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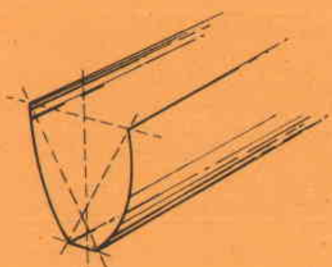
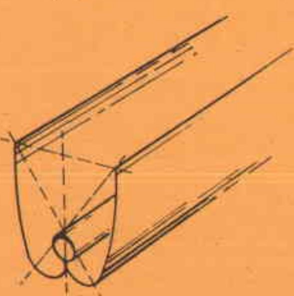
PREDICTED HEAT-TRANSFER PERFORMANCE OF AN EVACUATED GLASS-JACKETED CPC RECEIVER: COUNTERCURRENT FLOW DESIGN



by

George Thodos



p 25 Fig 10
How do you get 30%
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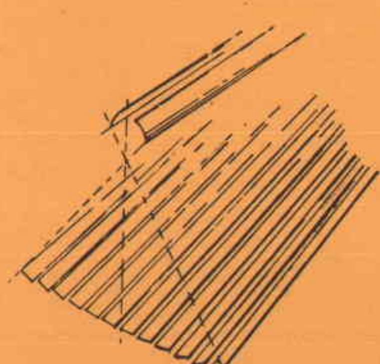

Solar Energy Group

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PREDICTED HEAT-TRANSFER PERFORMANCE OF AN EVACUATED GLASS-JACKETED CPC RECEIVER: COUNTERCURRENT FLOW DESIGN

by

George Thodos*

Solar Energy Group

May 1976

*Professor Thodos, Northwestern University, was the recipient of the AUA Distinguished Appointment Award for 1975.

TABLE OF CONTENTS

LIST OF FIGURES

NOMENCLATURE

NOMENCLATURE

Symbol	Description
A	Area of tube
A_c	Cross-section area
A_r	Receiver surface
A_g	Glass-jacket surface
dA	Differential peripheral area of tube associated with differential length, dx
dA_i	Differential inside area of tube wall associated with differential length, dx
dA_m	Differential mean area of tube wall associated with differential length, dx
dA_o	Differential outside area of tube wall associated with differential length, dx
c_p	Heat capacity of fluid
d_i	Inside diameter of inner tube
d_o	Outside diameter of inner tube
D	Tube diameter
D_e	Equivalent diameter for annular space
F	Geometrical configuration factor, Equation (18)
$\mathcal{F}$	Configuration factor of system for geometry and emittance, Equation (18)
g	Gravitational constant
G	Mass velocity
h	Heat-transfer coefficient
h_g	Convective heat-transfer coefficient of air surrounding glass jacket
h_i	Inside heat-transfer coefficient for fluid flowing through tube
h'_i	Heat-transfer coefficient for fluid in annular space adjacent to receiver surface
h_o	Heat-transfer coefficient for fluid flowing adjacent to tube wall in annular space

NOMENCLATURE

Symbol	Description
k	Thermal conductivity
k_1	Proportionality constant, rate of radiant-energy influx on receiver surface per unit length of receiver
k_2	Proportionality constant, Equation (3)
ℓ	Thickness of tube wall
L	Length of receiver
m	Mass flow rate of fluid
n_i	Average number of reflections from aperture to receiver of CPC
Q	Rate of heat flow, Btu/hr or cal/sec
Q_{conv}	Rate of heat flow due to convection
Q_{rad}	Rate of heat flow due to radiation
Q_x	Rate of heat exchange beyond point x
dQ_1	Radiant heat absorbed by external surface of receiver over distance dx
dQ_2	Thermal short-circuiting across wall of inner tube over distance dx
dQ_3	Sensible heat gain by fluid through annular space, flowing over distance dx
S	Insolation, Btu/hr ft^2 or watts/m^2
t	Temperature of fluid
t	Temperature of fluid in annular space of receiver
Δt	Temperature difference across liquid film
t_1	Temperature of fluid entering annular space of receiver
t_L	Temperature of fluid leaving annular space of receiver
T	Absolute temperature
T	Temperature of fluid within tube of receiver
T_1	Temperature of fluid leaving tube of receiver
T_g	Absolute temperature of glass jacket, °R

NOMENCLATURE

Symbol	Description
T_L	Temperature of fluid entering tube of receiver
T_r	Absolute temperature of receiver surface, °R
T_{sky}	Absolute temperature of sky, °R
T_{surr}	Absolute temperature of surroundings
U	Overall heat-transfer coefficient
x	Distance from entry point of receiver
dx	Differential length of receiver facility
α_i	Proportionality constant, $dA_i = \alpha_i\ dx$
α_m	Proportionality constant, $dA_m = \alpha_m\ dx$
α_o	Proportionality constant, $dA_o = \alpha_o\ dx$
α_s	Solar absorptivity of receiver surface
β	Coefficient of thermal expansion
γ	Fraction of total insolation accepted by CPC
ε	Emissivity of surface
ε_g	Emissivity of glass jacket
ε_r	Emissivity of receiver surface
η	Overall efficiency of collector
η_o	Optical efficiency of collector
θ	Time
μ	Viscosity
ρ	Density
ρ	Reflectivity of surface
ρ_R	Solar reflectivity of reflector
σ	Stefan-Boltzmann constant
τ	Effective solar transmissivity of system
τ_c	Solar transmissivity of a single cover
τ_g	Solar transmissivity of glass jacket

PREDICTED HEAT-TRANSFER PERFORMANCE OF AN EVACUATED GLASS-JACKETED CPC RECEIVER: COUNTERCURRENT FLOW DESIGN

by

George Thodos

ABSTRACT

The heat-transfer performance of an evacuated glass-jacketed CPC-receiver facility, free on one end and fixed onto the glass jacket at the other, has been carried out using heat-transfer relationships and the best information available in the literature. Specifically, the collector examined was a 3x-CPC facility, 8 ft (2.44 m) long, with an entrance aperture 4.5 in. (11.43 cm) wide covered with a single glass cover, and provided with an aluminum reflecting surface ($\rho = 0.88$). To maximize heat retention, a selectively treated receiver surface, $\varepsilon = 0.11$, was used. The optical efficiency of this CPC collector facility was calculated to be $\eta_o = 0.536$.

The heat reaching the surface of this receiver was transferred to liquid Dowtherm A flowing through this facility. The configuration within this receiver was of a reverse concentric flow design, with liquid Dowtherm A entering the annular space and leaving through the tube located concentrically to it. This countercurrent arrangement necessitated the development of specific mathematical expressions for the temperature of the fluid within the annular space and through the inside of the concentric tube.

A number of performance curves are presented that relate the overall efficiency of the collector with the temperature of the fluid entering and leaving this concentric receiver facility, using an incident insolation to the CPC collector of 200 Btu/hr ft^2 (630 watts/m^2) and an ambient air temperature of 100°F (37.8°C). In addition, related performance curves are presented for the average temperature of the glass jacket and receiver surface, and the highest temperature of the fluid prevailing at the entrance of the inner tube.

I. INTRODUCTION

In the conventional operation of CPC collectors*, attempts to minimize heat losses are ordinarily made with the introduction of transparent cover plates at the entrance aperture and the introduction of effective insulation behind the receivers of these collectors. These approaches, although effective in realizing a modest reduction in heat losses for collectors operating

*CPC is the acronym for Compound Parabolic Concentrator.

at moderate temperatures, do not lend themselves to minimizing heat losses for collectors operating significantly above ambient temperature conditions.

To overcome this limitation, it has been suggested that the receiver be surrounded by a glass jacket and that the space within the jacket be evacuated. This arrangement should prove effective in minimizing heat losses from the receiver due to conduction, convection, and radiation. More specifically, the presence of a high vacuum within the jacket containment (about 10^{-5} to 10^{-9} torr) should eliminate completely any losses from the receiver due to convection and at the same time should prove effective towards minimizing the overall losses due to conduction, convection, and radiation.

The confinement of the receiver in a glass tube, with the ends of the receiver permanently fixed to the glass jacket, introduces the problem of creating stress and strain on the glass during the operational phase of the collector. One of the suggested schemes for avoiding this problem involves the use of a floating-head countercurrently operating heat-exchanger facility. A cross-sectional view of such a receiver is presented in Fig. 1. The receiver consists of two concentric tubes, with the outer surface of the outer tube provided with external vertical fins. The fluid introduced into the receiver is made to flow through the annular space and reverses direction at the end of the receiver to flow through the inner tube before leaving the heat-exchange facility.

The receiver assembly within the evacuated glass jacket and its relative position within the reflector of a typical CPC collector is diagrammatically presented in Fig. 2. For optimum radiant energy interception, this

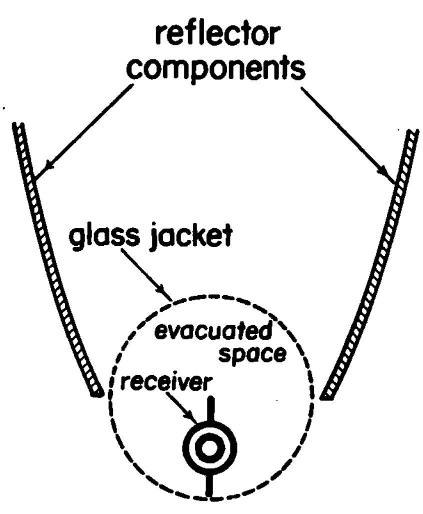
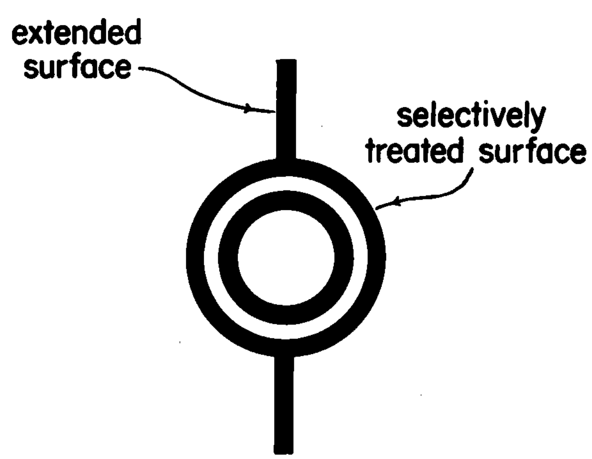


Fig. 2. Cross-sectional View of CPC Collector and Receiver Surrounded by Evacuated Glass Jacket.

Fig. 1. Cross-sectional View of Concentric Heat Exchange Facility.



arrangement places the receiver on the lower half of the cross-sectional area of the glass-jacketed space as shown in this figure.

A longitudinal view of the receiver assembly is presented in Fig. 3. This arrangement permits the steady penetration of solar energy through the glass jacket onto the combined tubular-fin receiver and its transport in the form of heat through the receiver wall and into the fluid flowing through the annular space of the concentric heat exchanger. As the fluid in the annular space moves to one end of the receiver, it is steadily heated, reaches a maximum temperature, and then reverses direction to enter the inner tube. With this reversal, the heated fluid begins to lose heat through the walls of the inner tube to the cooler fluid flowing through the annular space. This heat exchange is continued until the fluid within the inner tube reaches the end of the tube, where it is discharged from the unit. This thermal short-circuiting, although undesirable, becomes an integral part for the thermal analysis of this heat-exchange facility. Therefore, its contribution must be accounted for and must be treated quantitatively in order to establish the necessary parameters for proper assessment of the overall heat-transfer capability of such a CPC-receiver facility.

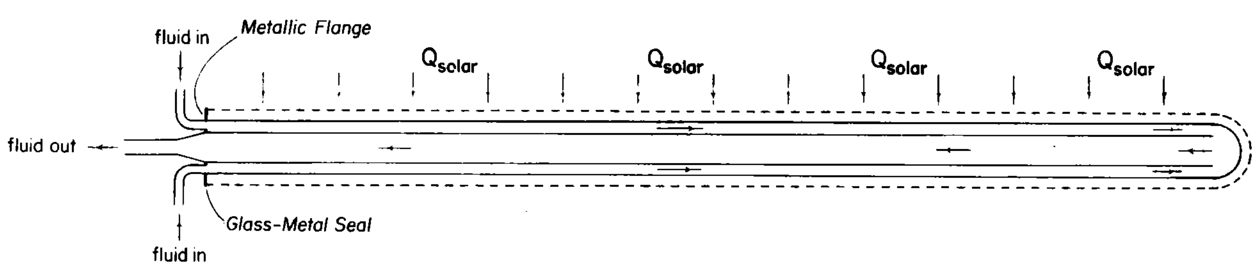


Fig. 3. Longitudinal View of Countercurrent Heat-exchange Facility for a CPC Receiver Surrounded by an Evacuated Glass Jacket.

II. DEVELOPMENT OF TEMPERATURE PROFILES FOR COUNTERCURRENT RECEIVER

The radiant energy transmitted through the glass jacket is absorbed by the receiver surface and is then transferred through the receiver walls to the fluid flowing through the annular space. The consequent thermal short-circuiting ensuing from the hot fluid flowing through the inside concentric tube gives rise to temperature profiles consistent with those presented in Fig. 4. A mathematical treatment of the salient variables is herewith presented for the establishment of the temperature profiles prevailing in the annular space and also within the tube. The temperature of the fluid within the tube is designated as T, while that in the annular space is referred to as t. At the end of the receiver, x = L, both temperatures become identical, and therefore, $T_L = t_L$. However, at any other point between x = 0 and x = L, T > t.

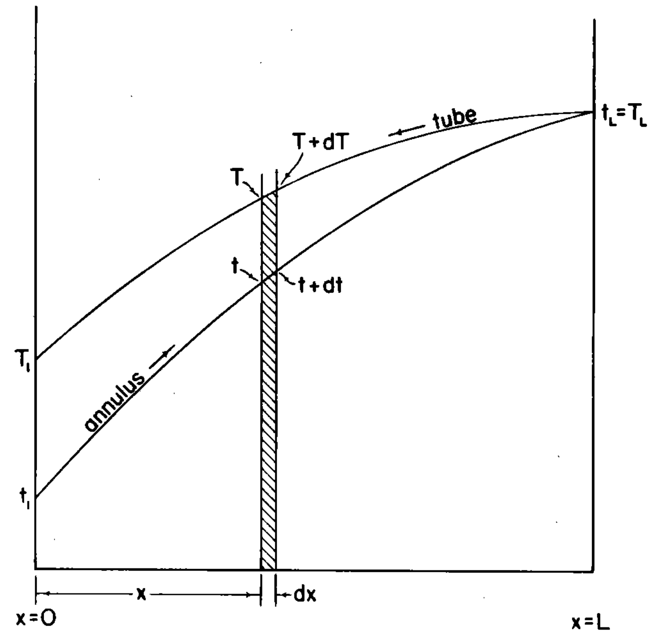


Fig. 4. Schematic Representation of Temperature Profiles of Fluid Flowing Countercurrently in a CPC Receiver.

With these designations, and also that at $x = 0$, $t = t_1$, and $T = T_1$, the heat exchange over an element of differential length, dx, follows. Considering the quantities of heat involved over a time element $d\theta$, the following can be formulated:

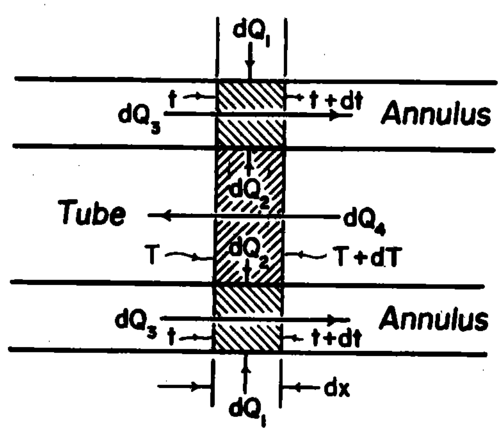


Fig. 5. Basic Heat Quantities and Temperatures Associated with Length dx of Countercurrent CPC-receiver Facility.

dQ_1 = radiant heat absorbed by external surface of receiver over distance dx.

dQ_2 = thermal short-circuiting due to heat flow across wall of inner tube over distance dx.

dQ_3 = sensible heat gain by fluid through annular space, flowing over distance dx.

A heat balance requirement on element of length dx requires that

$$dQ_1 + dQ_2 = dQ_3 \tag{1}$$

Since the amount of heat absorbed by the external surface of the receiver can be assumed to vary linearly with distance, it follows that

$$dQ_1 = k_1 dx \tag{2}$$

The thermal short-circuiting across the wall of the inner tube becomes

$$dQ_2 = UdA(T - t) = k_2(T - t)dx \qquad (3)^*$$

On the other hand, the sensible heat gain, over distance dx by the fluid flowing through the annular space is

$$dQ_3 = mc_p dt \tag{4}$$

Substituting Equations (2), (3), and (4) into Equation (1) results in the differential equation

$$k_1 dx + k_2(T - t)dx = mc_p dt \tag{5}$$

An overall heat balance over the entire receiver produces the relationship

$$Q_{total} = mc_p(T_1 - t_1) = k_1 L \tag{6}$$

However, the heat exchange Q_x beyond point x requires that

$$Q_x = mc_p(T - t) = k_1(L - x) \tag{7}$$

---

*In Equation (3), $\frac{1}{UdA} = \frac{1}{h_i dA_i} + \frac{\ell}{kdA_m} + \frac{1}{h_o dA_o} = \frac{1}{h_i \alpha_i dx} + \frac{\ell}{k\alpha_m dx} + \frac{1}{h_o \alpha_o dx}$

$= \left[\frac{1}{h_i \alpha_i} + \frac{\ell}{k\alpha_m} + \frac{1}{h_o \alpha_o}\right] \frac{1}{dx}$ where $\frac{1}{k_2} = \frac{1}{h_i \alpha_i} + \frac{\ell}{k\alpha_m} + \frac{1}{h_o \alpha_o}$

from which it follows that

$$T - t = \frac{k_1}{mc_p}(L - x) \tag{8}$$

Substituting Equation (8) into Equation (5), it follows that

$$k_1 dx + k_2 \frac{k_1}{mc_p}(L - x)\, dx = mc_p dt \tag{9}$$

or that $$\left[k_1 + \frac{k_1 k_2}{mc_p}(L - x)\right] dx = mc_p dt \tag{9a}$$

Integration of Equation (9a) between the limits $x = 0$, $t = t_1$ and point (x,t) produces t, the temperature distribution of the fluid flowing through the annular space as,

$$t = t_1 + \frac{k_1}{mc_p}\left[1 + \frac{k_2}{2mc_p}(2L - x)\right] x \tag{10}$$

The detailed steps associated with the development of Equation (10) are presented in the Appendix A, Section IA.

A comparable treatment for the establishment of T, the temperature distribution of the fluid flowing through the inner tube requires that

$$dQ_2 = dQ_4 \tag{11}$$

where $$dQ_2 = k_2(T - t)(-dx)$$

and $$dQ_4 = mc_p(-dT)$$

Thus, the heat requirement of Equation (11) yields

$$mc_p(-dT) = k_2(T - t)(-dx)$$

or

$$(T - t)dx = \frac{mc_p}{k_2} dT \tag{12}$$

Substituting Equation (12) into Equation (5) produces the relationship

$$k_1 dx + mc_p dT = mc_p dt \tag{13}$$

Integration of Equation (13) between the limits of $x = 0$, $T = T_1$, $t = t_1$, and point (x, t, T) yields the expression,

$$T - t = (T_1 - t_1) - \frac{k_1}{mc_p} x \tag{14}$$

Detailed steps associated with the development of Equation (14) are presented in Appendix A, Section IB.

To eliminate t from Equation (14), this relationship is combined with Equation (10) to yield,

$$T = T_1 + \frac{k_1}{mc_p} \frac{k_2}{mc_p} (2L - x) \frac{x}{2} \tag{15}$$

The development of Equation (15) from Equations (10) and (14) is also presented in Appendix A, Section IC.

The involvement of Equations (10) and (14) becomes very basic to the heat-transfer calculations associated with the performance of a floating-head receiver operating under countercurrent conditions. Their use requires that constants k_1 and k_2 be known. Constant k_1 can be fixed once the efficiency of the CPC receiver is designated while constant k_2 can be assumed and later can be checked to establish its consistency with the heat-transfer requirements of the receiver facility. Whereas the proper choice for constant k_2 can be resolved internally within the specifications of the problem, the selection of constant k_1 requires that the efficiency of the CPC receiver be assumed properly and checked at the conclusion of the calculations.

III. PROCEDURE OF CALCULATIONS

A 3x-CPC collector, of the dimensions presented in Fig. 6, has been selected for study. This CPC collector is specified with an entrance aperture of 4.5 in. (11.43 cm), an exit aperture of 1.5 in. (3.81 cm) and a length of 8 ft (2.44 m). The receiver of this collector is to be surrounded by an evacuated glass-jacketed containment and is to be maintained unattached on one end to accommodate for expansion and contraction without imparting stresses and strains to the glass jacket. The outer tube of this receiver has been arbitrarily selected to be a 3/8-in. (0.9525-cm) copper tube, while the inner tube, concentric to it, has been set to be a glass tube, 1/4 in. (0.635 cm) in outer diameter.*

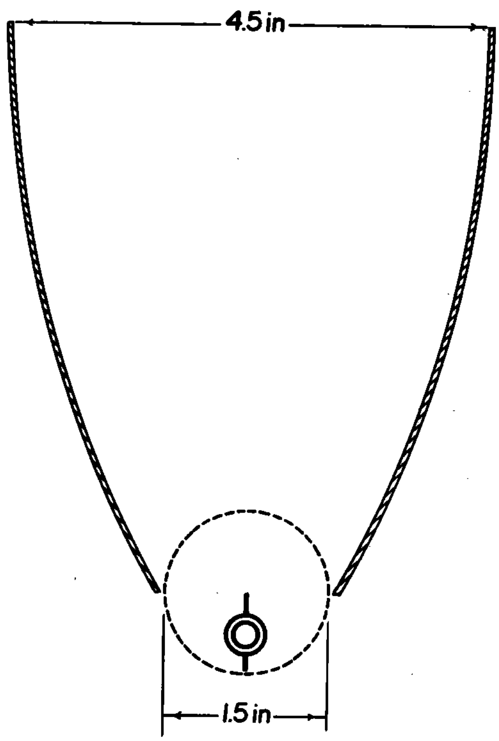


Fig. 6. Schematic Diagram of a 3x-CPC Collector with an Evacuated Glass-jacketed Receiver.

For this study, Dowtherm A has been selected as the heat-transfer fluid. This fluid is made to flow first through the annular space between the copper pipe and the glass tube and is then made to reverse direction at the end of the facility to enter the inner concentric tube.

For this arrangement, the efficiency of this collector will be calculated for conditions giving rise to varying temperature differences between the receiver surface and ambient air. Throughout these calculations, ambient air temperature has been taken to be 100°F (37.8°C).

A. Optical Efficiency

The calculation of the optical efficiency of this 3x-CPC collector has been based on the relationship

$$\eta_o = \gamma \, \rho_R^{\,n_i} \, \tau \, \alpha_s \quad (16)$$

*Although a glass tube has been selected in this case for minimizing the short-circuiting thermal losses, at the conclusion of these calculations it has been shown that this choice is immaterial and that the selection of a copper tube did not produce any significantly different results. This condition prevails from the dominant nature of the fluid films existing inside and outside of the tube.

reported by Ari Rabl[1]. The following values have been assigned to this particular collector configuration:

γ = 0.92 (fraction of total insolation S accepted by CPC)

ρ_R = 0.88 (reflectivity of reflector: vacuum-deposited aluminum on plastic)

τ = effective solar transmissivity of system, $\tau_c \tau_g$

τ_c = 0.90 (solar transmissivity of a single cover)

τ_g = 0.85 (solar transmissivity of glass jacket)

α_s = 0.90 (solar absorptivity of receiver surface)

n_i = 1.3 (average number of reflections from aperture to receiver of CPC)

These prescribed values produce the following optical efficiency for this facility,

$$\eta_o = 0.92\ (0.88)^{1.3}(0.85 \times 0.90)(0.90) = 0.536$$

This value implies that, for this particular CPC design, 53.6 percent of the insolation incident to the aperture of this collector finds its way to the receiver surface. However, additional heat losses associated with the receiver are concurrently present to produce a value for the effective efficiency that falls below this value of 53.6 percent.

B. Calculation of Heat Losses from Receiver

Using an insolation flux of 200 Btu/hr ft^2 (630 watts/m^2), the amount of heat incident to the aperture of the collector becomes

$$S = 200 \left[\frac{4.5}{12} \times 8\right] = 600 \text{ Btu/(hr)(CPC-8 ft long)}$$

$$= 1891 \text{ watts/(m)}^2 \text{(CPC-2.44 m long)}$$

Of this value, 0.536 x 600 = 322 Btu/hr (1015 watts/m^2) finds its way to the receiver, some of which is dissipated as a heat loss and the rest is directed as useful heat to raise the temperature of the fluid flowing through the receiver. Therefore, for the conditions of this problem,

$$Q_{useful} + Q_{losses} = 322 \text{ Btu/(hr)(CPC-8 ft long)}$$

$$= 1015 \text{ watts/(m}^2\text{)(CPC-2.44 m long)}$$

1. Physical Properties of Dowtherm A

For the heat-transfer calculations associated with this study, the physical properties of Dowtherm A presented in Table B-1 and Figs. B-1 and B-2 in Appendix B have been used throughout. Values of the physical properties of Dowtherm A have been extracted from this table and the two figures and have been used to develop the following analytical expressions for the indicated properties:

Density

$$\rho = 67.96 - 0.0263t - 6.0 \times 10^{-6}t^2 - 3.485 \times 10^{-14}t^{4.8}$$

where ρ = density, lb/ft^3

and t = temperature, °F

Specific Heat

$$c_p = 0.3508 + 3.717 \times 10^{-4}t$$

where c_p = specific heat, Btu/lb °F

and t = temperature, °F

Viscosity

$$\mu = 0.01142e^{3000/T}$$

where μ = viscosity, centipoises

and T = absolute temperature, °R

Thermal Conductivity

$$k = 0.0843 - 3.95 \times 10^{-5} t$$

where k = thermal conductivity, Btu/hr ft^2 °F/ft

and t = temperature, °F

In addition, for the calculation of the Grashof group, $D^3 \rho^2 g\beta\Delta t/\mu^2$, the dimensional modulus, $\rho^2 g\beta/\mu^2$, has been calculated for Dowtherm A for a number of temperatures and correlated to the expression,

$$\frac{\rho^2 g\beta}{\mu^2} = 6.657 \times 10^{-3} [t + 100]^{4.13}$$

where $\rho^2 g\beta/\mu^2$ = dimensional group, 1/ft^3 °F

and, t = temperature, °F

These relationships find utility in the programming of computational calculations and have been used as such in the subsequent heat-transfer analysis of the CPC receiver.

2. Establishment of Mean Temperatures of Fluid Within Annulus and Tube Sides

For this example, the case concerned with the heating of Dowtherm A from 200° to 400°F (93.3° to 204.4°C) is considered. The heat losses associated with this case are calculated in order to establish the efficiency of the collector as $\eta = Q_{useful}/S$. Assume efficiency of collector as $\eta = 0.24$ (this value is to be checked at the conclusion of calculations). Therefore,

$$Q_{useful} = 0.24 \times 600 = 144 \text{ Btu/(hr)(CPC)}$$

$$= 454 \text{ watts/(m}^2\text{)(CPC)}$$

Since, $Q = mc_p(T_1 - t_1)$, it follows that

$$mc_p = 144/(400 - 200) = 0.72 \text{ Btu/(hr)(°F)} = 0.412 \text{ watts/°C}$$

The parameters k_1 and k_2 for Equations (10) and (15) are now considered.

$$Q = k_1 L,\ 144 = k_1(8),\ k_1 = 18.00 \text{ Btu/(hr) (ft of receiver)}$$

$$= 0.173 \text{ watts/(cm of receiver)}$$

Assume k_2 = 0.32 Btu/(hr)(°F)(ft of receiver) = 0.005534 watts/(°C)(cm of receiver)(to be checked). Therefore, the temperature at the reversal point of the receiver, according to Equations (10) and (15) becomes

$$t_L = 200 + \frac{18.00}{0.72}\left[1 + \frac{0.32}{2(0.72)}(16 - 8)\right] 8 = 756°\text{F } (402°\text{C})$$

$$T_L = 400 + \frac{18.00}{0.72}\ \frac{0.32}{0.72}\ (16 - 8)\ \frac{8}{2} = 756°\text{F } (402°\text{C})$$

These are check calculations, since t_L must be equal to T_L. Thus, the temperature profiles become as indicated in Fig. 7. The temperature of 756°F (402°C), although realistic from calculational considerations, is not justified in practice since the normal boiling point of Dowtherm A is 495°F (257°C). A pressurized system could be used, but its use is not recommended above 750°F (399°C). The value of 756°F (402°C) at the end of the receiver produces average temperatures of T_{avg} = 578°F (303°C) in the annulus and t_{avg} = 478°F (248°C) inside the tube.

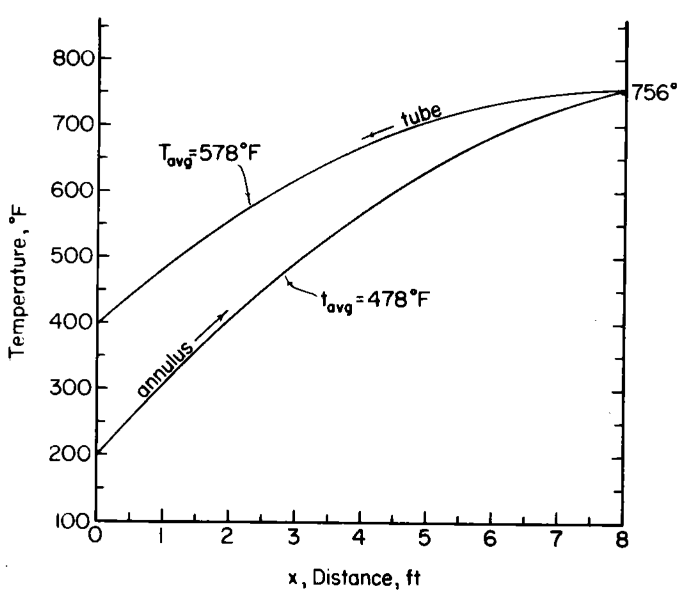


Fig. 7. Temperature Distribution of Dowtherm Flowing Through Annular and Tube-side Sections of CPC Receiver.

3. Basic Areas Associated With Receiver

The outer tube of the concentric receiver is a 3/8-in. (0.9525-cm) copper tube onto which vertical extended surfaces are provided for complete interception of the radiant energy beamed at the bottom of the collector. Also, the concentric inner tube has been selected to be 1/4-in. (0.635-cm) glass tubing. This size of tubing has been selected in order to produce cross-sectional areas for the inside tube and the annular space that are about the same. Assigning 0.035 in. (0.0889 cm) of thickness for each of the walls of the copper tubing and glass tubing, the basic dimensions of the receiver become those presented in Fig. 8. These dimensions account for inside diameters of $d_i = 0.375 - 2(0.035) = 0.305$ in. (0.775 cm) for the copper tube and $d_i = 0.250 - 2(0.035) = 0.180$ in. (0.457 cm) for the glass tube. These dimensions can be summarized as follows:

Copper Tube	Glass Tube
$d_o = 0.375$ in. (0.9525 cm)	$d_o = 0.250$ in. (0.635 cm)
$d_i = 0.305$ in. (0.775 cm)	$d_i = 0.180$ in. (0.457 cm)

Using these values, the following areas are calculated for use in the subsequent parts of this problem.

External Receiver Area:

$$A = \pi \frac{3/8}{12} (8) + 2 \left[2 \frac{3/16}{12} (8)\right] = 1.285 \text{ ft}^2 \ (1.193 \text{ m}^2)$$

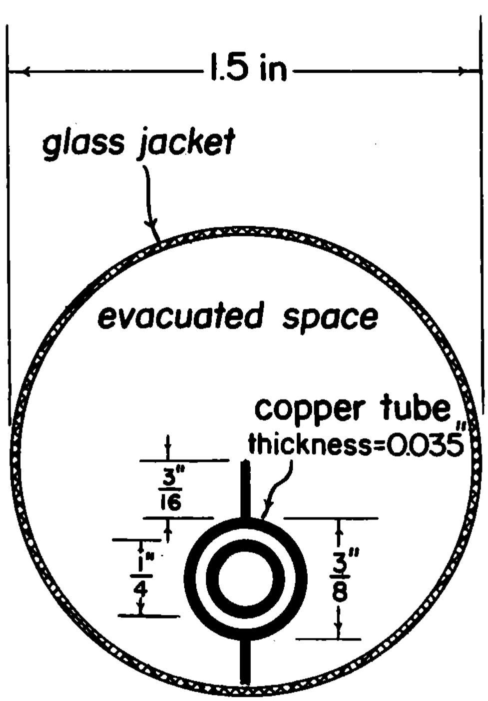


Fig. 8. Cross-sectional View of Glass Jacket and Concentric Receiver Facility Provided With External Extended Fin Surfaces.

Peripheral Areas (Inside Tube)

$$dA_i = \pi\left[\frac{0.25 - 2 \times 0.035}{12}\right]dx = 0.0471 \text{ dx ft}^2$$

$$dA_o = \pi\left[\frac{0.25}{12}\right]dx = 0.0654 \text{ dx ft}^2$$

$$dA_m = \frac{1}{2}\left[0.0471 \text{ dx} + 0.0654 \text{ dx}\right] = 0.05625 \text{ dx ft}^2$$

Cross-sectional Areas

Inside Tube: $A_c = \frac{\pi}{4}\left(\frac{0.180}{12}\right)^2 = 0.0001767 \text{ ft}^2 \ (0.164 \text{ cm}^2)$

Annulus: $A_c = \frac{\pi}{4}\left[\left(\frac{0.305}{12}\right)^2 - \left(\frac{0.250}{12}\right)^2\right] = 0.0001665 \text{ ft}^2$
$= (0.155 \text{ cm}^2)$

4. Inside Heat-transfer Coefficient (Inside Tube)

For the calculation of the inside heat-transfer coefficient, h_i, the Sieder-Tate relationship[2] is used

$$\frac{h_i D}{k} = 1.86 \left(\frac{DG}{\mu}\right)^{1/3} \left(\frac{c_p \mu}{k}\right)^{1/3} \left(\frac{D}{L}\right)^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

For mc_p = 0.72 Btu/(hr)(°F), it follows that m = 0.72/0.547 = 1.316 lb/hr = 0.1987 g/sec for c_p 528°F = 0.547 Btu/(lb)(°F). It then follows that, G = 1.316/0.0001767 = 7448 lb/(hr)(ft^2) = 2.069 lb/(sec)(ft^2) = 1.010 g/(sec)(cm^2).

Physical Properties of Dowtherm A at 578°F:

c_p = 0.566 Btu/(lb)(°F)

μ = 0.2055 x 0.000672 = 0.0001381 lb/ft sec

k = 0.0615 Btu/(hr)(ft)(°F)

Reynolds Number:

$$\frac{DG}{\mu} = \frac{(0.180/12)(2.069)}{0.0001381} = 224.7 \text{ (viscous flow)}$$

Prandtl Number:

$$\frac{c_p\mu}{k} = \frac{(0.566)(0.0001381)(3600)}{0.0615} = 4.58$$

$$\frac{h_i D}{k} = 1.86 \left(224.7\right)^{1/3} \left(4.58\right)^{1/3} \left(\frac{0.180/12}{8}\right)^{1/3} \left(\frac{0.2055}{0.252}\right)^{0.14}$$

$$= 2.25$$

where μ_s = 0.252 cp at 510°F (assumed temperature)

$$h_i = 2.25 \frac{0.0615}{0.180/12} = 9.22 \text{ Btu/hr ft}^2 \text{ °F}$$

5. Outside Heat-transfer Coefficient (Inside Tube)

For the calculation of the outside heat-transfer coefficient, the relationship of Chen, Hawkins and Solberg for annular flow in the laminar range[2] is applied;

$$\frac{h_o D_e}{k} = 1.02 \left(\frac{D_e G}{\mu}\right)^{0.45} \left(\frac{c_p\mu}{k}\right)^{0.5} \left(\frac{\mu}{\mu_s}\right)^{0.14} \left(\frac{D_e}{L}\right)^{0.4} \left(\frac{D_2}{D_1}\right)^{0.8} \left(\frac{D_e^3\rho^2 g\beta\Delta t}{\mu^2}\right)^{0.05}$$

where D_e = (0.305 - 0.250)/12 = 0.00458 ft (0.1396 cm), and $G = \frac{1.316}{0.0001665}$ = 7904 lb/(hr)(ft^2) = 2.196 lb/(sec)(ft^2) = 1.072 g/(sec)(cm^2).

Physical Properties of Dowtherm at 478°F

c_p = 0.528 Btu/(lb)(°F)

μ = 0.280 x 0.000672 = 0.0001879 lb/(ft)(sec)

k = 0.0654 Btu/(hr)(ft)(°F)

Reynolds Number:

$$\frac{D_e G}{\mu} = \frac{(0.00458)(2.196)}{0.0001879} = 53.5 \text{ (viscous flow)}$$

Prandtl Number:

$$\frac{c_p\mu}{k} = \frac{(0.528)(0.0001879)(3600)}{0.0654} = 5.46$$

The influence of the Grashof Group, $Gr = D^3\rho^2 g\beta\Delta t/\mu^2$, on the heat-transfer coefficient cannot be too significant because of the small value of the exponent associated with it. Nonetheless, in these calculations, its contribution is being included by utilizing, at 518°F, the relationship

$$\frac{\rho^2 g\beta}{\mu^2} = 6.657 \times 10^{-3}\ [478 + 100]^{4.13} = 1.698 \times 10^{9}/\text{ft}^3\ °\text{F}$$

Therefore, $Gr = (0.00458)^3(1.698 \times 10^9)(540 - 518) = 3589$, where t = 540°F (282°C) is assumed to be the receiver wall temperature. Thus, the Nusselt group for the annular flow of Dowtherm A at an average temperature of 478°F (248°C) becomes

$$\frac{h_o D_e}{k} = 1.02(53.5)^{0.45}(5.46)^{0.50}\left(\frac{0.280}{0.250}\right)^{0.14}\left(\frac{0.00458}{8}\right)^{0.4}\left(\frac{0.305}{0.250}\right)^{0.8}$$

$$(3589)^{0.05} = 1.02(5.995)(2.337)(1.016)(0.05048)(1.1724)$$

$$(1.5057) = 1.294$$

Therefore,

$$h_o = 1.294\ \frac{0.0654}{0.00458} = 18.48\ \text{Btu/(hr)(ft}^2\text{)(°F)}$$

$$= 0.1048\ \text{watts/(cm}^2\text{)(°C)}$$

With the calculation $h_i = 9.22$ and $h_o = 18.48$, it now becomes possible to check the assumed value $k_2 = 0.32$ Btu/(hr)(°F)(ft of receiver) using for glass, k = 0.61 Btu/hr ft °F.

$$\frac{1}{UdA} = \frac{1}{h_i dA_i} + \frac{\ell}{kdA_m} + \frac{1}{h_o dA_i} = \frac{1}{(9.22)(0.0471\ dx)} + \frac{0.035/12}{(0.61)(0.05625\ dx)}$$

$$+ \frac{1}{(18.48)(0.0654\ dx)} = \frac{1}{0.4343\ dx} + \frac{0.002917}{0.03431\ dx} + \frac{1}{1.2086\ dx}$$

$$= \frac{2.3026}{dx} + \frac{0.0850}{dx} + \frac{0.8274}{dx} = \frac{3.2150}{dx}$$

Therefore, $k_2 = 1/3.2150 = 0.311$ versus $k_2 = 0.32$ (assumed).

Although an adjustment for an assumed value of $k_2 = 0.311$ is justified, it will not be attempted on these calculations, since the computerized program to be adopted for them will bring these values within a prescribed consistency agreement.

6. Inside Heat-transfer Coefficient (Outside Tube)

In order to establish an average temperature for the surface of the receiver, it is necessary to calculate the film coefficient for Dowtherm A existing on the inside of this receiver. This is being carried out by assuming that the Sieder-Tate relationship for this inside heat-transfer coefficient applies on the assumption that the inside glass tube is absent. Thus,

$$\frac{h_i' D}{k} = 1.86 \left(\frac{DG}{\mu}\right)^{1/3} \left(\frac{c_p \mu}{k}\right)^{1/3} \left(\frac{D}{L}\right)^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

In this case, D = 0.375 - 2(0.035) = 0.305 in. (0.0254 ft),

$$G = \frac{1.316}{0.0001665} = 7904 \text{ lb/(hr)(ft}^2) = 2.196 \text{ lb/(sec)(ft}^2)$$

Physical Properties of Dowtherm A at 478°F (248°C)

$$c_p = 0.528 \text{ Btu/(lb)(°F)}$$

$$\mu = 0.280 \times 0.000672 = 0.0001879 \text{ lb/(ft)(sec)}$$

$$k = 0.0654 \text{ Btu/(hr)(ft)(°F)}$$

Reynolds Number:

$$\frac{DG}{\mu} = \frac{(0.0254)(2.196)}{0.0001879} = 297$$

Prandtl Number:

$$\frac{c_p \mu}{k} = \frac{(0.528)(0.0001879)(3600)}{(0.0654)} = 5.46$$

Therefore,

$$\frac{h_i' D}{k} = 1.86(297)^{1/3}(5.46)^{1/3}\left(\frac{0.0254}{8}\right)^{1/3}\left(\frac{0.280}{0.256}\right)^{0.14}$$

where μ = 0.256 cp at 505°F (assumed receiver temperature). Thus,

$$\frac{h'_i D}{k} = 1.86(6.67)(1.761)(0.1470)(1.013) = 3.254$$

and

$$h'_i = 3.254 \frac{0.0654}{0.0254} = 8.38 \text{ Btu/(hr)(ft}^2\text{)(°F)}$$
$$= 0.0475 \text{ watts/(cm}^2\text{)(°C)}$$

These conditions produce the following Δt across the film

$$Q = h'_i A \Delta t$$
$$144 = 8.38\ \pi \frac{0.305}{12} \times 8 \times \Delta t = 8.38(0.639)\Delta t,$$
$$\Delta t = 27°\text{F}, (15°\text{C})$$

Therefore, the average temperature of the receiver becomes

$$t_r = 478 + 27 = 505°\text{F} \ (T_r = 965°\text{R})$$

7. General Remarks on Heat Losses from Receiver to External Surroundings

The heated receiver, surrounded by the evacuated space, is losing heat almost entirely by radiation. The heat loss by convection is nil and that by conduction at the ends can be kept to a minimum. In this case, only heat losses by radiation are considered. The heat radiated from the receiver to the glass jacket will cause the temperature of the glass to rise and thus effect a net heat loss from the exterior of the glass jacket to the surroundings by the combined mechanisms of convection and radiation.

The involvement of heat transfer associated with this configuration is diagrammatically presented in Fig. 9, where

$$Q_{net(r \rightleftarrows g)} = Q_{conv} + Q_{rad(g \rightleftarrows sky)}$$

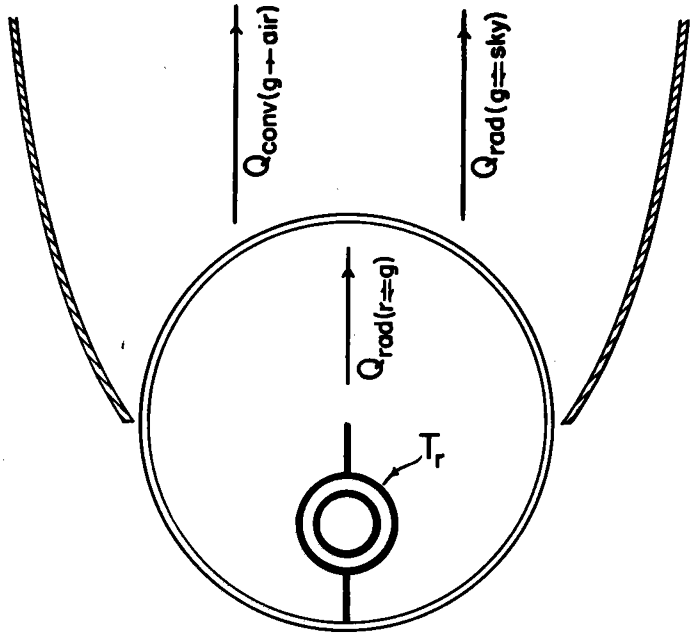


Fig. 9. Heat Losses and Their Transfer Mechanisms from Evacuated Glass-jacketed CPC-receiver Facility.

These quantities are interrelated and will be satisfied only if the proper temperature of the glass jacket is established.

8. <u>Radiant Heat Exchange Between Receiver and Glass Jacket</u>

Since the receiver and glass jacket constitute an enclosure, the standard radiant heat exchange relationship

$$Q_{(r \rightleftarrows g)} = \sigma A_r \mathfrak{F}_{rg} [T_r^4 - T_g^4] \tag{17}$$

applies. In this relationship, T_r and T_g are the absolute temperatures of the receiver and glass, respectively, A_r represents the area of the receiver surface and $\mathfrak{F}_{rg}$ is the configuration factor of the system that takes into account the geometrical orientation of the two surfaces and their deviation from ideal black body conditions. The definition of this factor considers the geometrical orientation and the emittance of these surfaces as follows:

$$\frac{1}{A_r \mathfrak{F}_{rg}} = \frac{1}{A_r F_{rg}} + \frac{1}{A_r}\left(\frac{1}{\varepsilon_r} - 1\right) + \frac{1}{A_g}\left(\frac{1}{\varepsilon_g} - 1\right) \tag{18}$$

where

A_r = receiver surface, $\pi\left(\frac{3/8}{12}\right)8 + 2\left[2\left(\frac{3/16}{12}\right)8\right] = 1.285\ \mathrm{ft}^2 (1194\ \mathrm{cm}^2)$

A_g = glass-jacket surface, $\pi\left[1.5 - 2\left(\frac{2.0}{25.4}\right)\right]\frac{8}{12} = 2.81\ \mathrm{ft}^2 (2611\ \mathrm{cm}^2)$

(where glass thickness is 2.0 mm)

ε_g = 0.94, for glass[3]

ε_r = 0.11, selective surface CuO on Al[4]

and

F_{rg} = 1.00 (the receiver sees all of the glass jacket)

Consequently,

$$\frac{1}{A_r \mathcal{F}_{rg}} = \frac{1}{1.285 \times 1.00} + \frac{1}{1.285}\left(\frac{1}{0.11} - 1\right) + \frac{1}{2.81}\left(\frac{1}{0.94} - 1\right) = 7.097$$

or

$$A_r \mathcal{F}_{rg} = \frac{1}{7.097} = 0.1409$$

and, therefore,

$$Q_{net(r \rightleftarrows g)} = 0.1714 \times 0.1409 \left[\left(\frac{T_r}{100}\right)^4 - \left(\frac{T_g}{100}\right)^4\right]$$

$$= 0.02415 \left[\left(\frac{T_r}{100}\right)^4 - \left(\frac{T_g}{100}\right)^4\right]$$

9. Heat Exchange Between Glass Jacket and Surroundings

The simultaneous heat loss by convection and radiation from the glass jacket to the surroundings can be established once the temperature of the glass jacket is specified. These losses become

$$Q_{conv} + Q_{rad} = h_g A_g [T_g - T_{surr}] + \sigma A_g \mathcal{F}_{g(sky)} [T_g^4 - T_{sky}^4] \qquad (19)$$

The radiative heat loss from the surface of the glass jacket to the surroundings requires that the approximate sky temperature be established. This is obtained from the relationship,

$$[T, °K]_{sky} = 0.0552\ [T, °K]_{air}^{1.5} \qquad (20)$$

suggested by Duffie and Beckman.[4] For an ambient temperature of 100°F (T_{surr} = 560°R = 311°K)

$$[T, °K]_{sky} = 0.0552(311)^{1.5} = 0.0552(5488) = 303°K = 545°R$$

The product $A_g \mathfrak{F}_{g(sky)}$ is calculated as follows:

$$\frac{1}{A_g \mathfrak{F}_{g(sky)}} = \frac{1}{A_g F_{g(sky)}} + \frac{1}{A_g}\left(\frac{1}{\varepsilon_g} - 1\right) + \cancel{\frac{1}{A_{sky}}\left(\frac{1}{\varepsilon_{sky}} - 1\right)} A_{sky} >> A_g$$

Since $A_g = \frac{1.5}{12} \times 8 = 3.14 \text{ ft}^2$, and assuming $F_{g(sky)} = 1.00$, it follows that

$$\frac{1}{A_g \mathfrak{F}_{g(sky)}} = \frac{1}{3.14 \times 1.00} + \frac{1}{3.14}\left(\frac{1}{0.94} - 1\right) = \frac{1}{3.14}\left(1 + \frac{1}{0.94} - 1\right)$$

$$= 0.339$$

or that $A_g \mathfrak{F}_{g(sky)} = 2.95$.

Since half of the tube sees the sky, it follows that

$$A_g \mathfrak{F}_{g(sky)} = \frac{1}{2}(2.95) = 1.475$$

The overall heat balance on the glass-tube jacket thus becomes,

$$\sigma A_r \mathfrak{F}_{rg}[T_r^4 - T_g^4] = h_g A_g [T_g - T_{surr}] + \sigma A_g \mathfrak{F}_{g(sky)}[T_g^4 - T_{sky}^4] \quad (21)$$

Assuming $h_g \approx 1.00$, it follows that

$$(0.1714)(0.1409)\left[\left(\frac{965}{100}\right)^4 - \left(\frac{T_g}{100}\right)^4\right] = (1.00)(3.14)[T_g - 560]$$

$$+ (0.1714)(1.475)\left[\left(\frac{T_g}{100}\right)^4 - \left(\frac{545}{100}\right)^4\right]$$

$$\overbrace{0.02415\left[8672 - \left(\frac{T_g}{100}\right)^4\right]}^{Q_1} = \overbrace{3.14[T_g - 560]}^{Q_2} + \overbrace{0.2528\left[\left(\frac{T_g}{100}\right)^4 - 882\right]}^{Q_3}$$

A trial-and-error solution for T_g produces the following:

T_g, °R	Q_1	Q_2	Q_3	$Q_2 + Q_3$	$\Delta Q = Q_1 - (Q_2 + Q_3)$
585	181	78	73	151	30
590	180	94	83	177	3
591	180	97	85	182	-2

Therefore, T_g = 590°R (130°F). Since a heat loss of Q_{loss} = 179 Btu/hr (52.42 watts), it follows that

$$Q_{useful} = 332 - 179 = 143 \text{ Btu/hr (41.87 watts)}$$

to yield a collector efficiency, $\eta = 143/600 = 0.238$ versus an assumed value of 0.24. Therefore, $\eta = 0.238$ for $\Delta t = t_{receiver} - t_{ambient} = 505 - 100 =$ 405°F.

There yet remains to verify that $h_g \approx 1.0$ Btu/hr ft^2 °F. Use

$$Nu = 0.53\ [Gr \cdot Pr]^{1/4} \qquad 10^4 < Gr \cdot Pr < 10^9$$

obtained from Welty, Wicks, and Wilson[5].

For air, at $t_f = \frac{1}{2}(100 + 130) = 115$°F, $\rho^2 g\beta/\mu^2 = 1.60 \times 10^6/(°F)(ft^3)$ and Pr = 0.701. Thus,

$$Gr \cdot Pr = \frac{1.5}{12}^3 (1.60 \times 10^6)(130 - 100)(0.701) = 43{,}800$$

then it follows that,

$$Nu = 0.53[43,800]^{1/4} = 7.67$$

and

$$h_g = 7.67 \frac{0.0796}{1.5/12} = 4.88 \text{ Btu/hr ft}^2 \text{ °F}$$

$$= 0.0277 \text{ watts/(cm}^2\text{)(°C)}$$

versus an assumed value of 1.00 Btu/hr ft^2 °F. This calculated value represents a relatively high coefficient for natural convection. Being that as it may, the use of $h_g = 1.00$ is probably justifiable, when considering that half of the glass tube area is subjected to natural convection, thus making the product $h_gA_g = (4.88)(3.14/2) = 7.66$ Btu/hr °F. Furthermore, since a cover-glass is present at the aperture of the collector, the air will not circulate as freely within the collector cavity and, therefore, h_g will be significantly lower. Assuming the actual value of h_g to be about one-half that calculated, this coefficient reduces to 2.44 Btu/hr ft^2 °F and with its proper area involvement, the product h_gA_g becomes 2.44 (3.14/2) = 3.83 Btu/hr °F compared to 3.14 Btu/hr °F used. Under the present conditions, it is difficult to resolve this difference in an exacting manner and, therefore, the assumed value of $h_g = 1.00$ Btu/hr ft^2 °F will be assumed to be close to the actual performance value.

IV. SUMMARY OF CALCULATIONS

The context of the calculational procedure was programmed and used to produce a number of cases in which the fluid discharge temperature, T_1, was varied from 150 to 600°F in 50° increments. Whenever conditions permitted, the fluid temperature entering the receiver was made equal to 100°F; however, with increasing discharge temperatures, this was not always possible. The results obtained from this computerized program are presented graphically in Figs. 10-14.

In Fig. 10, relationships are presented of collector efficiency versus fluid-inlet temperature t_1, for parameters of T_1, constant discharge fluid temperature. All of these temperature parameters, T_1, terminate on the efficiency versus receiver temperature curve which represents the condition arising for infinite fluid flow through the receiver facility. Under these flow conditons, the inlet and discharge temperatures of the fluid become the same. The relationships of Fig. 10 bear out the logical conclusion that with increasing temperature demands on the receiver, the efficiency of the collector decreases. Also, with such temperature requirements, the extent of variation between inlet and outlet fluid temperatures becomes more restrictive. Thus, whereas for a discharge temperature of T_1 = 300°F, the inlet temperature can vary 200°F, from 100°F to 300°F; this temperature variation decreases to 90°F, from 460° to 550°F for a fluid discharge temperature of T_1 = 550°F. In this context, it should be pointed out that the initial inlet fluid temperatures associated with a discharge temperature, T_1, are limiting and that these values should not be extrapolated below their indicated values. Thus, for T_1 = 450°F, a value of t_1 = 285°F corresponds to a realistic limiting temperature condition which represents the lowest temperature the inlet fluid should possess if it is to reach a discharge temperature of 450°F, corresponding to slow fluid flow conditions within the receiver.

Figure 11 is substantially a cross-plot representation of Fig. 10 and relates collector efficiency with fluid discharge temperature for parameters of constant inlet fluid temperature. In this figure, the parametric curves are initiated for conditions corresponding to infinite flow rates and terminate at the limiting discharge temperature represented by the end of each of these parametric relationships.

Figure 12 presents the glass-jacket temperature as a function of fluid inlet temperature for parameters of constant discharge temperature. Except for the lower temperature levels of $T_1 < 300$°F, these relationships exhibit a significant rise with increasing temperature differences, $T_1 - t_1$. For infinite fluid flow rates, the glass-jacket temperature corresponding to each parametric discharge temperature relationship approaches the lowest possible value to generate from all such values, the curve describing the dependence of t_g on t_1. For this case, $t_1 = T_1 \approx t_{rec}$. The relationships of Fig. 12 indicate that under extreme conditions, the glass-jacket can reach a temperature of 150°F, but for conditions of more common encounter, a temperature of 120-130°F appears to be a suitable compromise.

Auxiliary temperature information associated with the receiver surface and maximum fluid temperature within the receiver is presented in Figs. 13 and 14. The dependence of the average receiver surface temperature on t_1, the fluid inlet temperature, is presented in Fig. 13 for constant fluid discharge temperatures. The parametric relationships of this figure terminate at infinite fluid flow conditions when $t_1 = T_1$ to generate the linear dependence of t_r, the receiver surface temperature, with t_1, the fluid inlet temperature. Figure 14 shows the excessively high temperatures that prevail at the end of the receiver where the fluid reverses direction and enters the inner tube of the receiver. These temperatures become unrealistic above 495°F, the normal boiling point of Dowtherm A. Nonetheless, these calculations presuppose no phase change and the applicability of the physical property relationships of Dowtherm A into this high temperature region. This figure shows that when the temperature rise from t_1 to T_1 is modest, usually below 50°F, the corresponding value of t_L does not become unreasonably high. However, larger differences of $T_1 - t_1$ produce excessively high fluid temperatures t_L that reflect correspondingly on high receiver temperatures and consequently greater heat losses. Therefore, it could prove more expeditious if the temperature rise within a single CPC unit were limited to temperature rises of $T_1 - t_1 \leq 40$°F, and that such units be connected in series if the heat duty demanded overall temperature increases above this value.

V. RECOMMENDATIONS

These calculations indicate that the operation of a countercurrently operating receiver facility can become critical if large temperature differences are imposed on the flowing fluid. This condition can become prohibitive if, in addition, the outlet temperature of the fluid is expected to be high. However, if reasonably small temperature differences are imposed across the flowing fluid, this condition can be avoided to permit the efficiency to assume a respectable value, $\eta > 0.40$. Temperature differences of less than 250°F between receiver and air must prevail to maintain the efficiency within this level of expectancy. If the temperature of the fluid leaving is to rise above 350°F, then efficiencies of $\eta < 0.40$ must be tolerated, regardless of the temperature rise imposed on the flowing fluid.

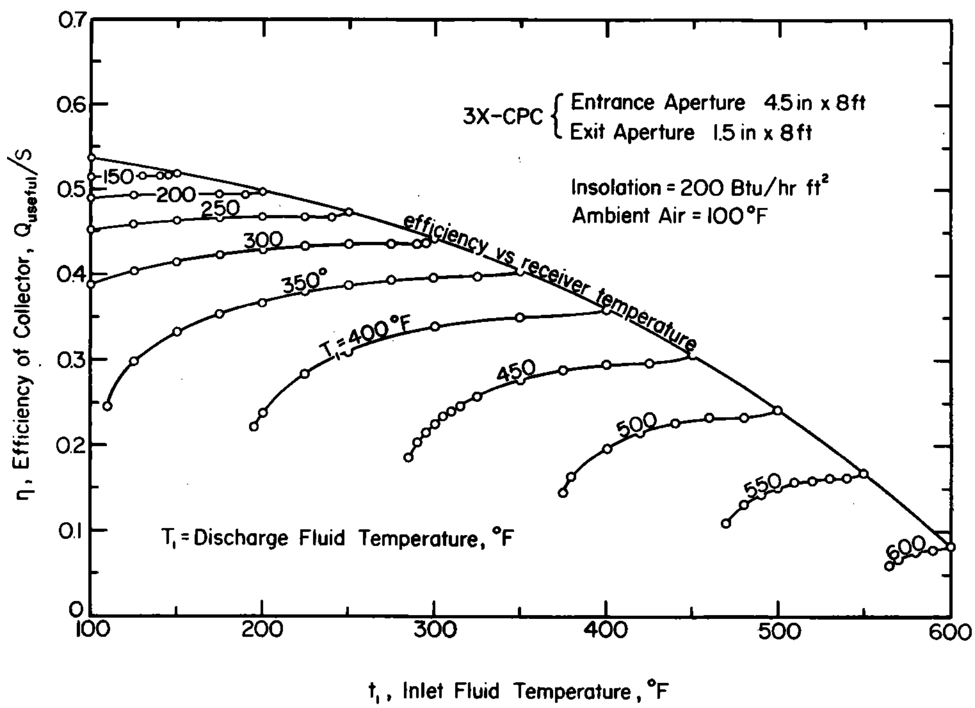


Fig. 10. Efficiency Dependence on Inlet and Discharge Temperatures of Fluid Flowing Through Concentric Tube Receiver Facility.

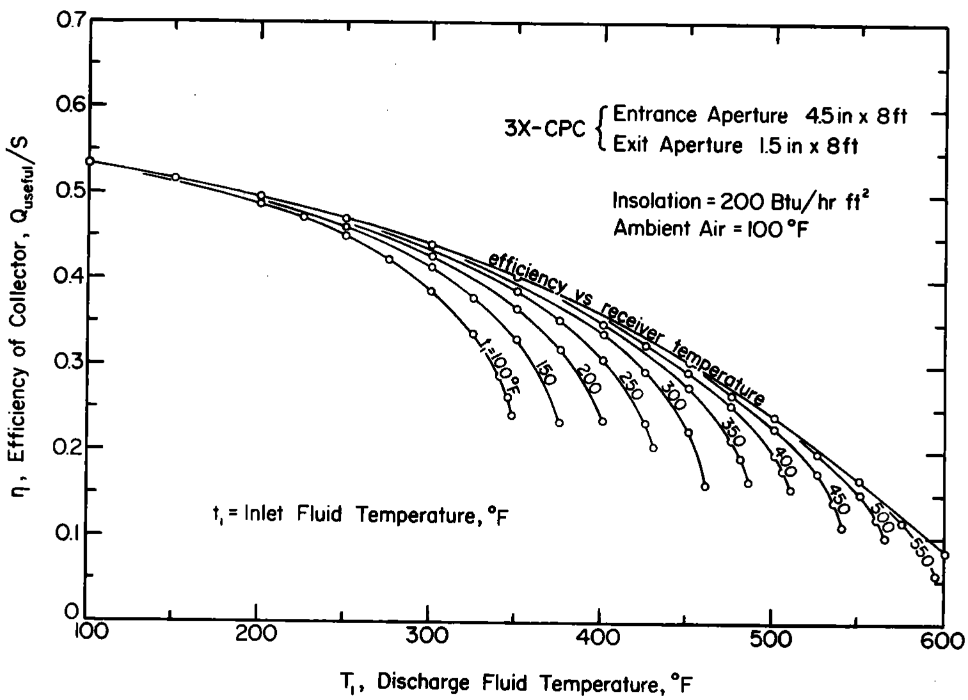


Fig. 11. Efficiency Dependence on Discharge and Inlet Temperatures of Fluid Flowing Through Concentric Tube Receiver Facility.

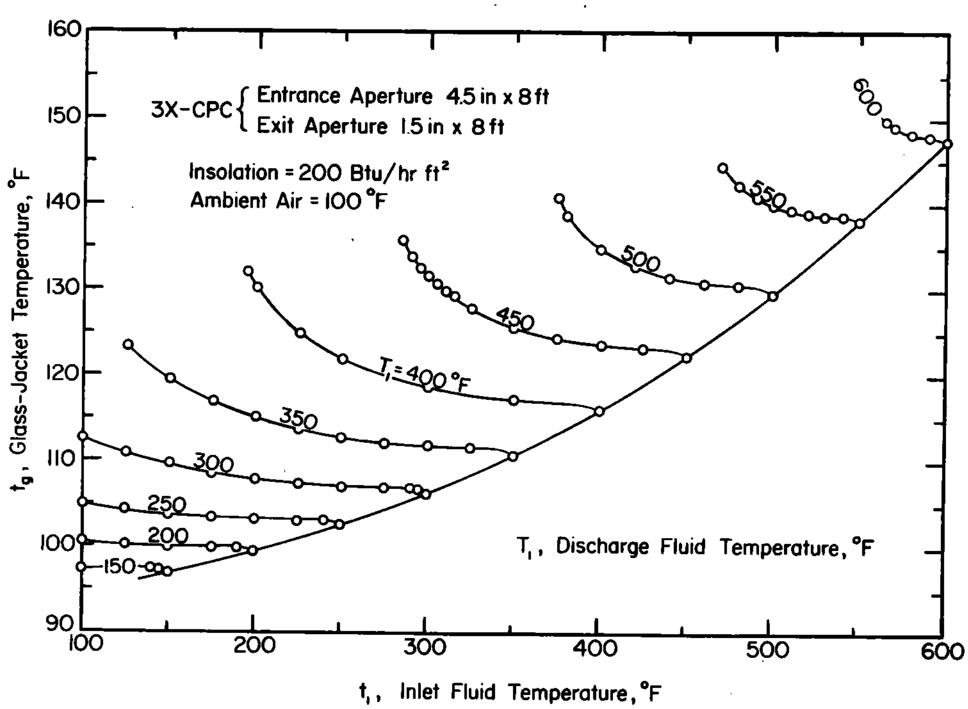


Fig. 12. Dependence of Glass-jacket Temperature on Fluid Temperature Conditions.

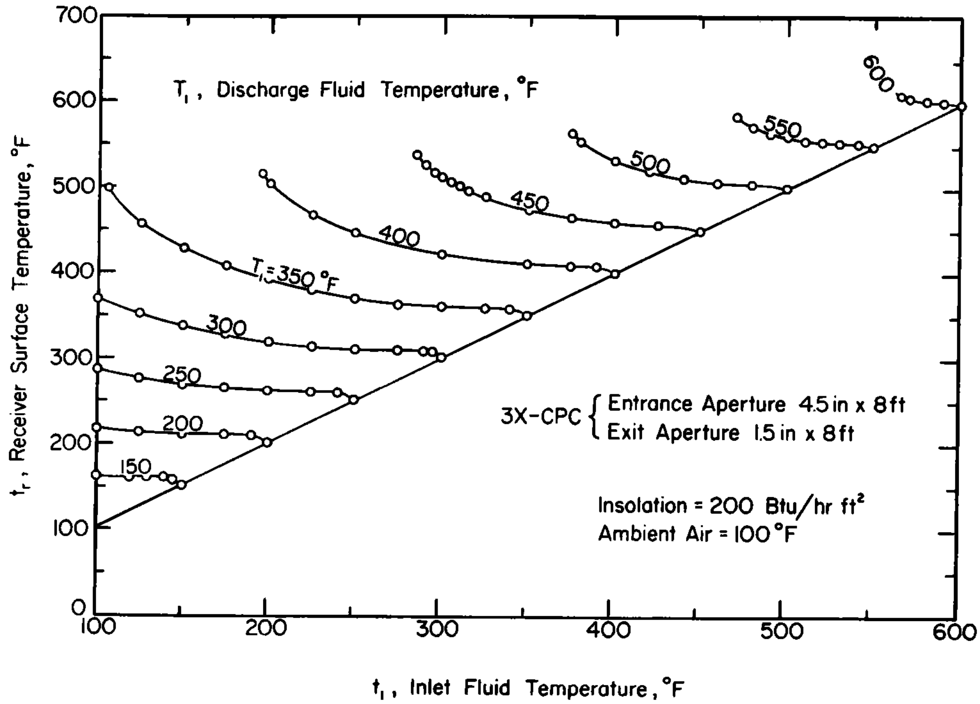


Fig. 13. Dependence of Receiver Surface Temperature on Temperatures of Fluid Flowing Through Receiver.

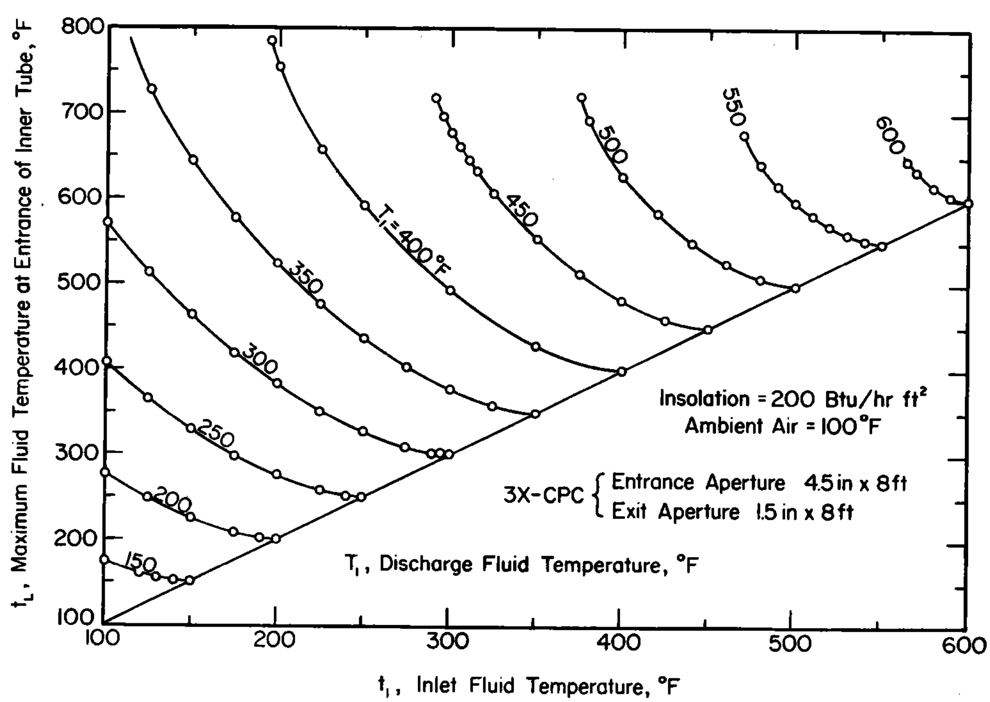


Fig. 14. Maximum Fluid Temperature at Entrance of Inner Tube of Receiver.

APPENDIX A

Mathematical Treatment for Development of Equations (10) and (15)

Section IA:

Integration of Equation (9a)

$$mc_p dt = \left[k_1 + \frac{k_1 k_2}{mc_p}(L - x)\right] dx \qquad (9a)$$

Let $\alpha = k_1$, and $\beta = \frac{k_1 k_2}{mc_p}$

$$\int_{t=t_1}^{t} mc_p dt = \int_{x=0}^{x} [\alpha + \beta(L - x)]\, dx \qquad (IA-1)$$

$$mc_p(t - t_1) = \alpha x + \beta \int_0^x (L - x)\, dx \qquad (IA-2)$$

Let $z = L - x$; then $dz = -dx$,

$$mc_p(t - t_1) = \alpha x - \beta \int_L^z z dz = \alpha x - \frac{\beta}{2} z^2 \Big|_L^{z=L-x}$$

$$= \alpha x - \frac{\beta}{2}\left[(L - x)^2 - L^2\right] = \alpha x - \frac{\beta}{2}\left[L^2 - 2Lx + x^2 - L^2\right]$$

$$= \alpha x - \frac{\beta}{2}(-2Lx + x^2) = \alpha x + \frac{\beta}{2} x(2L - x)$$

$$= [\alpha + \frac{\beta}{2}(2L - x)]x$$

$$mc_p(t - t_1) = \left[k_1 + \frac{k_1 k_2}{2mc_p}(2L - x)\right] x$$

$$t - t_1 = \left[\frac{k_1}{mc_p} + \frac{k_1 k_2}{2(mc_p)^2}(2L - x)\right] x = \frac{k_1}{mc_p}\left[1 + \frac{k_2}{2mc_p}(2L - x)\right] x$$

$$t = t_1 + \frac{k_1}{mc_p}\left[1 + \frac{k_2}{2mc_p}(2L - x)\right] x \qquad (IA-3)$$

Equation (IA-3) yields at $x = 0$, $t = t_1$, and at $x = L$, it produces

$$t_L = t_1 + \frac{k_1}{mc_p}\left[1 + \frac{k_2}{2mc_p}(2L - L)\right]L$$

or that

$$t_L - t_1 = \frac{k_1}{mc_p}\left[1 + \frac{k_2 L}{2mc_p}\right]L$$

Since, from an overall heat balance, Equation (6)

$$T_1 - t_1 = \frac{k_1 L}{mc_p}$$

it follows that

$$\frac{t_L - t_1}{T_1 - t_1} = 1 + \frac{k_2 L}{2mc_p} \tag{IA-4}$$

<u>Section IB</u>:

Integration of Equation (13)

$$k_1 dx + mc_p dT = mc_p dt \tag{13}$$

$$k_1 \int_0^x dx + mc_p \int_{T_1}^{T} dT = mc_p \int_{t_1}^{t} dt \tag{IB-1}$$

$$k_1 x + mc_p(T - T_1) = mc_p(t - t_1) \tag{IB-2}$$

$$mc_p(T - T_1) - mc_p(t - t_1) = -k_1 x$$

$$mc_p(T - T_1 - t + t_1) = -k_1 x$$

$$(T - t) - (T_1 - t_1) = -\frac{k_1}{mc_p} x$$

$$T - t = (T_1 - t_1) - \frac{k_1}{mc_p} x \qquad \text{(IB-3)}$$

Equation (IB-3) at $x = 0$ becomes $T - t = T_1 - t_1$ and at $x = L$

$$T - t = (T_1 - t_1) - \frac{k_1 L}{mc_p} = (T_1 - t_1) - (T_1 - t_1) = 0$$

<u>Section IC</u>:

Combination of Equations (10) and (14):

$$t = t_1 + \frac{k_1}{mc_p}\left[1 + \frac{k_2}{2mc_p}(2L - x)\right] x \qquad (10)$$

$$T - t = (T_1 - t_1) - \frac{k_1}{mc_p} x \qquad (14)$$

Substitution of Equation (10) into Equation (14) and rearranging yields

$$T = t_1 + \frac{k_1}{mc_p}\left[1 + \frac{k_2}{2mc_p}(2L - x)\right] x + (T_1 - t_1) - \frac{k_1}{mc_p} x \qquad \text{(IC-1)}$$

$$T = t_1 + \frac{k_1 x}{mc_p} + \frac{k_1}{mc_p}\frac{k_2}{mc_p}\frac{x}{2}(2L - x) + T_1 - t_1 - \frac{k_1}{mc_p} x \qquad \text{(IC-2)}$$

$$T = T_1 + \frac{k_1}{mc_p}\frac{k_2}{mc_p}(2L - x)\frac{x}{2} \qquad \text{(IC-3)}$$

Equation (IC-3) at $x = 0$ yields $T = T_1$ and at $x = L$, it produces

$$T_L = T_1 + \frac{k_1}{mc_p}\frac{k_2}{mc_p}\frac{L^2}{2} = T_1 + \frac{k_1 L}{mc_p}\frac{k_2}{mc_p}\frac{L}{2} \tag{IC-4}$$

Since

$$T_1 - t_1 = \frac{k_1 L}{mc_p}$$

it follows that

$$T_L = T_1 + (T_1 - t_1)\frac{k_2}{mc_p}\frac{L}{2}$$

or that

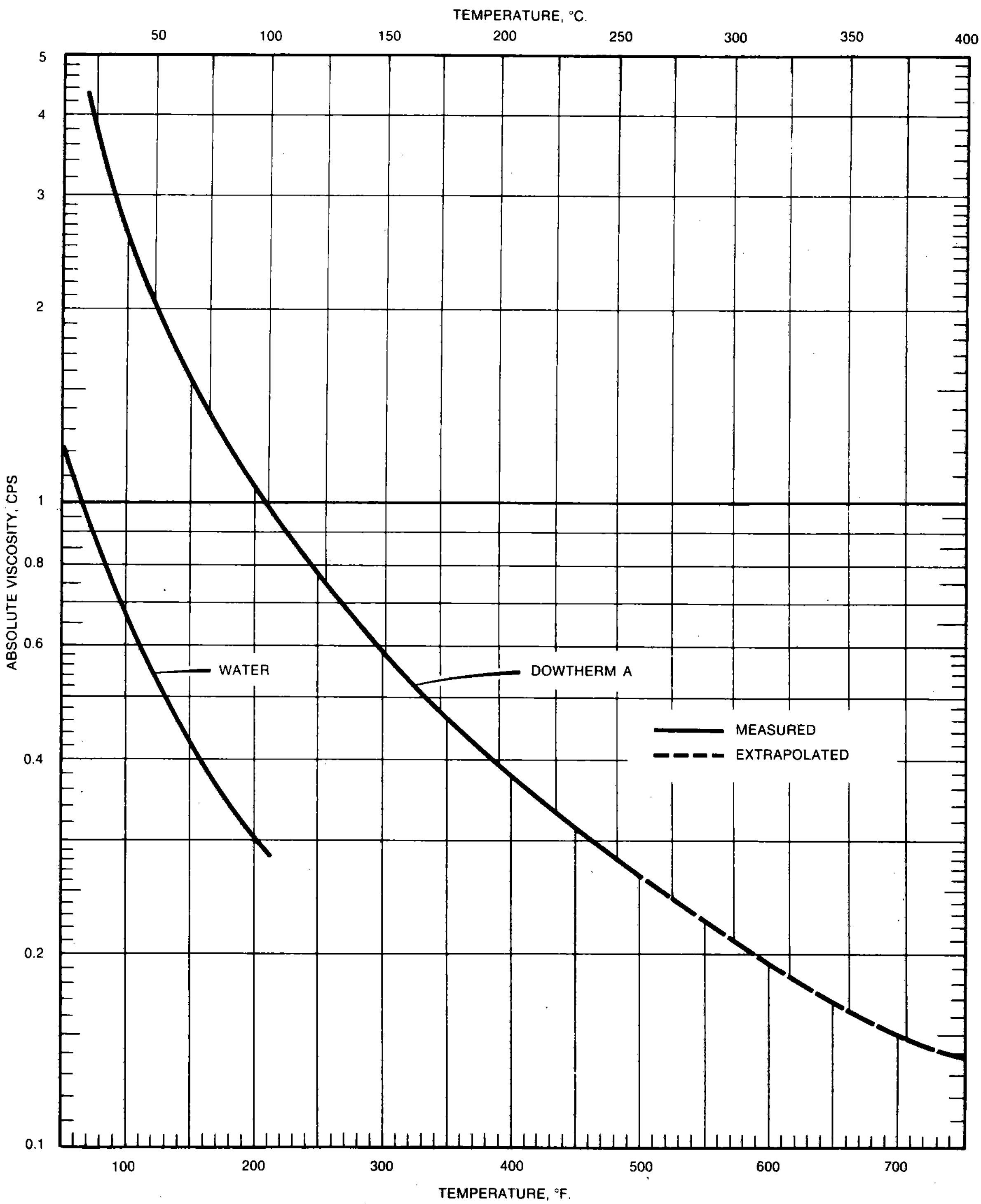
$$\frac{T_L - T_1}{T_1 - t_1} = \frac{k_2}{mc_p}\frac{L}{2} \tag{IC-5}$$

Table B-1. Saturation Properties of Dowtherm A (Metric Units).*

TEMPERATURE		VAPOR PRESSURE		ENTHALPY			SPECIFIC HEAT	DENSITY		SPECIFIC GRAVITY
°F.	°C.	Absolute	Gauge	Liquid	Latent	Vapor	Liquid	Liquid	Vapor	Liquid
		kg/cm²		kcal/kg			cal/(g) (°C)	g/cm³	kg/m³	t/25°C.
53.6	12.0	0.0000		0.0	97.3	97.3	0.371	1.066	0.000	1.069
60	15.6	0.0000		1.3	96.9	98.2	0.374	1.063	0.000	1.066
70	21.1	0.0000		3.4	96.2	99.6	0.377	1.059	0.000	1.062
80	26.7	0.0000		5.5	95.6	101.1	0.381	1.054	0.000	1.057
90	32.2	0.0001		7.7	94.8	102.5	0.385	1.050	0.000	1.053
100	37.8	0.0001		9.8	94.2	104.0	0.388	1.046	0.000	1.049
110	43.3	0.0001		11.9	93.6	105.5	0.392	1.041	0.002	1.044
120	48.9	0.0002		14.2	92.9	107.1	0.396	1.037	0.002	1.040
130	54.4	0.0003		16.4	92.2	108.6	0.400	1.032	0.002	1.035
140	60.0	0.0005		18.6	91.6	110.2	0.403	1.028	0.003	1.031
150	65.6	0.0007		20.8	91.0	111.8	0.407	1.023	0.005	1.026
160	71.1	0.0010		23.1	90.4	113.5	0.411	1.019	0.006	1.022
170	76.7	0.0014		25.4	89.8	115.2	0.414	1.014	0.008	1.017
180	82.2	0.0019		27.7	89.1	116.8	0.418	1.010	0.011	1.013
190	87.8	0.0026		30.1	88.6	118.6	0.422	1.005	0.014	1.008
200	93.3	0.0036		32.4	87.9	120.3	0.426	1.001	0.019	1.003
210	98.9	0.0048		34.8	87.3	122.1	0.429	0.996	0.026	0.999
220	104.4	0.0064		37.2	86.8	123.9	0.433	0.991	0.034	0.994
230	110.0	0.0084		39.6	86.2	125.8	0.437	0.987	0.043	0.990
240	115.6	0.0112		42.1	85.6	127.6	0.440	0.982	0.054	0.985
250	121.1	0.0141		44.5	85.0	129.5	0.444	0.977	0.070	0.980
260	126.7	0.0183		46.9	84.4	131.4	0.448	0.972	0.088	0.975
270	132.2	0.0232		49.4	83.9	133.3	0.451	0.968	0.111	0.971
280	137.8	0.0288		52.0	83.3	135.3	0.455	0.963	0.138	0.966
290	143.3	0.0359		54.5	82.7	137.2	0.459	0.958	0.170	0.961
300	148.9	0.0443		57.1	82.2	139.2	0.463	0.953	0.207	0.956
310	154.4	0.0548		59.7	81.6	141.2	0.466	0.948	0.251	0.951
320	160.0	0.0675		62.3	81.0	143.3	0.470	0.943	0.306	0.946
330	165.6	0.0823		64.9	80.4	145.3	0.474	0.939	0.367	0.941
340	171.1	0.0991		67.5	79.9	147.4	0.477	0.934	0.439	0.936
350	176.7	0.1195		70.2	79.3	149.5	0.481	0.928	0.522	0.931
360	182.2	0.1427		72.9	78.7	151.6	0.485	0.923	0.617	0.926
370	187.8	0.170		75.6	78.2	153.8	0.488	0.918	0.727	0.921
380	193.3	0.201		78.3	77.7	155.9	0.492	0.913	0.852	0.916
390	198.9	0.236		81.1	77.1	158.1	0.496	0.908	0.993	0.911
400	204.4	0.278		83.8	76.4	160.3	0.500	0.903	1.153	0.906
410	210.0	0.324		86.6	75.9	162.5	0.503	0.898	1.334	0.901
420	215.6	0.337		89.4	75.3	164.7	0.507	0.893	1.536	0.895
430	221.1	0.437		92.2	74.7	166.9	0.511	0.887	1.762	0.890
440	226.7	0.504		95.1	74.2	169.2	0.514	0.882	2.015	0.885
450	232.2	0.580		97.9	73.6	171.5	0.518	0.877	2.294	0.879
460	237.8	0.663		100.8	72.9	173.8	0.522	0.871	2.605	0.874
470	243.3	0.757		103.7	72.4	176.1	0.526	0.866	2.947	0.868
480	248.9	0.860		106.7	71.7	178.4	0.529	0.860	3.325	0.863
490	254.4	0.974		109.6	71.1	180.7	0.533	0.855	3.740	0.857
494.8	**257.1**	**1.033**	**0.000**	**111.1**	**70.8**	**181.8**	**0.535**	**0.852**	**3.957**	**0.854**
500	260.0	1.100	0.067	112.6	70.5	183.1	0.537	0.849	4.194	0.851
510	265.6	1.238	0.205	115.6	69.8	185.4	0.541	0.834	4.692	0.846
520	271.1	1.390	0.357	118.6	69.2	187.8	0.545	0.838	5.233	0.840
530	276.7	1.556	0.523	121.7	68.5	190.2	0.549	0.832	5.824	0.834
540	282.2	1.737	0.704	124.7	67.8	192.6	0.554	0.826	6.467	0.828
550	287.8	1.934	0.901	127.8	67.1	194.9	0.558	0.820	7.163	0.823
560	293.3	2,147	1.114	130.9	66.4	197.4	0.562	0.814	7.918	0.816
570	298.9	2.378	1.345	134.1	65.7	199.8	0.567	0.808	8.733	0.811
580	304.4	2.628	1.595	137.3	64.9	202.2	0.571	0.802	9.616	0.804
590	310.0	2.897	1.864	140.4	64.2	204.6	0.575	0.796	10.56	0.798
600	315.6	3.187	2.154	143.7	63.4	207.1	0.579	0.790	11.59	0.792
610	321.1	3.498	2.465	146.9	62.6	209.5	0.582	0.783	12.69	0.785
620	326.7	3.832	2.799	150.1	61.8	211.9	0.586	0.777	13.88	0.779
630	332.2	4.189	3.156	153.4	61.0	214.4	0.589	0.770	15.16	0.772
640	337.8	4.572	3.539	156.7	60.2	216.9	0.593	0.764	16.53	0.766
650	343.3	4.979	3.946	160.0	59.3	219.3	0.596	0.757	18.00	0.759
660	348.9	5.413	4.380	163.3	58.4	221.8	0.599	0.750	19.59	0.752
670	354.4	5.875	4.842	166.7	57.6	224.3	0.602	0.743	21.27	0.745
680	360.0	6.367	5.334	170.1	56.7	226.7	0.605	0.736	23.09	0.738
690	365.6	6.887	5.854	173.4	55.7	229.2	0.608	0.729	25.05	0.731
700	371.1	7.438	6.405	176.8	54.8	231.6	0.611	0.721	27.15	0.723
710	376.7	8.029	6.996	180.3	53.8	234.1	0.615	0.714	29.41	0.716
720	382.2	8.647	7.614	183.7	52.8	236.6	0.619	0.706	31.84	0.708
730	387.8	9.301	8.268	187.2	51.7	239.0	0.623	0.698	34.45	0.700
740	393.3	9.990	8.957	190.8	50.7	241.4	0.628	0.690	37.27	0.692
750	398.9	10.72	9.687	194.3	49.6	243.8	0.633	0.682	40.31	0.684
760	404.4	11.48	10.447	197.9	48.4	246.3	0.640	0.673	43.61	0.675
770	410.0	12.29	11.257	201.5	47.2	248.7	0.647	0.665	47.19	0.667
780	415.6	13.15	12.117	205.2	45.9	251.0	0.655	0.656	51.09	0.658
790	421.1	14.04	13.007	208.8	44.5	253.3	0.664	0.646	55.36	0.648
800	426.7	14.99	13.957	212.6	43.1	255.7	0.675	0.637	60.05	0.638

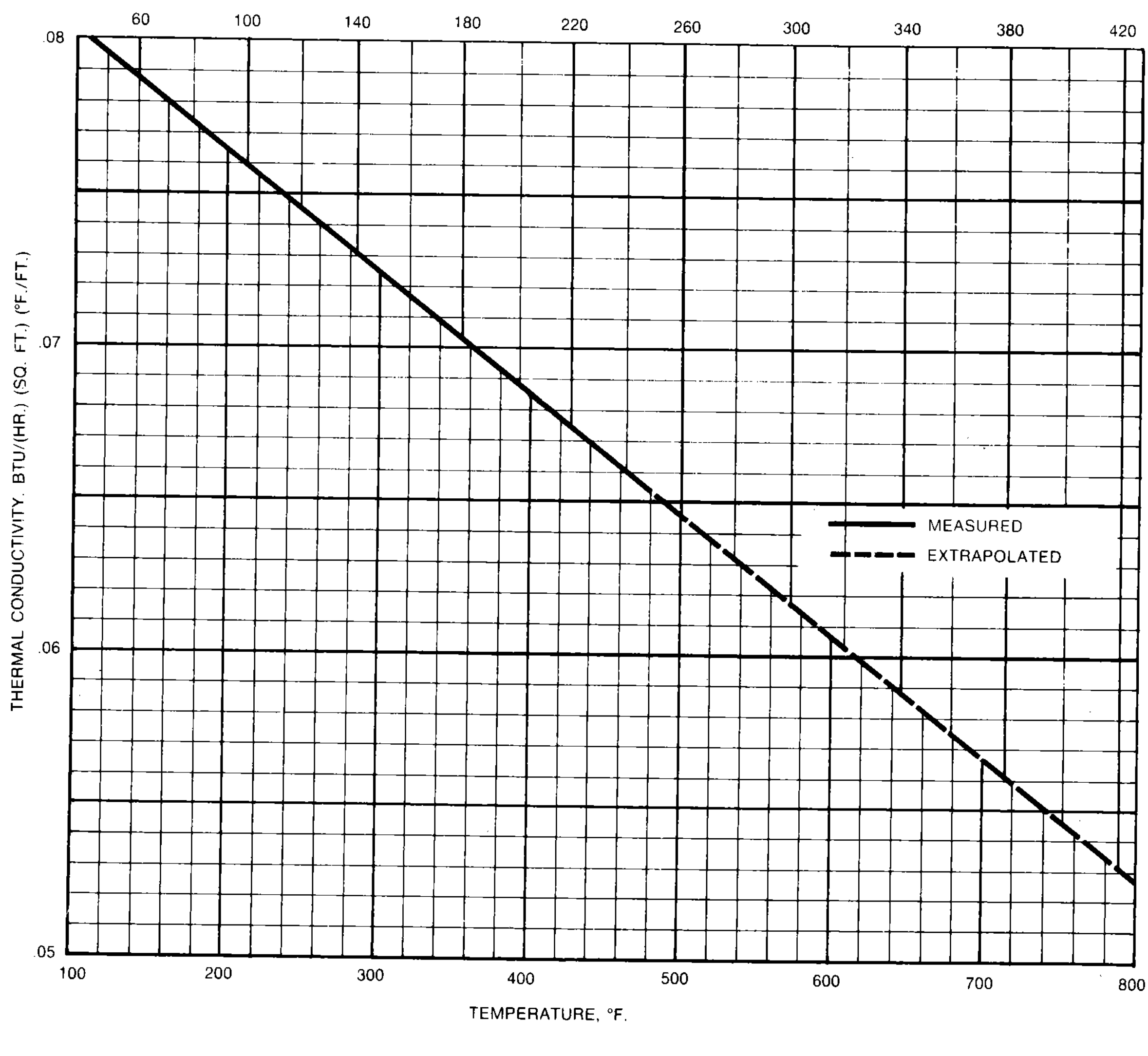
*Dowtherm Heat Transfer Fluids, Dow Chemical Co. (1971).

Fig. B-1. Liquid Viscosity of Dowtherm A.*



*Dowtherm Heat Transfer Fluids, Dow Chemical Co. (1971).

Fig. B-2. Thermal Conductivity of Liquid Dowtherm A.*



*Dowtherm Heat Transfer Fluids, Dow Chemical Co. (1971).

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