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Effects of Bending on the Fatigue Life of Solar Receiver Tubes Subjected to One-Sided Heating

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EFFECTS OF BENDING ON THE FATIGUE LIFE OF SOLAR RECEIVER TUBES SUBJECTED TO ONE-SIDED HEATING

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ABSTRACT

The strains due to thermal gradients in central receivers are commonly analyzed as two-dimensional, assuming the results to be conservative with respect to the realistic three-dimensional problem. It is the purpose of this report to examine this assumption in detail. The Department of Energy's Barstow Pilot plant is taken as a representative system and is studied using simple beam theory, with a simple bar model for comparison. The results indicate that the tube supports designed for protection from winds and earthquakes effectively prevent any bending, except in regions of high axial thermal gradients. Thus a generalized plane-strain analysis should accurately reflect the stresses and strains except in those regions.

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EFFECTS OF BENDING ON THE FATIGUE LIFE OF SOLAR RECEIVER TUBES SUBJECTED TO ONE-SIDED HEATING

Introduction

Since the oil embargo of 1973, a substantial amount of effort has been directed toward the development of solar-powered electric generating facilities. One of the principal approaches that has been investigated is the central receiver concept, in which energy is concentrated by a large field of heliostats and collected by a large receiver. The receiver which is placed in this focused sunlight must absorb energy at very high flux levels and transfer it to a working fluid. In all of the designs proposed to date this impingement of solar energy occurs on only one side of the receiver tube, resulting in a very large temperature gradients from the front to the back of the tube. The temperature gradients themselves are not of great concern, but they produce very high bending stresses (in some designs substantially above the yield strength of the tube) which are likely to produce fatigue damage as the plant goes through its daily cycles.

To date these thermal stress problems have mostly been analyzed with a generalized plane-strain formulation applied to a cross section normal to the axis of the tube,[1,2,3] with the hope that this two-dimensional analysis would prove to be conservative when a three-dimensional analysis was performed. Specifically, some reduction of the thermal stresses due to bending or bowing of the tubes was expected in the more elaborate analysis. However, since the production of energy is a very cost-conscious endeavor, a new entry into the field can ill afford any unnecessary conservatism. In order to assess the degree to which bending of the heat-absorbing tubes of the receiver reduces the fatigue damage from diurnal cycling, the Department of Energy's Barstow Pilot Plant[4] receiver-tube configuration has been used to compare the results from simple beam theory with those of a simple bar model.

The system under study may be regarded as a large number (10-20) of discrete supports placed along a thin tube ($50 < \ell/r_0 < 200$) which is heated on one side and free to expand axially. The key questions to answer are: Do the supports provide enough resistance to lateral bending so that the tube can be regarded as being continuously constrained so that a two-dimensional analysis may be safely applied? Further, if it is found that the restraint does not justify two-dimensional analysis, what effect does the bending have? If a two-dimensional analysis is conservative, how much additional performance can be gained by conducting a more elaborate analysis; or, if it is unconservative,(1) how much shorter will the actual life be?

Method of Solution

It was desirable to model the effects of axial variations in the incident solar flux and in the heat transfer coefficient at the boundary between the tube and the heat transfer fluid, as well as the changes in the local fluid temperature. Since these quantities were available only in numerical form, a computerized model of the receiver tube was necessary. However, finite-element programs seemed unsuitable since a full three-dimensional analysis was too costly and an analysis using beam-type elements failed to use all of the available thermal information. Instead, a model was developed based upon simple beam theory with a thermal loading as described by Burgreen.[5] This technique seemed appropriate since it incorporated bending of the tubes (the effect of interest) and allowed calculation of the axial stresses (the largest stresses in most solar receiver tubes) without resorting to a linearized thermal distribution.

To treat the problem, the beam was broken into each of its component spans and the thermal loads applied as if the beam were simply supported. First, the curvature equation for each span,

$$\frac{d^2 w_1}{dx^2} = \frac{\alpha}{I} M_t \ , \tag{1}$$

where

$$M_t = \int_A \int \Delta T(y,z) z \, dy \, dz \ ,$$

was intergrated and pinned-pinned boundary conditions applied to give

$$w_1(x) = -\int_0^x \int_0^\zeta \frac{\alpha}{I} M_t(\eta) \, d\eta \, d\zeta + \frac{x}{L} \int_0^L \int_0^\zeta \frac{\alpha}{I} M_t(\eta) \, d\eta \, d\zeta \ . \tag{2}$$

---

(1)A simple unconservative case can be found by considering a continuous beam with three equal length spans and a pinned restraint at each of the four supports. If such a beam is subjected to a linear front-to-back temperature gradient which is constant along its length, the beam develops a curved shape which has constant curvature in the middle span. This curvature is in the opposite direction from that caused by the temperature gradient in an unrestrained beam, indicating that the resulting strains are higher than if the beam had simply been held straight.

Then the integrals in the axial direction were evaluated numerically from closed-form expressions for the first moment of the temperature, M_t (see below). Finally, the connection between the spans was considered and continuity of slopes at each support was enforced by superposition of a solution of the form

$$w_2 = a_3 x^3 + a_2 x^2 + a_1 x + a_0 \quad (3)$$

Included in the continuity calculations were the effects of a rotational spring (spring constant K_i) so that the final solution was appropriate for the beam shown in Figure 1. Once the deflections were found, the longitudinal strain (ε_x) could be calculated at any point using

$$\varepsilon_x = \frac{\sigma_x}{E} = \alpha\left(\bar{T} + \frac{M_t z}{I} - \Delta T\right) - \frac{Mz}{I}, \quad (4)$$

where $M = -EI\frac{d^2w}{dx^2} - \alpha E M_t$ and $w = w_1 + w_2$.

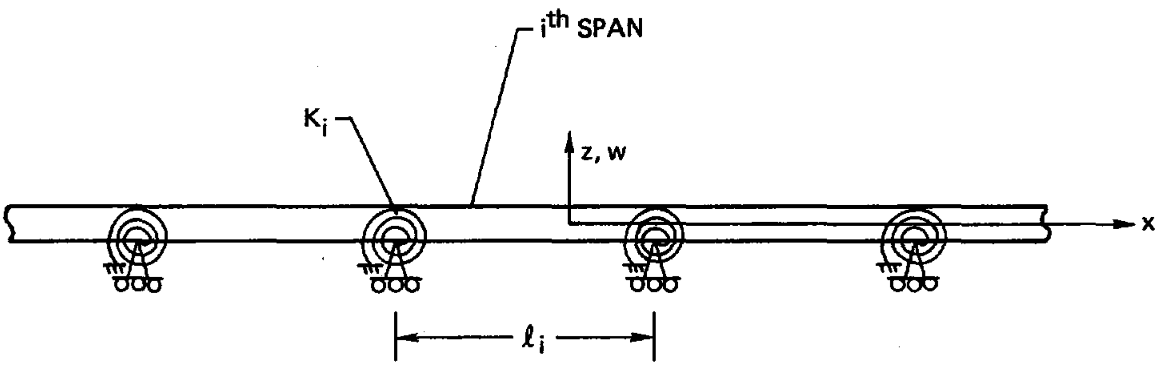

Figure 1. Conceptual Diagram of a Component in the Beam Model

For comparison a simple bar model was also used. Since the bar model permits expansion in the axial direction, but does not allow bending, there is some relief of stresses by thermal expansion in the axial direction, but not as much as in the beam model. Thus the bar model produces the stress distribution for a tube which is constrained against lateral motion along its entire length and produces results similar to a generalized plane-strain analysis. For this model the strain can be calculated from

$$\varepsilon_x = \frac{\sigma_x}{E} = \alpha(\bar{T} - T) \,. \quad (5)$$

The primary input to each of these models which remains to be discussed is the temperature distribution in the tube. The temperature distribution $T(r, \theta)$ is given by a two-dimensional Fourier series for the steady-state temperatures in a tube cross section heated by a cosine flux distribution on the front half and insulated on the back half, and which has a uniform heat transfer coefficient on the inside wall (see Fig. 2). Thus

$$T(r,\theta) = \frac{q_p r_o}{k\pi}\left[\ln\left(\frac{r}{r_i}\right) + \frac{k}{hr_i}\right] + T_f + \frac{q_p r_o}{2k}\,\frac{(r/r_i) + (r_i/r)\,\beta_1}{(r_o/r_i) + (r_i/r_o)\,\beta_1}\cos\theta$$

$$- \frac{q_p r_o}{k\pi}\sum_{n = 2,4,6\ldots}^{\infty}\frac{2(-1)^{n/2}}{n(n^2 - 1)}\left[\frac{(r/r_i)^n + (r_i/r)^n\,\beta_n}{(r_o/r_i)^n + r_i/r_o)^n\,\beta_n}\right]\cos n\theta \qquad (6)$$

where

$$\beta_n = \frac{1 - hr_i/kn}{1 + hr_i/kn} ,$$

from which the average temperature and first moment are easily shown to be, respectively,

$$\bar{T} = \frac{q_p r_o}{2k}\,\frac{1}{\pi(r_o^2 - r_i^2)}\left[\left(\frac{k}{hr_i} - \frac{1}{2}\right)(r_o^2 - r_i^2) + r_o^2\ln\frac{r_o}{r_i}\right] + T_f \qquad (7)$$

and

$$M_t = \frac{q_p r_o^2\,\pi(r_o^2 - r_i^2)}{8k}\;\frac{\frac{1}{2}(r_o^2 + r_i^2) + r_i^2\,\beta_1}{r_o^2 - r_1^2\,\beta_1} . \qquad (8)$$

The heat transfer coefficient and bulk fluid temperature needed in Eqs. (6), (7), and (8) were calculated by HYDRAULIC (an in-house computer code which models the thermal-hydraulic performance of solar receivers) for a specified heat flux distribution and tube parameters.

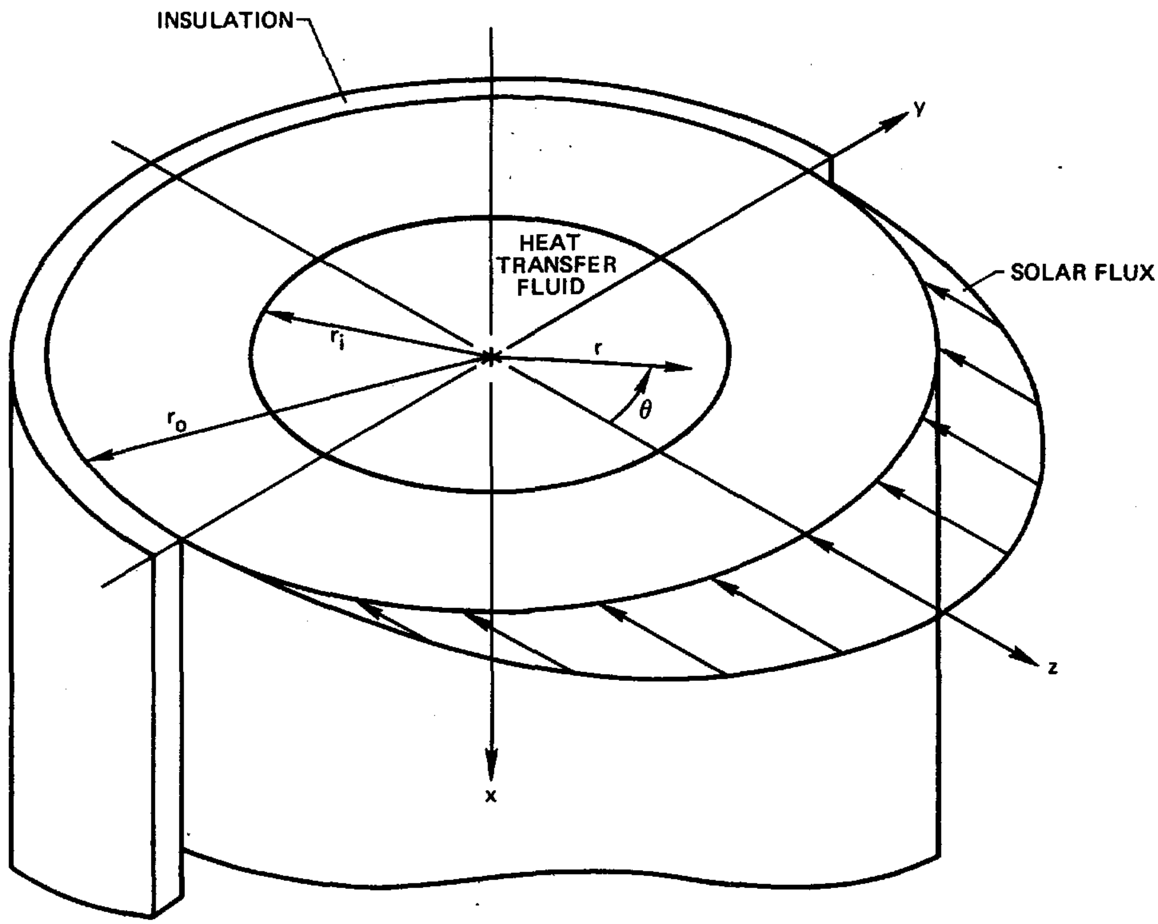

Figure 2. Schematic Drawing of the Solar Central Receiver Tube with One-Sided Solar Flux Heating

Results and Discussion

The results shown in Figure 3 give the longitudinal strains at the crown of a typical tube in a north panel of the DOE Barstow Pilot Plant. The tube itself is 12.5 m (41 feet) long with lateral supports at 0.9-m (3-foot) intervals (the support interval is dictated by wind and seismic loads). The results for three different cases are shown.

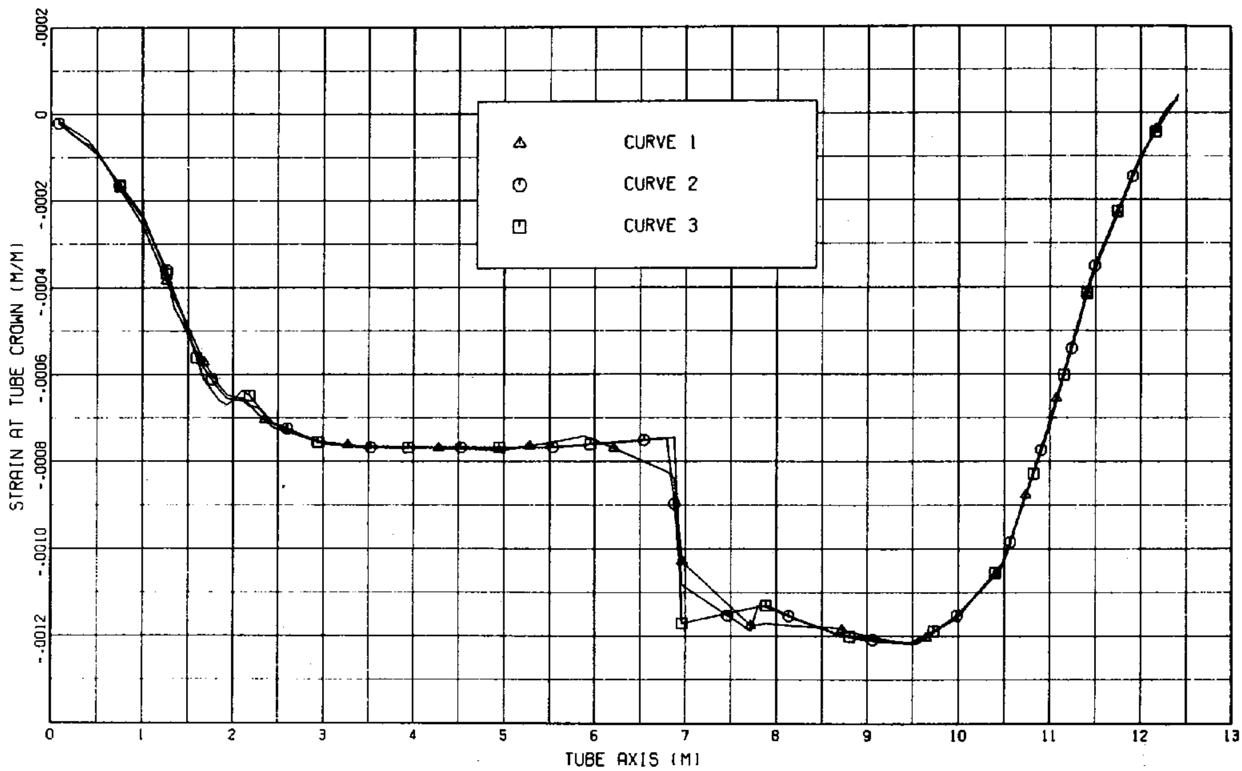

Figure 3. Strain at the Center Front of the Tube versus Axial Location (Peak Flux = 0.3 MW/m^2, Heat Transfer Fluid = Water, Pressure = 11 MPa, Inlet Temp = 288 C, Exit Temp = 516 C)

Curve 1 is the result of the analysis treating the tube as a continuous beam with no rotational restraints at the supports. Curve 2 is the result of increasing the rotational restraints of the continuous beam of Curve 1 to an infinite value, making the tube into a number of 0.9-m spans with no rotation allowed at the ends. These two cases were selected because they bound the possible range of rotational restraint provided by the actual tube mounting method. Curve 3 is the result of the bar model run for comparison with the bending analysis.

Examination of Figure 3 shows no appreciable difference between the three curves except in the region between 6 and 8 m (20 and 25 feet). Even in this area the differences are in the details of transition between two levels with no great errors in the magnitudes of the strain levels. This region of relatively rapid transition between two strain levels is associated with the departure from nucleate boiling which occurs at 7 m (23 feet) and causes very rapid longitudinal changes in the tube temperatures.

The similarity in results for the beam and bar models suggest that a two-dimensional analysis is adequate as long as the temperature gradients along the tube are small compared to those across the tube. Is there any analytical reason to believe such a conjecture? Consider for a moment the equations of equilibrium in terms of the displacements in the presence of nonuniform temperatures:

$$\frac{E}{2(1+\nu)}\left[\lambda^2 u_{,\bar{x}\bar{x}} + u_{,\bar{y}\bar{y}} + u_{,\bar{z}\bar{z}}\right] + \frac{E}{2(1+\nu)(1-2\nu)}$$

$$\left[\lambda^2 u_{,\bar{x}\bar{x}} + \lambda v_{,\bar{x}\bar{y}} + \lambda w_{,\bar{z}\bar{x}}\right] - \frac{E\alpha\ \lambda \Delta T_{,\bar{x}}}{1-2\nu} = 0 \quad , \tag{9}$$

$$\frac{E}{2(1+\nu)}\left[\lambda^2 v_{,\bar{x}\bar{x}} + v_{,\bar{y}\bar{y}} + v_{,\bar{z}\bar{z}}\right] + \frac{E}{2(1+\nu)(1-2\nu)}$$

$$\left[\lambda u_{,\bar{x}\bar{y}} + v_{,\bar{y}\bar{y}} + w_{,\bar{y}\bar{z}}\right] - \frac{E\alpha}{1-2\nu}\Delta T_{,\bar{y}} = 0 \tag{10}$$

$$\frac{E}{2(1+\nu)}\left[\lambda^2 w_{,\bar{x}\bar{x}} + w_{,\bar{y}\bar{y}} + w_{,\bar{z}\bar{z}}\right] + \frac{E}{2(1+\nu)(1-2\nu)}$$

$$\left[\lambda u_{,\bar{x}\bar{z}} + v_{,\bar{y}\bar{z}} + w_{,\bar{z}\bar{z}}\right] - \frac{E\alpha}{1-2\nu}\Delta T_{,\bar{z}} = 0 \quad . \tag{11}$$

For the geometry of the Barstow Pilot Plant $\lambda = r_o/\ell$ has a value of 1/144, suggesting an asymptotic solution for the displacements:

$$u = \sum_{i=0}^{\infty} \lambda^i u_i, \text{ etc.} \tag{12}$$

Looking at the zeroth order equations one finds

$$u_{o,\bar{y}\bar{y}} + u_{o,\bar{z}\bar{z}} = o \tag{13}$$

$$\frac{E}{2(1+\nu)}\left[v_{o,\bar{y}\bar{y}} + v_{o,\bar{z}\bar{z}}\right] + \frac{E}{2(1+\nu)(1-2\nu)}\left[v_{o,\bar{y}\bar{y}} + w_{o,\bar{y}\bar{z}}\right]$$

$$- \frac{E\alpha\ \Delta T_{,\bar{y}}}{1-2\nu} = 0 \quad , \tag{14}$$

$$\frac{E}{2(1+\nu)}\left[w_{o,\bar{y}\bar{y}} + w_{o,\bar{z}\bar{z}}\right] + \frac{E}{2(1+\nu)(1-2\nu)}\left[v_{o,\bar{y}\bar{z}} + w_{o,\bar{z}\bar{z}}\right]$$

$$- \frac{E\alpha\ \Delta T_{,\bar{z}}}{1-2\nu} = 0 \quad . \tag{15}$$

Equations (14) and (15) are precisely the equations of a generalized plane-strain analysis and Eq. (13) (together with the appropriate boundary conditions) provides for rotation and translation of the cross section. The first order correction to this solution is driven by the longitudinal derivatives of the zeroth order solution and the longitudinal temperature gradient so that a substantial modification to the generalized plane-strain solution should be expected only in regions of high axial thermal gradients. This conclusion is consistent with the numerical results.

Conclusion

This investigation into the bending of tubes in solar receivers due to one-sided heating of the tubes indicates that the supports provided to protect the tubes from wind and earthquake damage effectively prevent any bending from occurring except in regions of high axial thermal gradients. Therefore, a generalized plane-strain analysis should accurately reflect the state of stress and strain except in those regions where thermal conditions are rapidly changing. One example of rapidly changing thermal conditions is the boiling transition; others could be associated with mechanical attachments or other details of the design. However, until such thermal gradients are identified and analyzed, an analysis which considers bending seems to have little to offer for a better understanding of thermal stresses in the Barstow Pilot Plant Receiver due to one-sided heating.

GLOSSARY

Symbol	Meaning
E	Young's modulus
h	heat transfer coefficient at inner wall
I	moment of inertia of tube
K_i	rotational spring constant
k	thermal conductivity of tube
ℓ_i	length of the i^{th} span
M_t	first moment of temperature
M	mechanical moment
q_p	peak incident heat flux
r, θ	cylindrical coordinates of tube
r_I	inside radius of the tube
r_o	outside radius of the tube
$\overline{T}$	mean temperature deviation from strain free state
T_f	local fluid temperature
ΔT	temperature change from strain free state
u,v,w	displacements in x,y,z directions
$u,_{\overline{x}\,\overline{x}}$	$\partial^2 u/\partial \overline{x}^2$
w_1	lateral deflection of the tube from thermal loads
w_2	lateral deflection from end constraints of each span
x	axial coordinate
$\overline{x}$	x/ℓ
y	tube coordinate normal to solar flux
$\overline{y}$	y/r_o

z	tube coordinate in direction from which solar flux arrives
$\bar{z}$	z/r_0
α	coefficient of thermal expansion
ε_x	axial strain
λ	r_0/ℓ
ν	Poisson's ratio
σ_x	axial stress

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