344$$

PV of recurring costs:

$$OM = .015 \cdot 6000 = 90$$

$$PVMISC = 90 \cdot PVF(.1, .05, 20) \cdot (1.05)^1$$

$$= 90 \cdot 12.11 \cdot 1.05 = 1145$$

$$PVPROP = 0$$

$$PVENER = 100 \cdot PVF(.1, .07, 20) \cdot (1.07)^1$$

$$= 100 \cdot 14.16 \cdot 1.07 = 1515$$

Given the above PVs, it is now possible to calculate the LCC of the SES:

$$TPV_R = PVSYS + PVMISC + PVENER + (1 - t) \cdot PVPROP - t \cdot PVINT - PVITC - PVSV$$

$$= 5707 + 1145 + 1515 + 0 - .3 \cdot (3344) - 0 - 0 = 7364$$

$$AC_R = \frac{7364}{PVF(.1, 0, 20)} = \frac{7364}{8.51} = 865$$

Given either \$7364 at the beginning of system operation or \$865/year for 20 years, the homeowner could fully pay for his SES.

LCC of Conventional Energy System -- The same PVs must be calculated to find the LCC of the conventional system, using conventional costs as inputs:

PV of initial system costs:

$$IC = 1800 \cdot (1.05)^1 = 1890$$

$$PVLOAN = (1 - .2) \cdot 1890 \cdot \left[\frac{PVF(.1, 0, 20)}{PVF(.085, 0, 30)} + \frac{(1.085)^{30} - (1.085)^{20}}{(1.1)^{20} \cdot \left[(1.085)^{30} - 1\right]}\right]$$

$$= (1 - .2) \cdot 1890 \cdot \left[\frac{8.51}{10.75} + \frac{11.56 - 5.11}{6.73 \cdot [11.56 - 1]}\right] = 1334$$

$$PVSYS = .2 \cdot 1890 + 1334 = 1712$$

PV of yearly interest payments:

$$PVINT = (1 - .2) \cdot 1890 \cdot \left[PVF(.1, .085, 20) \cdot \left(.085 - \frac{1}{PVF(.085, 0, 30)}\right)\right.$$

$$\left. + \frac{PVF(.1, 0, 20)}{PVF(.085, 0, 30)}\right] = (1 - .2) \cdot 1890 \cdot \left[16.01 \cdot (.085 - .093)\right.$$

$$\left. + \frac{8.51}{10.75}\right] = 1003$$

PV of recurring costs:

$$OM = .015 \cdot 1800 = 27$$

$$PVMISC = 27 \cdot PVF(.1, .05, 20) \cdot (1.05)^1 = 27 \cdot 12.11 \cdot 1.05 = 343$$

$$PVPROP = 0$$

$$PVENER = 250 \cdot PVF(.1, .1, 20) \cdot (1.1)^1 = 250 \cdot 18.18 \cdot 1.1 = 5000$$

Given the above PVs, it is now possible to calculate the LCC of the conventional system:

$$TPV_R = PVSYS + PVMISC + PVENER + (1 - t) \cdot PVPROP - t \cdot PVINT - PVITC - PVSV$$

$$= 1712 + 343 + 5000 + 0 - .3 \cdot 1003 - 0 - 0 = 6754$$

$$AC_R = \frac{6754}{PVF(.1, 0, 20)} = \frac{6754}{8.51} = 794$$

Given either \$6754 at the beginning of system operation or \$794/year for 20 years, the homeowner could fully pay for the conventional energy system.

Economic Feasibility -- To determine economic feasibility, we must look at the solar savings term:

$$\text{Solar Savings} = LCC_{conventional} - LCC_{solar} = 6754 - 7364 = -610$$

Since the solar savings term is negative, the SES in this particular example is not economically feasible, based on a life-cycle costing criterion, and the homeowner would save money by installing the conventional system.

Commercial Application

Let's now assume instead that the above systems are being considered for installation on a small office building. The cost inputs are the same, with commercial values for the inputs in Appendix A being used.

LCC of Solar Energy System --

PV of initial system cost:

$$IC = 6000 \cdot (1.05)^1 = 6300$$

$$PVLOAN = (1 - .3) \cdot 6300 \cdot \left[\frac{PVF(.1, 0, 20)}{PVF(.08, 0, 25)} + \frac{(1.08)^{25} - (1.08)^{20}}{(1.1)^{20} \cdot [(1.08)^{25} - 1]}\right]$$

$$= (1 - .3) \cdot 6300 \cdot \left[\frac{8.51}{10.67} + \frac{6.85 - 4.66}{6.73 \cdot [6.85 - 1]}\right] = 3763$$

$$PVSYS = .3 \cdot 6300 + 3763 = 5653$$

PV of yearly interest payments:

$$PVINT = (1 - .3) \cdot 6300 \cdot \left[PVF(.1, .08, 20) \cdot \left(.08 - \frac{1}{PVF(.08, 0, 25)}\right)\right.$$

$$\left. + \frac{PVF(.1, 0, 20)}{PVF(.08, 0, 25)}\right] = (1 - .3) \cdot 6300 \cdot \left[15.36 \cdot (.08 - .0937)\right.$$

$$\left. + \frac{8.51}{10.67}\right] = 2589$$

PV of yearly depreciation charges:

Assume SOYD depreciation

$$PVDEPD = 6300 \cdot \frac{2 \cdot [20 - PVF(.1, 0, 20)]}{20 \cdot 21 \cdot (.1)} = 6300 \cdot \frac{2 \cdot [20 - 8.51]}{20 \cdot 21 \cdot (.1)} = 3447$$

PV of investment tax credit:

$$I = .1 \cdot 6300 = 630$$

$$PVITC = \frac{630}{1.1} = 573$$

PV of recurring costs:

PVMISC, PVPROP, and PVENER have the same values as in the homeowner case.

It is now possible to calculate the LCC of the SES for this commercial application:

$$TPV_C = PVSYS + (1 - t) \cdot (PVMISC + PVPROP + PVENER) - t \cdot (PVDEPD + PVINT)$$
$$- PVITC - PVSV = 5653 + (1 - .5) \cdot (1145 + 0 + 1515) - .5 \cdot (3447 + 2589)$$
$$- 573 - 0 = 3392$$

$$AC_C = \frac{3392}{PVF(.1, 0, 20)} = \frac{3392}{8.51} = 399$$

<u>LCC of Conventional Energy System</u> --

PV of initial system cost:

$$IC = 1800 \cdot (1.05)^1 = 1890$$

$$PVLOAN = (1 - .3) \cdot 1890 \cdot \left[\frac{PVF(.1, 0, 20)}{PVF(.08, 0, 25)} + \frac{(1.08)^{25} - (1.08)^{20}}{(1.1)^{20} \cdot [(1.08)^{25} - 1]}\right]$$

$$= (1 - .3) \cdot 1890 \cdot \left[\frac{8.51}{10.67} + \frac{6.85 - 4.66}{6.73 \cdot (6.85 - 1)}\right] = 1129$$

$$PVSYS = .3 \cdot 1890 + 1129 = 1696$$

PV of yearly interest payments:

$$PVINT = (1 - .3) \cdot 1890 \cdot \left[PVF(.1, .08, 20) \cdot \left(.08 - \frac{1}{PVF(.08, 0, 25)}\right) + \frac{PVF(.1, 0, 20)}{PVF(.08, 0, 25)}\right]$$

$$= (1 - .3) \cdot 1890 \cdot \left[15.36 \cdot (.08 - .0937) + \frac{8.51}{10.67}\right] = 777$$

PV of yearly depreciation charges:

Assume SOYD depreciation

$$PVDEPD = 1890 \cdot \frac{2 \cdot [20 - PVF(.1, 0, 20)]}{20 \cdot 21 \cdot (.1)} = 1890 \cdot \frac{2 \cdot [20 - 8.51]}{20 \cdot 21 \cdot (.1)} = 1034$$

PV of investment tax credit:

$$I = .1 \cdot 1890 = 189$$

$$PVITC = \frac{189}{1.1} = 172$$

PV of recurring costs:

PVMISC, PVPROP, and PVENER have the same values as in the homeowner case.

It is now possible to calculate the LCC of the conventional energy system:

$$TPV_C = PVSYS + (1 - t) \cdot [PVMISC + PVPROP + PVENER]$$

$$- t \cdot (PVDEPD + PVINT) - PVITC - PVSV = 1696 + .5 \cdot (343 + 0 + 5000)$$

$$- .5 \cdot (1034 + 777) - 172 - 0 = 3290$$

$$AC_C = \frac{3290}{PVF(.1, 0, 20)} = \frac{3290}{8.51} = 387$$

Economic Feasibility -- We can now determine the economic feasibility of this commercial solar installation:

$$\text{Solar Savings} = LCC_{conventional} - LCC_{solar} = 3290 - 3392$$

$$= -102$$

Again the SES in this example is not economically feasible given a life-cycle costing criterion.

APPENDIX D

Treatment of Inflation in an Economic Analysis

Introduction

There are several different methods of dealing with inflation in an economic analysis. Inflation is an economic fact of life, yet its treatment in various studies is by no means standardized. The issues involved in treating inflation when analyzing an investment decision will be discussed in this Appendix. In order to illustrate the effect of these issues a hypothetical capital investment will be examined. It will be shown that the evaluation of an investment decision can vary depending on the method chosen for dealing with inflation.*

Theoretical Concepts

It is essential to define several concepts before proceeding with the method discussion, the first being the difference between current and constant dollars. Current dollars are dollars that reflect the inflation present in the economy. Society pays for its goods and services in current dollars. Constant dollars are dollars from which inflation has been removed--dollars that have been deflated or adjusted for changes in the price level. Constant dollars reflect only real changes in the relative price of goods and services and are always expressed in terms of some reference price year.

This can be illustrated by looking at the Gross National Product (GNP) of the United States. The GNP for 1977 in current 1977 dollars was $1889.6 billion, 10.7% higher than 1976's current dollar GNP of $1706.5 billion. Much of that increase was due to an overall rise in the prices of the goods and services that make up the GNP, rather than an actual increase in the volume of those goods and services. One year's inflation can be removed from the 1977 GNP in order to express the 1977 GNP in constant 1976 dollars.** This becomes $1789.4 billion. Therefore, the real increase in the volume of goods and services over 1976 was 4.9%. By expressing the GNP in constant dollars, all effects of inflation are removed, and it is possible to look at the real increase in the volume of goods and services produced.

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*For a more rigorous treatment of the methods of handling inflation, see "Engineering Economy and the Two Rates of Return--Mixed Mode Computations," Sanford Baum, AIIE Transactions, March 1978.

**Current dollars are adjusted to constant dollars by dividing by $(1 + g)^N$ where g is the inflation rate and N is the number of years of inflation being removed. The value of g from 1976-1977 was 5.6%.

The second concept that is important in understanding inflation is the difference between nominal or market interest rates and real interest rates. Nominal or market interest rates are rates that include a component for inflation. The nominal rates prevailing in today's market are high due to the high inflation rates the US has been experiencing in recent years. Real interest rates are inflation-free. They represent the real time cost of money--the rates that would prevail if there were no inflation. Inflation is primarily responsible for the difference between real interest rates of 2-3% and the nominal interest rates of 8-9% that occur in today's market.*

Current vs Constant Dollar Analysis

An analysis that treats inflation may be done in either current or constant dollars. In the current-dollar analysis, inflation is included in all cost estimates. All costs are expressed in current dollars; i.e., the actual out-of-pocket dollar costs for the investor in the year in which they occur. Nominal interest and nominal discount rates are used.** The analysis is illustrated by the following example:

Assume a commercial enterprise exists that is interested in making a capital investment. The data are as follows:

Capital investment, CI	=	$10,000
Miscellaneous yearly expense, OM	=	$100 at the beginning of year 1
Downpayment, D	=	10%
Nominal interest rate, i	=	10%
Nominal discount rate, d	=	10%
Real discount rate, d′	=	3.774%
Inflation rate, g	=	6%
Income tax rate, t	=	50%
Number of years, N	=	5
Depreciation	=	straight line

(These data are for illustrative purposes only and do not reflect any estimate of true economic values.)

*Given a nominal interest rate, i, and an inflation rate, g, it is possible to find the real interest rate, i′, as follows:

$$i' = \frac{1 + i}{1 + g} - 1$$

The assumption is that the only difference between nominal and real interest rates is that due to inflation. This neglects any other components, such as risk, that may have some influence on this difference.

**In this discussion, the term "interest rate" will be used to refer to the cost of borrowing money. The term "discount rate" will refer to the rate of return an investor wants on his investment. Both may be expressed in either nominal or real terms.

We want to look at the total cost of this investment over its lifetime. This may be expressed in two ways--either as a present value (PV) or as a levelized annual cost ($\overline{AC}$). The PV is determined by discounting each yearly cost back to year zero and then summing the discounted values. The levelized $\overline{AC}$ is determined by multiplying the PV by the capital recovery factor (CRF).*

The yearly cash flows are as follows:

Current Dollars ($)

Year	Loan Payment	Interest	Depreciation	Taxes Saved	Miscellaneous Expenses	Yearly Cost in Current $	PV of Yearly Cost
0						1000	1000
1	2374	900	2000	1503	106	977	888
2	2374	753	2000	1433	112	1053	870
3	2374	590	2000	1355	119	1138	855
4	2374	412	2000	1269	126	1231	841
5	2374	216	2000	1175	134	1333	828
						Total PV =	5282

Miscellaneous expenses have been inflated yearly at 6% and are in current dollars, as are the other payments. Each yearly cost is obtained as follows:

Yearly cost = Loan payment + Miscellaneous expenses - Taxes saved

Taxes saved = .5 · (Interest + Depreciation + Miscellaneous expenses)

The total present value of the yearly costs in year zero is $5282. The levelized annual cost, $\overline{AC}$, obtained by multiplying by the CRF for a 10% discount rate, is $1393.

Suppose instead the analysis is done in terms of constant dollars. Inflation is not included in the miscellaneous expenses. A real rather than nominal discount rate must be used to discount the yearly costs. A more subtle effect appears in the loan payments, interest, and depreciation charges. These payments were initially expressed in current dollars, but the real value of these payments declines through time if inflation is present. To express these payments in constant dollars, they must be deflated to year zero by the inflation rate of 6%. (Year zero in this context becomes both the reference year for constant dollars and the year as of which all PVs are calculated.) The cash flows become

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*The technique called "discounting" is accomplished by multiplying each yearly cost by the discount factor, $1/(1+d)^t$, where t is the number of years from year zero. The levelized annual cost is determined by multiplying the PV by the capital recovery factor,

$$[d \cdot (1+d)^N]/[(1+d)^N - 1] .$$

These terms are more fully described in Principles of Engineering Economy, Grant, Ireson and Leavenworth.

Constant Dollars ($)

Year	Loan Payment	Interest	Depreciation	Taxes Saved	Miscellaneous Expenses	Yearly Cost in Constant $	PV of Yearly Cost
0						1000	1000
1	2240	849	1887	1418	100	922	888
2	2113	670	1780	1275	100	938	870
3	1993	496	1679	1138	100	955	855
4	1880	326	1584	1005	100	975	841
5	1774	161	1495	878	100	996	828
						Total PV =	5282

The PV of the yearly costs in this constant dollar case, using the real discount rate of 3.774% is again $5282. The levelized annual cost, AC_o, obtained by multiplying by the CRF for 3.774%, is $1179.

Both the current and constant dollar analyses yield identical present values, although the levelized annual costs are different. There is a direct relationship between these two levelized costs. The current dollar levelized cost, $\overline{AC}$, is the amount of current dollars needed in each year of system operation to pay for all system costs. The constant dollar levelized cost, AC_o, is the amount of constant zero-year dollars needed in each year of system operation to pay for all system costs. However, AC_o may also be viewed as the amount of money needed in year zero, which if escalated yearly at the inflation rate would pay for all systems costs. The yearly costs, AC_j, represent the current yearly expenditures needed to cover all system costs and are derived as follows:

$$AC_j = AC_o \cdot (1 + g)^j \text{ for } j = 1, \ldots, N$$

The yearly amounts necessary in this example are

j	AC_j
1	1250
2	1325
3	1404
4	1488
5	1578

The PV of the AC_j terms using a 10% discount rate again equals $5282. The relationship between AC_o, AC_j, and $\overline{AC}$ is shown graphically in Figure D-1.*

*Much of this discussion was adopted from information presented in "The Cost of Energy from Utility Owned Solar Electric Systems, A Required Revenue Methodology for ERDA/EPRI Evaluations," Jet Propulsion Lab, June 1976, ERDA/JPL-1012-76/3.

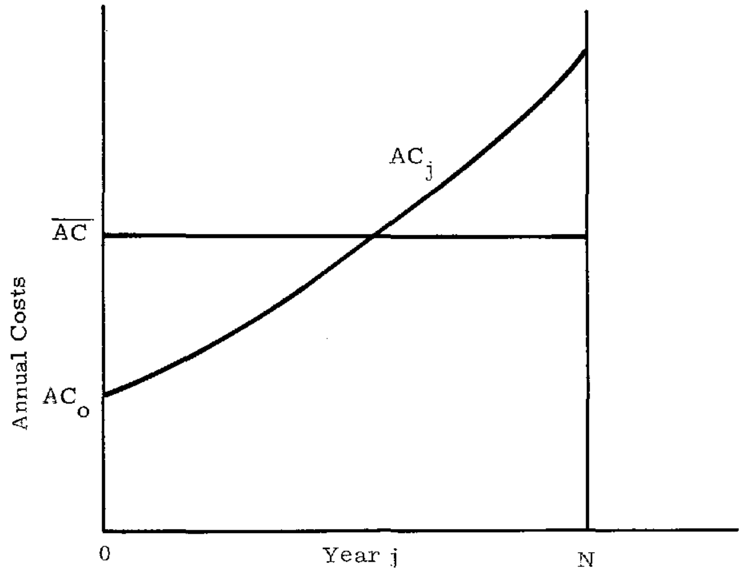

Figure D-1. Relationship Between AC_o, AC_j, and $\overline{AC}$

Given the relationships stated above, it is possible to calculate either AC_o or $\overline{AC}$ given the other. We know that the PVs are equal in both the current and constant dollar cases. Therefore:

$$\frac{AC_o}{CRF_{d'}} = \frac{\overline{AC}}{CRF_d}$$

$$AC_o = \frac{\overline{AC} \cdot CRF_{d'}}{CRF_d} \qquad \text{(D-1)}$$

This relationship is especially useful for calculating AC_o given $\overline{AC}$, thereby avoiding the need to calculate the constant dollar cash flows.

It is illustrative to view another way of arriving at the above relationship. We know that the PV of the levelized annual costs, $\overline{AC}$, and of the AC_j terms using discount rate, d, are equal. That is

$$\sum_{j=1}^{N} \frac{AC_j}{(1 + d)^j} = \frac{\overline{AC}}{CRF_d}$$

Therefore:

$$\sum_{j=1}^{N} \frac{AC_o \cdot (1 + g)^j}{(1 + d)^j} = \frac{\overline{AC}}{CRF_d}$$

$$AC_o \cdot \sum_{j=1}^{N} \frac{(1+g)^j}{(1+d)^j} = \frac{\overline{AC}}{CRF_d}$$

Summing the geometric series:

$$AC_o \cdot \frac{(1+g)}{(d-g)} \cdot \left[1 - \left(\frac{1+g}{1+d}\right)^N\right] = \frac{\overline{AC}}{CRF_d} \tag{D-2}$$

For Eqs. (D-1) and (D-2) to be equivalent, one must show that

$$\frac{(1+g)}{(d-g)} \cdot \left[1 - \left(\frac{1+g}{1+d}\right)^N\right] = \frac{1}{CRF_{d'}} .$$

Now $CRF_{d'} = \dfrac{d' \cdot (1+d')^N}{(1+d')^N - 1}$

Also d′ was derived from d and g as follows:

$$d' = \frac{1+d}{1+g} - 1$$

Therefore:

$$\frac{(1+g)}{(d-g)} \cdot \left[1 - \left(\frac{1+g}{1+d}\right)^N\right] = \frac{\left(\frac{1+d}{1+g}\right)^N - 1}{\left(\frac{1+d}{1+g} - 1\right) \cdot \left(\frac{1+d}{1+g}\right)^N}$$

$$= \frac{\left(\frac{1+d}{1+g}\right)^N - 1}{\left(\frac{d-g}{1+g}\right) \cdot \left(\frac{1+d}{1+g}\right)^N}$$

$$= \frac{(1+g)}{(d-g)} \cdot \left[1 - \left(\frac{1+g}{1+d}\right)^N\right] \quad \text{QED}$$

The Zero Inflation Rate Case

The use of either the current or constant-dollar method of dealing with inflation assumes that inflation is a fact of life that must be dealt with in economic studies. A popular method of dealing with inflation in many recent studies is to assume that there is no inflation whatsoever in the economy; i.e., that the inflation rate is and will continue to be zero. Under this scenario, current and constant dollars are equivalent since all dollars, regardless of the year in which they occur, have the same purchasing power. This method requires that both real interest and discount rates be used which are free of inflation. The analysis can be illustrated by continuing with the example above. A real interest rate, i′, must be calculated:

$$i' = \frac{1 + 0.1}{1 + 0.06} - 1 = 0.03774$$

The yearly cash flows in the zero inflation case are

Zero Inflation ($)

Year	Loan Payment	Interest	Depreciation	Taxes Saved	Miscellaneous Expenses	Yearly Cost	PV of Yearly Cost
0						1000	1000
1	2009	340	2000	1220	100	889	857
2	2009	277	2000	1189	100	920	854
3	2009	211	2000	1156	100	953	853
4	2009	143	2000	1122	100	987	851
5	2009	73	2000	1087	100	1022	849
						Total PV =	5264

The PV of the yearly cash flows using the real discount rate of 3.774% is $5264. The levelized annual cost is $1175.

The zero-inflation case answers are very close to those calculated using the constant-dollar analysis. This leads many analysts to assume zero inflation, since it is simpler to do and does not require deflating of current dollar payments. However, since this method does assume a zero-inflation rate, it can only be considered an approximation to reality when inflation is present in the economy. The magnitude of the error introduced into the analysis by this assumption may be difficult to estimate, and it is often no more difficult to treat inflation rigorously as it is to set it equal to zero.

There is one special case where the constant-dollar and zero-inflation methods yield identical results. This is in the case of an investment in a nondepreciable asset that is financed completely without borrowing. In this situation, the fixed current dollar loan payments and depreciation charges are nonexistent, and there is no necessity to deflate them to constant dollars. However, in the more usual case of a depreciable asset that is partially financed by borrowing, the two methods yield different results.

Conclusion

We have seen three different methods of treating inflation--the current-dollar, constant-dollar, and zero-inflation approaches. The first two methods assume that inflation exists in the economy, while the third assumes inflation equals zero. The current- and constant-dollar analyses will always yield identical PVs. The zero-inflation case will yield a PV different from that in the other two cases, except in certain cases where the PV will again be identical.

Any of these methods may be used when analyzing an investment decision. If it is desired to treat inflation rigorously, the analysis should be done in either current or constant dollars. If one chooses to disregard inflation, the zero-inflation case method may be used to obtain an approximation to the other methods, although the limitations of this approach must always be kept in mind. In all cases, the analyst should make clear the assumptions he is making regarding inflation.

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