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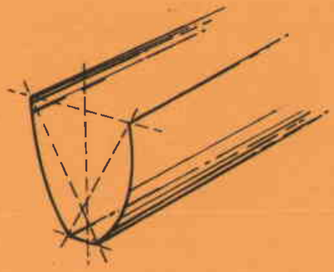
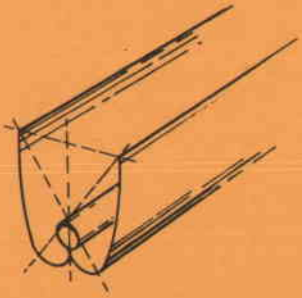
COMPOUND PARABOLIC CONCENTRATORS FOR SOLAR-THERMAL POWER

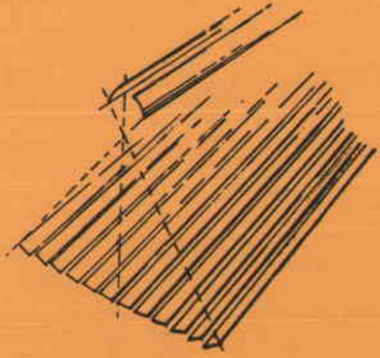
Semiannual Progress Report for the Period January–June 1975

by

Raymond M. Giugler, Ari Rabl, Kent Reed,
Vaclav J. Sevcik, and Roland Winston







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ABSTRACT

This report covers the development of Compound Parabolic Concentrators (CPC) during the period January 1, 1975 to June 30, 1975. The construction and testing of a tenfold concentrator was accomplished, and the optical and thermal performance was in excellent agreement with the mathematical model of the collector. The angular acceptance of the unit was measured by measuring the thermal output as a function of solar elevation. The results indicated that small imperfections in the shape of the collector did not have significant consequences.

A test of the angular acceptance of prototype mirror sections fabricated by various methods has been devised. This apparatus is simple to use in the laboratory, and does not require construction of a complete collector.

The optical design of a Compound Parabolic Concentrator with a fin receiver is presented in the report, and the relationships for reflector area and reflector height are given for this geometry. This geometry is of interest because of the decreased need for insulation and the reduction in the amount of metallic absorber needed.

I. INTRODUCTION

A. Proof-of-concept Experiments

Field testing of a CPC solar collector with tenfold concentration has been completed. The collector, which intercepts 8 ft^2 of solar radiation, was tested at the ANL outdoor testing facility. The conclusion is that the performance is accurately predicted by the computer model of Kreith and Kreider. These tests are reported in Sec. II.

B. Measurements of Reflector Properties

1. Surface Reflectance

Reflectivities of various candidate reflector materials have been measured (see Sec. III.A.).

2. Angular Acceptance

The angular-acceptance characteristics of several CPC collectors have been determined in order to check the accuracy of the production process. The technique and the results are described in Sec. III.B.

C. Analytical and Design Studies

1. CPC with Fin Receiver

A CPC with fin receiver offers two advantages over an ordinary CPC: (a) the absorber is irradiated from both sides (only half as much absorber material is needed), and (b) no heat is lost through back of collector (saves insulating material). The optical and thermal properties of this type of collector have been studied, and the results are presented in Sec. IV.

2. Comparison of Solar Concentrators

To help provide a rational basis for deciding which concentrator type is best suited for a particular application, various solar concentrators (both conventional and CPC) have been compared on the basis of their most important general characteristics, namely, concentration, acceptance angle, sensitivity to mirror errors, size of reflector area, and average number of reflections. This study has been accepted for publication by Solar Energy, and the abstract of the paper is as follows:

Abstract

Even though most variations of solar concentrators have been studied or built at some time or other, an important class of concentrators has been overlooked until very recently. These novel concentrators have been called ideal because of their optical properties, and an example, the compound parabolic concentrator, is being developed at Argonne National Laboratory. Ideal concetrators differ significantly from conventional instruments such as focusing parabolas. They act as a radiation funnel and do not form an image. For a given acceptance angle, their concentration surpasses that of other solar concentrators by a factor of two to four, but a rather large reflector area is required. The number of reflections varies with angle of incidence, with an average value around one in most cases of interest. In order to help provide a rational basis for deciding which concentrator type is best suited for a particular application, we have compared a variety of solar concentrators in terms of their most important general characteristics, namely concentration, acceptance angle, sensitivity to mirror errors, size of reflector area and average number of reflections.

The connection between concentration, acceptance angle and operating temperature of a solar collector is analyzed in simple intuitive terms, leading to a straightforward recipe for designing collectors with maximal concentration (no radiation emitted by the absorber must be allowed to leave the concentrator outside its acceptance angle). We propose some new concentrators, including the use of nonimaging concentrators as a second stage concentrator for conventional parabolic or Fresnel mirrors. Such combination approaches the performance of an ideal concentrator without demanding a large reflector; it has been shown to offer significant advantages for high temperature solar systems.

3. Second-stage Concentrators

We have developed the basic principles for designing second-stage concentrators following the nonimaging concentrator concept. This work is included as Appendix A.

D. Subcontracts

Supplemental funding for bringing industrial involvement into the development program, as well as for supporting the in-house work, was recently obtained. The objectives of industrial involvement have been implemented by awarding subcontracts in a number of areas; these are as follows:

1. A goal study for technical development and economic evaluation of the compound parabolic concentrator for solar-energy collector applications. Subcontracts have been awarded to Arthur D. Little, Inc. and to Bechtel Corporation.

2. Fabrication of concentrating flat-panel solar collectors (CPC design). Subcontracts have been awarded to Chamberlain Manufacturing Corporation and to American Science and Engineering, Inc.

3. Application of CPC solar concentrators for photovoltaic conversion. Subcontracts have been awarded to Mobil-Tyco Solar Energy Corporation and to Spectrolab, Inc.

4. Assistance in interpretation of test data, mathematical modeling, design, and economic analysis. Subcontracts have been awarded to Environmental Consulting Services, Inc. (Dr. F. Kreith and Dr. J. Kreider), who provided ANL with the original computer model of CPC collectors which has been used to interpret our field performance data.

II. PROOF-OF-CONCEPT EXPERIMENTS

A. Introduction

A solar-thermal collector of the compound-parabolic-concentrator (CPC) design[1] with a concentration ratio of 10 has been constructed at Argonne National Laboratory. The theoretical full-acceptance angle of 10° implies that angular reorientation at most once a day is sufficient to ensure acceptance of direct solar radiation. The outdoor test results show good agreement with a detailed theoretical analysis[2,3] and point to substantial improvement in high-temperature performance relative to flat-plate collectors.

B. Collector Construction

The collector consisted of two identical modules, each comprising a receiver, a pair of mirrors, and a Plexiglas cover. The mirrors were 0.020-in. Alzak sheets (specular reflectivity averaged over the solar spectrum $\approx$ 75%) held to the compound parabolic trough profile by exterior wood ribs mounted to 3/16-in.-thick aluminum base plates. To minimize the effect of the short collector length, the ends of the troughs were closed by flat Alzak sheets. Each module was 4 ft long and 3 ft high, and each had an input-aperture width of 12 in. and an output-aperture width of 1.2 in, for a concentration ratio of 10. The total input-aperture area was 8 ft^2. The 1/8-in. Plexiglas cover transmitted 89% of the incident solar radiation.

The receiver for each module was a semicylindrical trough made from 1-1/4-in.-dia copper tube cut in half lengthwise, with a 3/8-in. dia fluid channel brazed down the center. The receivers were painted with a non-selective black (absorbtivity $\approx$ 90%). The receiver was cradled in position below the mirror base plates by a 2-3/4-in.-wide, 1-3/4-in.-deep insulating block of glass foam which in turn was covered on the sides and bottom by 1-in.-thick Styrofoam. During the tests more insulation was added, as described in Sec. II.D.

C. Description of Experiments

Outdoor tests were conducted at Argonne National Laboratory during January-May 1975. An adjustable mounting platform was used to maintain the collector orientation normal to the sun's rays. The incident solar radiation was monitored by an Eppley pyranometer on the tilted collector plane, and by an Eppley normal-incidence pyrheliometer. A preheated mixture of ethylene glycol and water (50 volume %) was used as the collecting fluid, and the heat output of the collector was determined by measuring the flow rate and the temperature rise in the fluid from inlet to outlet. At typical flow rates of 0.25-0.5 gpm, the receivers were fairly isothermal. Temperature measurements along the receivers and the mirrors, as well as wind speed, wind direction, and ambient temperature, were monitored for diagnostic purposes. Useful data were obtained only for clear-sky conditions with stable pyranometer readings and steady-state temperatures on the collector.

D. Experimental Results

It is convenient to consider the efficiency η as a function of $\Delta T/A$, where

$\eta = Q_{out}/Q_{in}$,

Q_{out} = heat output of collector (Btu/hr),

$Q_{in} = S \times A$ (Btu/hr),

S = solar insolation (total on tilted collector plane) (Btu/hr-ft^2),

A = collector input area (ft^2),

$\Delta T = T_c - T_A$ (°F)

T_c = average receiver temperature (°F)

and

T_A = ambient temperature (°F).

One expects an approximately linear relationship $\eta = \eta(0) - U\Delta T/S$, where $\eta(0)$ is the no-thermal loss ("optical") efficiency, and U is the heat-loss coefficient (Btu/hr-ft^2-°F).

The uninsulated mirror configuration was used to determine the optical efficiency of the collector. From the efficiencies measured for $\Delta T/S$ in the range 0.2-0.65 °F/Btu/hr-ft^2, the optical efficiency was determined to be $\eta(0) = 0.50 \pm 0.01$, and the heat-loss coefficient $U = 0.49 \pm 0.02$ Btu/hr-ft^2-°F. The experimental optical efficiency agreed satisfactorily with the value 0.51 predicted from Kreith and Kreider's computer model. However, the heat-loss coefficient was more than twice the value (0.19) predicted from the model, which considers frontal convection and radiation losses alone.

Insulation was then added in stages to minimize back conduction and convection losses. To reduce losses through the metal mirrors, a minimum of 1-1/2 in. of polyurethane foam was applied between the wood ribs, and 2 in. of Styrofoam was added to the ends of the collectors. The efficiency was remeasured (at $\Delta T/S = 0.41$ °F/Btu/hr-ft^2), and an improved heat-loss coefficient ($U = 0.39 \pm 0.03$ Btu/hr-ft^2-°F) was obtained.

Polyurethane foam was then added to isolate the aluminum base plates of the mirrors from the ambient air, and the efficiency was redetermined. The heat-loss coefficient was found to be reduced to $U = 0.30 \pm 0.02$ Btu/hr-ft^2-°F. The data are shown in Fig. 1. The contribution to the U-value from back losses was determined by measuring the heat loss of the collector with the mirror troughs filled with Styrofoam beads, with the result $U_{back} = 0.141 \pm 0.003$ Btu/hr-ft^2-°F. Considering a residual contribution of 0.02 Btu/hr-ft^2-°F from the beads, the value $U_{front} = 0.18 \pm 0.02$ Btu/hr-ft^2-°F was obtained, in good agreement with 0.19 Btu/hr-ft^2-°F predicted from Kreith and Kreider's model. Correcting the model results for the measured back loss results in the solid curve in Fig. 1. The systematic positive deviation of the data at small $\Delta T/S$ occurs because, in the model, solar radiation absorbed by the mirrors is assumed lost, when, in fact, part of this energy ultimately reaches the receivers through radiative transfer.

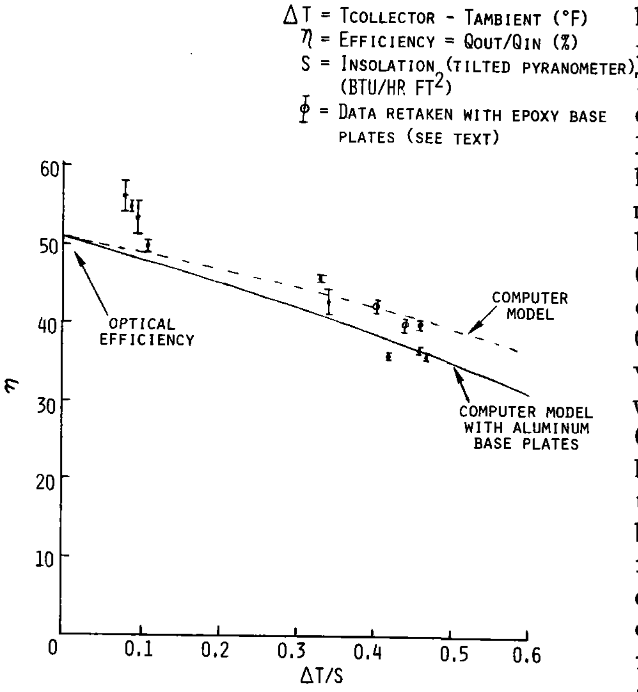


Fig. 1. Efficiency of 10x CPC Concentrator, 8 ft^2 - One Cover.

An analysis of the conduction losses indicated that the major remaining parasitic element in the system was the 3/16-in. aluminum base plates. As a final test, these plates were replaced with G-10 glass epoxy plates, and the efficiency measured for $\Delta T/S$ in the range 0.40-0.46°F/Btu/hr-ft^2. The data (see Fig. 1) imply that $U = 0.22 \pm 0.01$ Btu/hr-ft^2-°F. With the troughs again filled with Styrofoam beads, it was determined that $U = 0.05 \pm 0.01$ Btu/hr-ft^2-°F. Allowing for a contribution from the beads as before gave $U_{back} = 0.03 \pm 0.01$ Btu/hr-ft^2-°F, so that $U_{front} = 0.19 \pm 0.01$ Btu/hr-ft^2-°F, in excellent agreement with Kreith and Kreider's computer model. The model results, including the new back loss, are shown as the dashed curve in Fig. 1.

The angular acceptance of the collector was determined by adjusting its elevation relative to the sun and measuring the heat output. The results (see Fig. 2) show the collector efficiency to be essentially constant over the theoretical angular acceptance ($2\theta_{max} = 10°$).

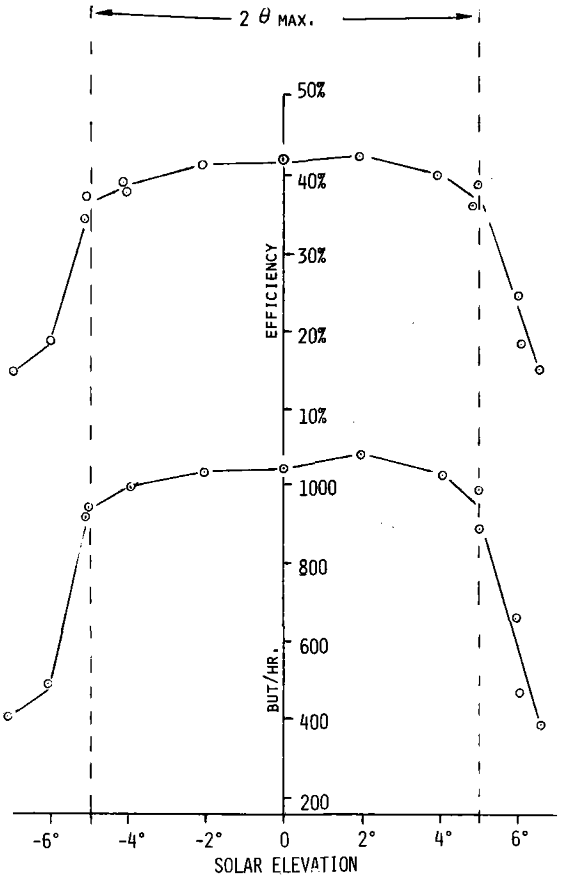


Fig. 2. Angular Acceptance of 10x CPC (θ max = 5°).

E. Conclusions

The 10x CPC collector performs in accordance with theoretical predictions, possessing an optical efficiency $\eta(0) = 0.50$ and a total heat-loss coefficient $U = 0.22$ Btu/hr-ft^2-°F. With improved absorbers (absorptivity $\approx$ 95%) and better aluminum mirrors (reflectance averaged over the solar spectrum $\approx$ 85%), the optical efficiency would improve to $\eta(0) \approx 0.60$. Coupled with the small heat-loss coefficient characteristic of concentration, a 10x CPC collector with such an optical efficiency would perform with 50% efficiency at 150°F above ambient temperature, a useful range for space conditioning where flat-plate collectors are marginal.

III. MEASUREMENTS OF REFLECTOR PROPERTIES

A. Surface-reflectance Measurements

The selection of a suitable reflector surface material is being guided by an experimental program to determine the effective reflectance of various samples. The spectral reflectance is measured in the range of 0.3-2.0 μm using a Cary 14 recording spectrophotometer, and then averaged over the spectral distribution of solar insolation to obtain the effective total reflectance. Table I lists current results.

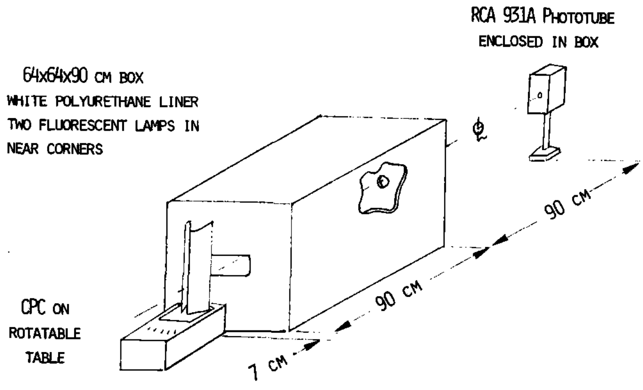
TABLE I. Total Reflectance at Normal Incidence

Sample	Surface	Total Reflectance,[a] Percent
Anodized Aluminum (ALCOA Alzak)	1st	77.2
Bright-dipped Aluminum	1st	72.6
Aluminized Glass	1st	87.5
Aluminized Glass	2nd	80.0
Aluminized Mylar	1st	81.0
Aluminized Epoxy	1st	86.0
Aluminized Plexiglass	2nd	79.1
Aluminized Teflon (Schjeldahl Film)	2nd	83.9
Silvered Glass	1st	90.1
Silvered Glass	2nd	82.8
Silvered Glass, Resin Overcoat	1st	88.3

[a] Obtained from spectral reflectance measured over the range 0.3-2.0 μm and integrated over the spectral distribution of solar insolation.

B. Angular Acceptance Measurements

The angular field of view of various CPC models has been determined experimentally by measuring the reflectivity as a function of angle using the light box apparatus shown in Fig. 3.



Defining apertures: Light Box- 10x20 entrance, 2.5 cm dia. exit
Phototube- 0.3 cm dia. entrance

Fig. 3. Light Box Apparatus for CPC Reflectivity Measurements.

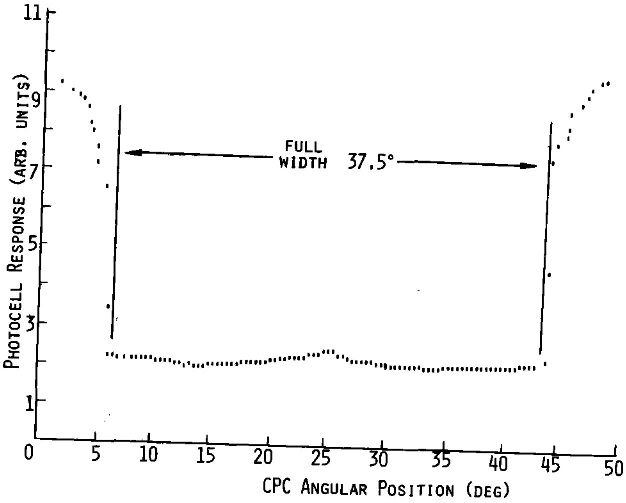
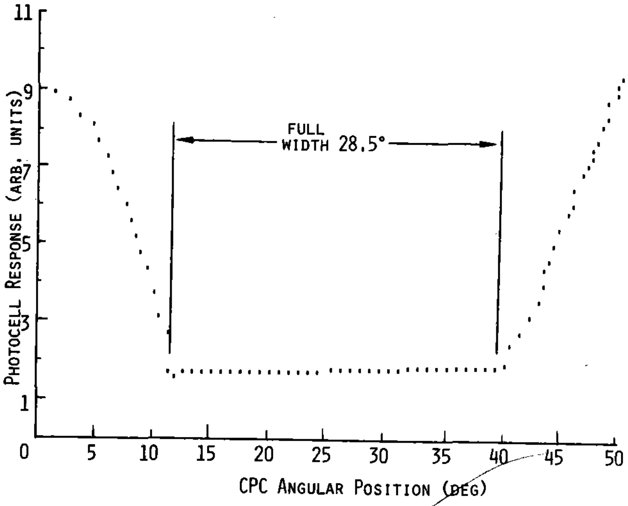


Fig. 4. Reflectivity of 3x CPC - Polished Steel Mold Tooling.

Fig. 5. Reflectivity of 3x CPC - First Surface Aluminized Glass Mirrors Formed by Sagging Glass over Steel Mold Tooling.



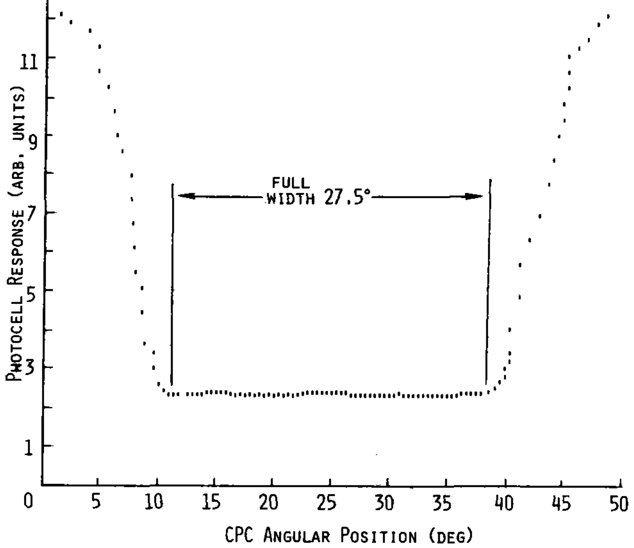


Fig. 6. Reflectivity of 3x CPC - Second Surface Aluminized Glass Mirrors Formed by Sagging Glass over Steel Mold Tooling.

This apparatus illuminates the CPC reflector with totally diffuse light, and measures the amount of light that is reflected back to the phototube. The CPC unit may be rotated on the turntable to vary the angle of incidence. The polished steel mold tooling that was used for fabricating the 3x CPC collector reported in the previous semiannual progress report (ANL-75-42) was tested on the light box. Results for the polished steel mold tooling are shown in Fig. 4. The less light reflected back to the photocell, the lower will be the photocell response. For the 3x CPC configuration, the acceptance angle should be 37.5°, and as can be seen from Fig. 4, an acceptance angle of 37.5° is sharply defined, indicating that the mold tooling has the proper shape.

The tests were repeated with glass mirrors that were formed by sagging glass (at high temperature) over the mold tooling to obtain the CPC shape. These glass shapes were coated with aluminum by vapor deposition and tested on the light box apparatus, repeating the experiment described above for the mold tooling. The results for the first-surface aluminized glass mirrors are shown in Fig. 5. The main difference from the test of the mold tooling is that the acceptance angle has been reduced from 37.5° to 28.5° and the transition is not sharp or abrupt. This indicates that the shape of the mirror was not proper.

The second-surface (aluminized) glass mirror was then evaluated, since this was the surface of the glass that would be in contact with the mold tooling during the sagging procedure. The results are shown in Fig. 6; no improvement over the first-surface aluminized mirror was noted. This means that the shape of the glass did not fully conform to the mold tooling during the sagging operation, and the fabrication technique is not satisfactory.

This measurement technique represents a simple, fast method for determining how well the reflecting surfaces have been made to conform to the correct CPC shape, and will be used to evaluate future fabrication techniques.

IV. CPC WITH FIN RECEIVER

In 1974, Winston and Hinterberger showed that ideal cylindrical concentrators can be designed with absorbers of arbitrary (convex) cross section. The example shown in Fig. 7 has a finlike receiver and is particularly promising for solar energy applications for the following reasons:

1. The absorber is irradiated from both sides. (Compared to an ordinary CPC with one-sided absorber, the requirement for absorber material is reduced to one-half.
2. No heat is lost through the back of the collector. (No insulating material is needed.)

The optical and thermal analysis of this collector is essentially the same as for an ordinary CPC. The most important parameters, namely height-aperture ratio, reflector/aperture ratio, and average number of reflections for a CPC with fin receiver, are plotted as function of

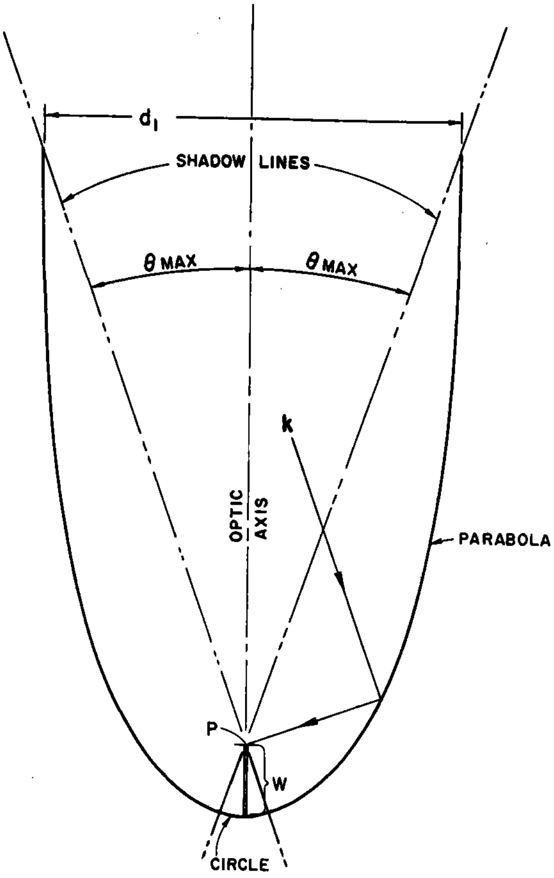


Fig. 7. CPC with Fin Receiver.

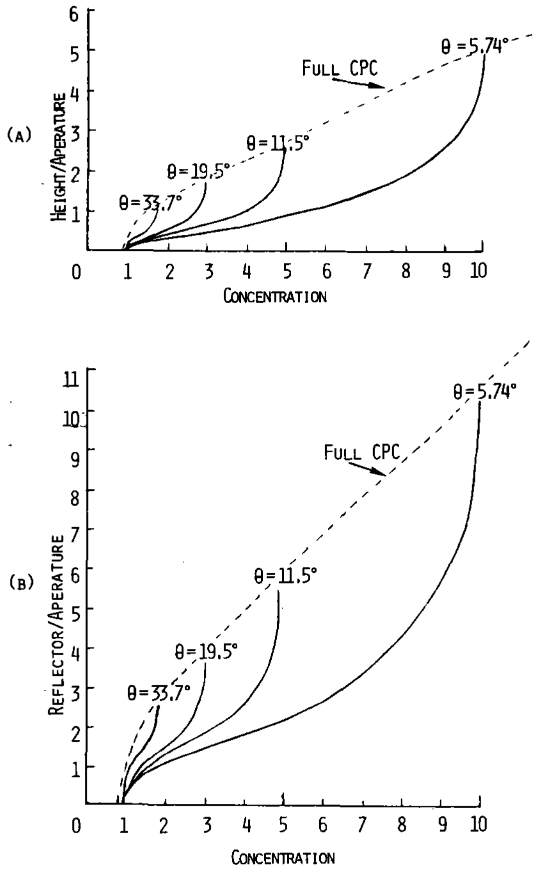


Fig. 8. Height (A) and Reflector Area (B) as Function of Concentration (Truncation) for Various Acceptance Half-angles θ.

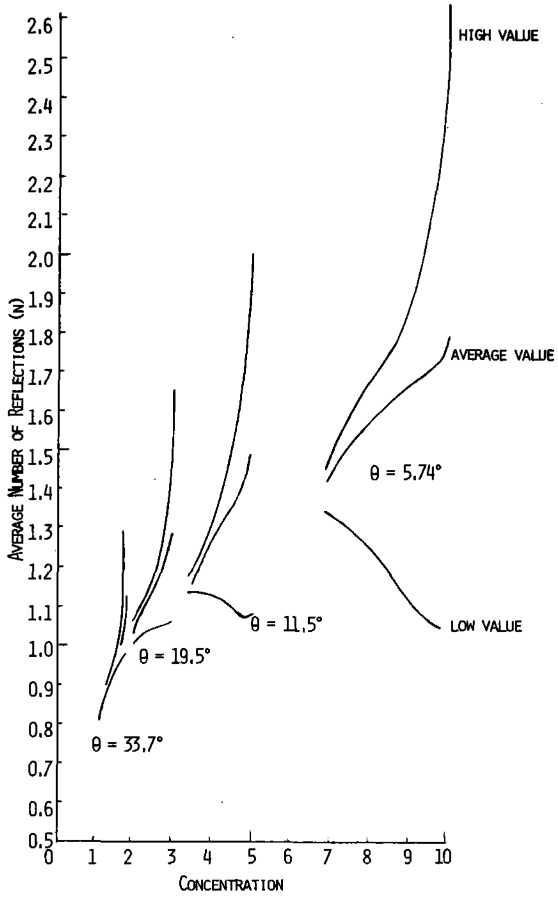


Fig. 9. Average Number of Reflections for CPC with Fin Receiver. Middle Curve Shows Average Value; Upper and Lower Curves Show Extreme Values.

concentration in Figs. 8 and 9, for acceptance half angles 5.7, 11.5, 19.5, and 33.7, both for full and for truncated concentrators. In Fig. 9, the middle curve for each acceptance half-angle θ shows the average overall angles of incidence $|\theta_{in}| < \theta$; the upper and lower curves show the extreme values attained for particular angles of incidence. For example, an untruncated collector with θ = 19.5° has a concentration 1/sin 19.5° = 3.0 and requires 3.6 m^2 reflector for each square meter of aperture; the number of reflections ranges from 1.06 to 1.65. If this collector is truncated to a concentration of two, it needs only 1.5 m^2 reflector for each square meter of aperture and the number of reflections range from 1.00 to 1.07 with an average of 1.04. The average number of reflections <n> determines (approximately) the fraction of radiation transmitted through the concentrator by the simple formula $\tau = \rho^{<n>}$, where ρ is the reflectivity of the concentrator wall. For a fin CPC, <n> is about 50% larger than for an ordinary CPC of comparable concentration, and, in practice, it will range from 0.9 to 1.3. This is not a serious disadvantage since good reflector materials (with $\rho \gtrsim 85\%$) are available. The flux distribution at the absorber is similar to that for the ordinary CPC, i.e., totally diffuse when averaged over time. The frontal heat loss is estimated to be approximately the same as for the ordinary CPC.

* * * *

REFERENCES

1. R. Winston, Solar Energy *16*, 89 (1974); and V. J. Sevcik, et al., ISES, U. S. Section Meeting, Fort Collins, Colo., August 1974.

2. J. F. Kreider, Environmental Consulting Services, Inc., private communication, 1974.

3. A. Rabl, *Optical and Thermal Properties of Compound Parabolic Concentrators*, ANL-SOL 75-01 (1975); to be published in Solar Energy.

APPENDIX A

IDEAL LIGHT COLLECTORS IN CYLINDRICAL GEOMETRY

The purpose of this note is to give a general method of solution to the problem of nonimaging light collection, a problem that has been worked on by one of the authors and others for some time.[1] The method in its general form is applicable only to cylindrical geometry which is, however, a case of considerable practical importance in solar energy collectors.[2,3] As discussed in Ref. 2, the three dimensional properties of a system with cylindrical symmetry are specified by the behavior of light rays in the transverse plane. Therefore, with no loss of generality we will consider the problem of collecting meridional rays. Moreover, for simplicity we treat the case of constant refractive index only (say, n - 1); the generalization to distinct indices of refraction is strightforward and is discussed in Ref. 2.

The problem that we address is the maximal concentration of radiation from an isotropic source onto a receiver surface of general shape. With reference to Fig. 1 we seek to collect all radiation from the source that impinges on the entrance aperture (C,C') and to concentrate it onto the receiver E. Moreover, we wish to minimize the arc length S of the profile curve of the receiver E.

We treat, in the initial example, a system that is symmetric about the optic axis (z). It is first necessary to establish the maximum possible concentration, i.e., the minimum value of S. This is conveniently done by using a hamiltonian description of the light ray trajectories propagating in the z direction. Introducing the direction cosine of the light ray k_x conjugate to x, the conserved phase space is given by

$$\int_{z\,=\,\text{constant}} dx\,dk_x \quad \text{is conserved.} \tag{1}$$

This quantity is related to the throughput or étendue used by other authors.[4] Evaluating the phase space at the entrance aperture we obtain the simple result

$$1/2 \int dx\,dk_x = (q - p) \tag{2}$$

the difference in distance between an edge of the source and the edges of the entrance aperture. Equivalently, this is the difference in distance between an edge of the entrance aperture and the edges of the source. Note that in the limit of a distant source subtending an angle $\pm\,\theta_{max}$, Eq. (2) tends to $\overline{CC'}\sin\theta_{max}$, where $\overline{CC'}$ is the entrance aperture width in agreement with Ref. 1.

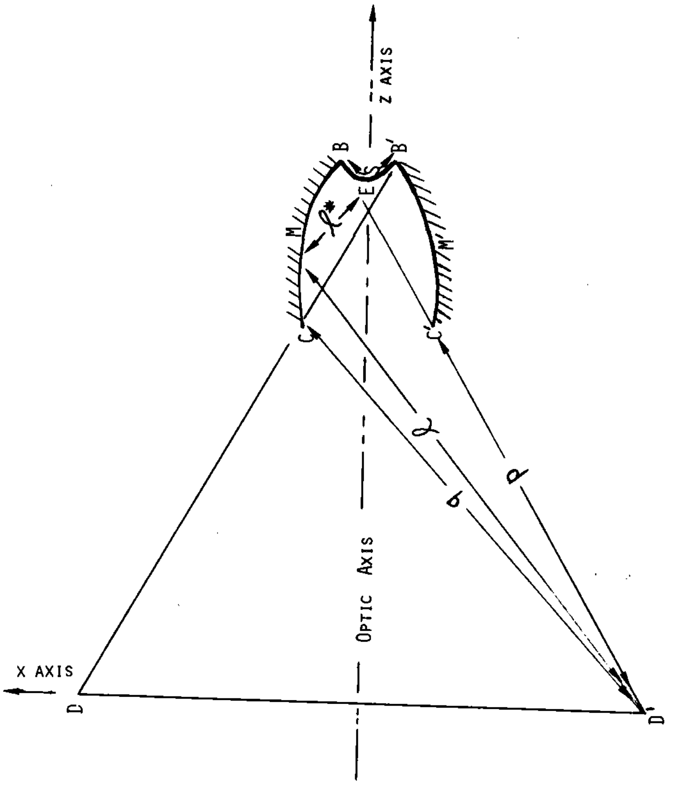


Fig. 1-A. Symmetric light collector. Receiver is tangent to extreme rays at B, B'.

Figure 1-A. is the transverse cross section of a cylindrical mirror (M,M') which concentrates light from a ribbon source (D,D') onto a receiver surface E. To achieve maximal concentration, it is necessary to exclude stray light trajectories originating outside the source from reaching the receiver. We therefore require the profile curve of the receiver E to be tangent to the extreme directions (D'C'), (DC) at points B,B', respectively. The sole condition imposed on E is that it be convex and in this instance symmetric about the optic axis. The convex requirement prevents a tangent crossing the receiver boundary. We assert the solution is to so choose the profile curve M that singly reflected rays originating from D' are tangent to E. This means E becomes the envelope of such rays. In other words, the receiver surface E is a caustic surface.[5] We denote the optical path length from D' to M by ℓ, from M to the conjugate point of tangency on the caustic by ℓ^* and the arc length along the caustic by s. Then, employing the methods of Ref. 3, we find that our construction imposes a specific relation between these quantities as follows:

$$d(\ell + \ell^*) = ds \; . \qquad (3)$$

Integrating Eq. (3) over the profile curve M, we obtain

$$S = \int_B^C d(\ell + \ell^*) = (q + \ell^*_C) - (p + \ell^*_{C'}) = (q - p) \qquad (4)$$

demonstrating that our solution indeed minimized the receiver perimeter S consistent with phase space conservation. We note that for the special case of a flat receiver (E parallel to D'D) the caustic collapses to a point and the profile curve of M is an ellipse with foci at D' and B'. For the same situation but with the source at infinity and subtending an angle $\pm\, \theta_{max}$, the profile curve M is a parabola with focus at B' and axis inclined at angle θ_{max} to the optic axis.[6] The flat receiver should therefore be considered the limiting case of a general convex receiver.

The solution shown in Fig. 1-A. can be readily adapted to a variety of less restrictive assumptions about the relationship of source to receiver; we give two examples in Figs. 2-A and 3-A. In Fig. 2-A we have extended

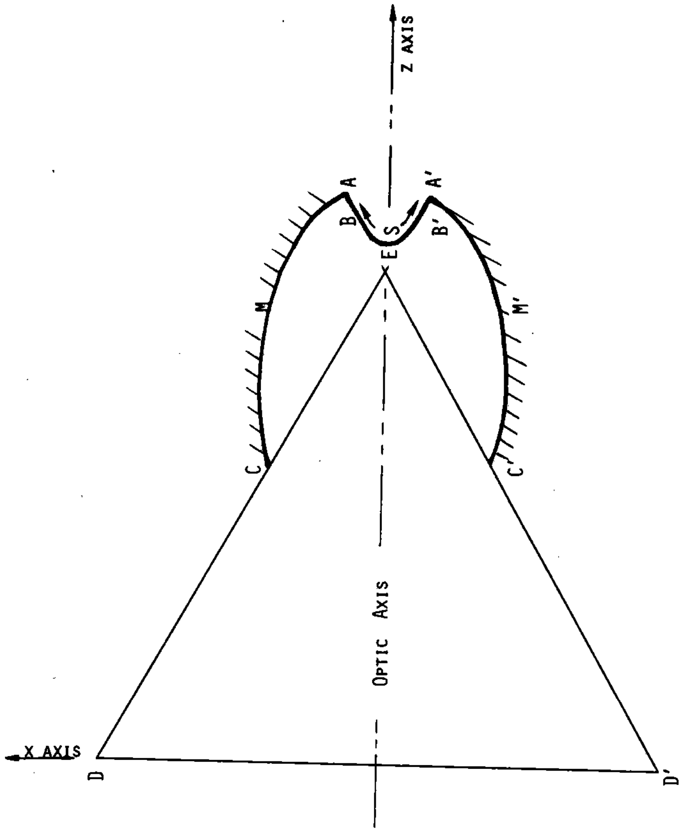
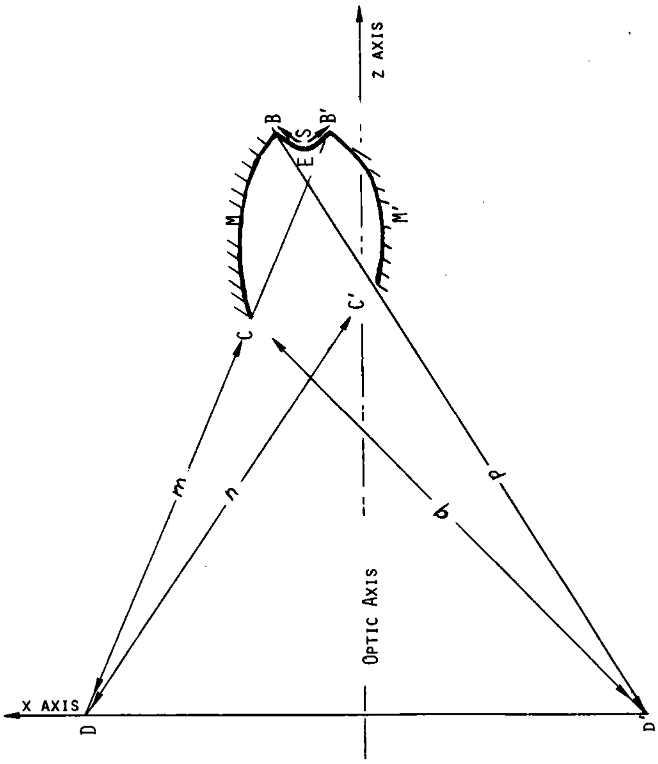


Fig. 2-A. Symmetric light collector. Receiver is extended backward along the extreme rays to A, A'.

Fig. 3-A. Asymmetric light collector. Receiver is tangent to extreme rays at B, B'.



the receiver curve E backward along the lines of tangency to points A, A'. A straightforward application of Eq. (3) again gives the maximum concentration condition of Eq. (4). In Fig. 3-A we have permitted the receiver to be asymmetrically disposed relative to the source. For this case, the phase space at the entrance aperture becomes

$$1/2 \int dx dk_x = 1/2 \quad [(q - p) + (n - m)] \tag{5}$$

which is a natural generalization of Eq. (2). To solve the asymmetric problem, we choose the profile curve M so that singly reflected rays from D' form the caustic curve E as before. Similarly we choose the profile curve M' so that singly reflected rays from D form the same caustic curve E. Let us denote by ℓ' and ℓ'^* the optical path length from D to M' and of the reflected ray from M' to the caustic curve E. Then, integrating along M we obtain

$$S = \int_B^C d(\ell + \ell^*) = (q - p) + (\ell_C^* - \ell_{C'}'^*) \, . \tag{6}$$

Integrating along M' (with the point D as origin) we obtain

$$S = \int_{B'}^{C'} d(\ell' + \ell'^*) = (n - m) + (\ell_{C'}'^* - \ell_C^*) \, . \tag{7}$$

Therefore, adding Eqs. (6) and (7), we find

$$S = 1/2[(q - p) + (n - m)] \tag{8}$$

which is the maximal concentration condition required by Eq. (5).

In conclusion we recall that the condition for ideal imaging of rays from a point 0 to a conjugate point 0' after reflection from a cartesian surface is given by Fermat's principle

$$\delta(\ell + \ell^*) = 0 \, . \tag{9}$$

Here ℓ, ℓ^* are optical path lengths from 0,0' to the reflecting surface and δ denotes a comparison of neighboring trajectories. By comparing neighboring trajectories from D' to the conjugate points along the caustic, we may reinterpret Eq. (3) as

$$\delta(\ell + \ell^*) = \delta s \tag{10}$$

where δs is the separation of neighboring conjugate points along the caustic. This suggests that the solution for an ideal light collector which performs maximal concentration obeys a condition [Eq. (10)] which generalizes Fermat's principle to a nonimaging situation.

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REFERENCES - APPENDIX A

1. R. Winston, J. Opt. Soc. Am., 60, 245 (1970).

2. R. Winston, Solar Energy, 16, 89 (1974).

3. R. Winston and H. Hinterberger, Solar Energy, (to be published).

4. W. B. Joyce, Phys. Rev. D9, 3234 (1974).

5. This solution was inereent in the early work of the author. See Ref. 1 which also contains the earlier references.

6. H. Hinterberger and R. Winston, Rev. Sci. Instr., 37, 1094 (1966); V. K. Baranov, Geliotekhnika, 2, 11 (1966).