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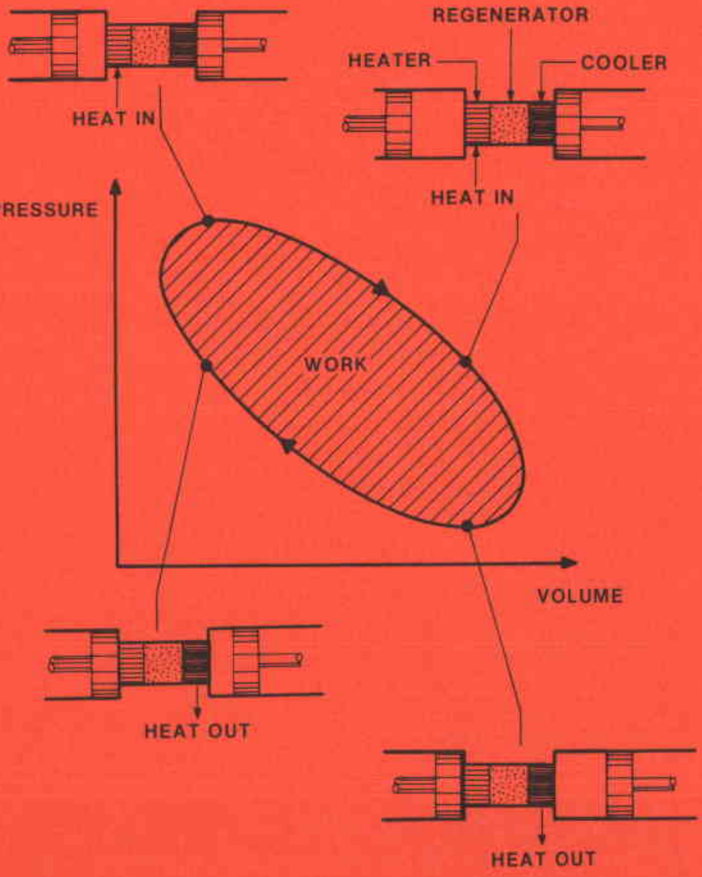
SEAMOPT: A STIRLING ENGINE PERFORMANCE OPTIMIZATION CODE

by

T. J. Heames, J. G. Daley,
and M. Minkoff



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SEAMOPT: A STIRLING ENGINE PERFORMANCE OPTIMIZATION CODE

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PREFACE

This report has been prepared as part of a project at Argonne National Laboratory to improve the technology base for development of Stirling engines through release of computer analysis codes, carrying out experiments that improve understanding of cycle thermodynamics, and exploration of advanced concepts. This project is sponsored by the Energy Conversion and Utilization Technologies Division, Office of Energy Utilization Research, United States Department of Energy.

The computer code described in this report is intended for public release and unrestricted use. Further improvements are necessary before distribution will be made, however, principally in the areas of generalizing the code to more easily accept new engine geometries and validating against known conditions. Comments and criticisms are therefore especially valuable at this time and should be directed to J. G. Daley.

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ABSTRACT

A computer code for Stirling engine research and design is described. The code system, SEAMOPT, has been used to: optimize component and engine performance, modify an existing engine to meet new application requirements, and identify design methods that lead to performance improvement and simplified engine design. SEAMOPT consists of a full Stirling engine simulation linked to a rigorous optimization code through an interface module which defines performance objectives and constraints which might limit values of design variables. Calculated results are presented from two example problems using the GPU-3 Stirling engine as a base design. The first example shows how regenerator dimensions can be changed to achieve three different performance objectives. The second example shows changes in the entire thermodynamic section needed to increase power by a factor of 8 while maintaining efficiency. The code, which requires 65K words of memory, executed problem 1 in 45 seconds and problem 2 in 10 minutes on an IBM 3033.

1. INTRODUCTION

The Stirling engine is remarkable for the many design variants that have been developed and the many applications that have been shown to be technically feasible. This flexibility follows from the large number of variables a designer can specify independently such as the choice of working fluid, working fluid pressure, geometry of heat exchangers and type of drive mechanism. Conventional heat engines do not offer a similar opportunity for creative design since (1) the number of variables that can be modified to achieve a desired performance goal are much more limited and (2) engine design flexibility is limited by combustion-related requirements in reciprocating internal combustion engines and in gas turbine engines by metallurgical and dimensional tolerance requirements.

SEAMOPT was developed to provide a systematic tool for determining values of Stirling engine design variables which enable the engine to meet all of a set of performance requirements. A base engine geometry must first be fully specified as well as the set of performance requirements that must be met (such as engine power, physical size, speed, etc.) at the design point. SEAMOPT then uses optimization theory to vary engine geometry and operating conditions (such as mean pressure) until a solution is found which meets the performance requirements while not exceeding specified engineering constraints. SEAMOPT is the first publically available code which offers this capability, although some private firms have stated they perform Stirling engine optimization with proprietary codes.

This report addresses the major aspects of SEAMOPT. Chapter 2 is an overview of the code system, including general discussions on optimization and Stirling engine simulation. Chapter 3 outlines how to use the code, including use of objective functions, use of constraint functions, input data requirements, modeling of the GPU-3 engine, and implementation of a new base engine design. Chapter 4 describes the major sections of the code and what the code does. Implementation of a new optimization algorithm is discussed and a general description given of code output. Chapter 5 gives the results of two sample problems; the first problem shows regenerator changes calculated by SEAMOPT in order to satisfy three different performance requirements, while the second problem shows use of SEAMOPT for power scaling. Engine geometry changes resulting from the second example increase the power output of the base design by a factor of eight.

2. STIRLING ENGINE DESIGN OPTIMIZATION

Design optimization is a form of computer aided engineering that is recently finding practical application for purposes as diverse as minimizing the amount of metal in automobile parts to improving energy efficiency of electric power plants. What is meant here by "optimization" is the automated process by which a solution to the general nonlinear programming problem is found. This is stated as

Choose the variables $x = (x_1, \ldots, x_N)$ to minimize the objective function

$$f(x) = f(x_1, \ldots, x_N) \tag{1A}$$

subject to the equality constraints

$$c_i(x) = 0 \quad i = 0,\ldots,k \tag{1B}$$

and the inequality constraints

$$c_i(x) \geq 0 \quad i = k+1,\ldots,m, \tag{1C}$$

where the objective function, f, and the constraint functions c_i, are nonlinear functions of n variables. The objective function for Stirling engine optimization would express desired performance—efficiency, power, size, or some combination of desired characteristics. The equality constraints are used to specify a required performance value, and the inequality constraints to ensure that engineering limits, such as an maximum allowable stress in materials, are not exceeded.

Optimization theory provides techniques for handling nonlinear relationships between design variables and performance and for proceeding from an initial set of solution values to a final set that minimizes the objective function while satisfying the constraints. Use of optimization theory in Stirling engine design was found to be advantageous and to produce results that could not realistically be obtained otherwise due to the large number of variables to be manipulated and to the conflicting requirements that typically have to be satisfied in a real design problem.

Figure 1 illustrates use of optimization to automate a portion of the design process. The designer interacts as shown in Fig. 1, first by providing the design objective. Although the concept may be simple, such as the desire to maximize efficiency or power, a mathematical expression is necessary which relates these concepts to the design variables. For example, a design which minimizes entropy generation has favorable efficiency and power characteristics and is thus a good design objective. No Stirling analysis model directly computes entropy generation, but all compute the heat flow into and out of the system. By dividing these computed values by appropriate temperatures, an equation for entropy production is developed which can be used as an objective function. Several other objective functions are discussed in Section 3.1 of this report.

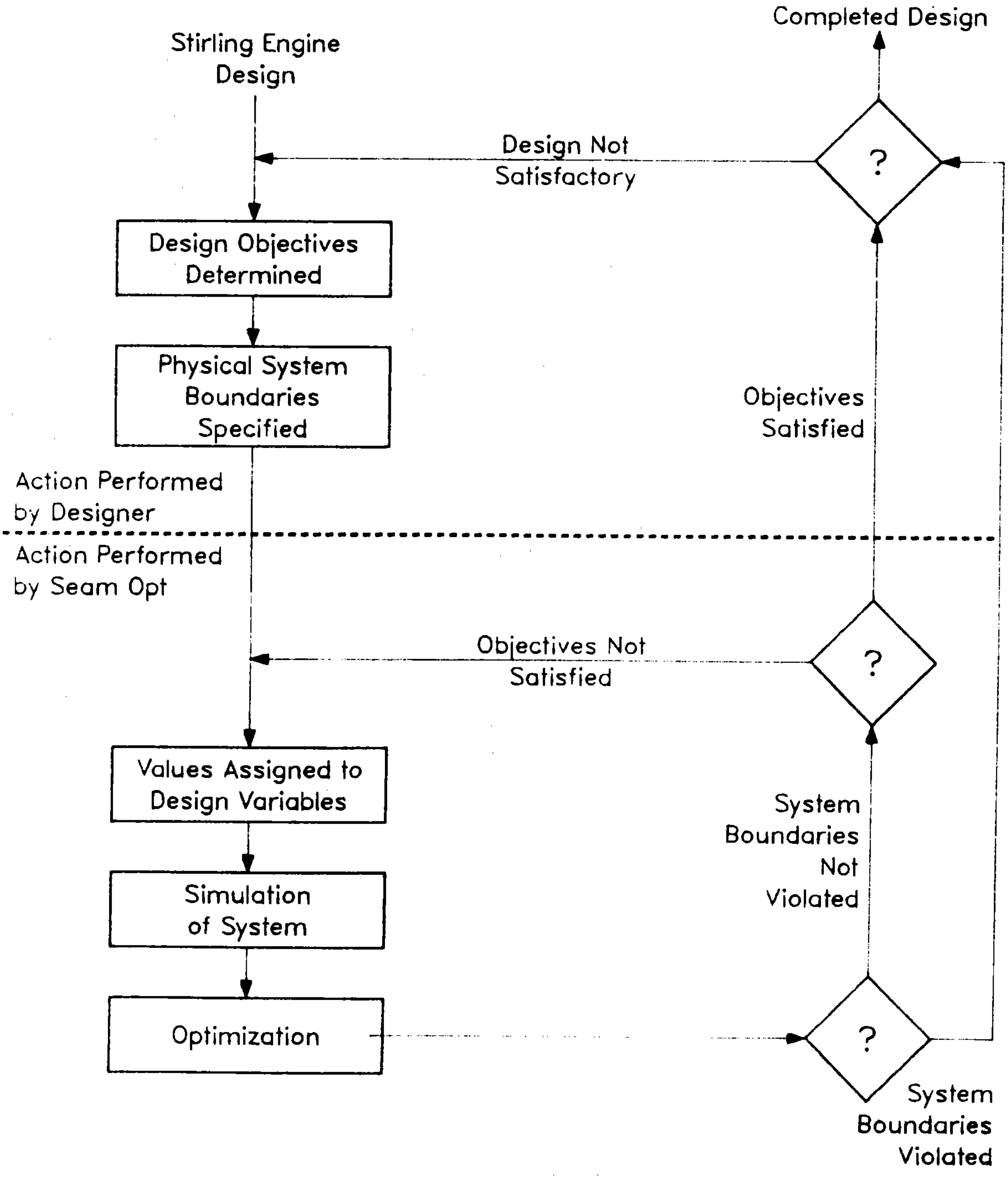


Fig. 1. Process of Stirling Design Using Optimization Program

The "physical system boundaries" or constraint equations shown in Fig. 1 refer to calculated parameters such as maximum stress in individual components, or the largest allowable dimension of a component. These boundaries must be defined and modified by the designer, as indicated by Fig. 1, until an acceptable solution to the design objectives is found. The code currently has 29 constraints, contained in Subroutine LIMITS and described in Section 3.2.

There are three circumstances under which the optimization program discontinues computation and requires some action by the designer.

- The original objectives are satisfied,
- An optimum solution was not found but a criterion for maximum number of evaluations has been exceeded, and
- The system boundaries (constraints) are violated for all solutions.

The designer must decide whether to accept the present design as satisfactory, or redefine the design objectives and/or the system boundaries and continue.

Thus, optimization is an important tool that can greatly expand a designer's capability, but its successful application depends on the skill and creativity of the designer. The designer must participate in the optimization process to evolve a design that best meets his application needs.

2.1 STIRLING ENGINE SIMULATION

Stirling engine performance depends on a large number of design variables. Some of these are operating parameters such as engine speed, mean pressure, and heater head temperature. Others are structural, geometrical or material properties such as stress (in components), piston diameter and type of working fluid (whether hydrogen or helium, for example). The relationship between design variables and performance must be well understood for improved designs to evolve without a long period of trial and error. Use of computer aided design tools, such as SEAMOPT, further requires that this relationship is analytically expressed. This analytical expression, or system simulation, is the key to achieving predicted performance improvement in practice.

The major challenge of Stirling engine simulation is that steady state (or quasi-steady state) is never approached since the working fluid continuously reciprocates between pistons. Cyclic steady state has therefore been defined to describe a condition in which the change between variables from one cycle to the next is "small". Rigorous multidimensional solution of the governing equations (conservation of mass, energy, and momentum) throughout the engine during the cycle is not presently being pursued by any Stirling engine analyst since:

- Large amounts of computer time would need to be expended,
- Convergence to cyclic steady state may not occur since even one-dimensional simplified simulations often do not converge,

- There is presently no way to validate such calculations since experimental data are not available, and

- The errors introduced by other assumptions (example: use of steady-state heat transfer correlations and steady-state entrance and exit loss correlations) are unknown, but are likely to outweigh the computational improvement due to a rigorous numerical solution.

Accordingly, Stirling engine simulations all contain simplifications that make convergence possible within an acceptable amount of computation time. A discussion of Stirling engine simulations that have appeared in the literature is given in [1].

Several Stirling engine computer codes have been obtained and modified at ANL to be operated with common sets of input data; in order that performance predictions could be compared on a common basis. Standard input and output routines were written as well as standard library routines to provide common determination of heat transfer correlations and working fluid properties. The SEAM (Stirling Engine Analysis Module) system shown in Table 1 eventually resulted from this work. Two simulations, described below, were selected as best meeting the two requirements of most Stirling engine researchers of:

- A rapidly converging simulation that calculates power, efficiency and cycle-averaged energy flows in the engine, and

- A multi-cell simulation that accounts for mass and energy throughout the engine during the cycle and converges after many cycles of detailed computation.

SEAM1 is based on work originally performed at MIT [2] to simulate a Stirling cycle refrigeration device. This model was extended by Martini [3] and others [4,5] to Stirling prime movers. The original code tested at Argonne was supplied by Martini. The present code is described in [6] with validation against kinematic engine data. Validation against free-piston engine data is given in [7]. SEAM1 allows both isothermal and adiabatic spaces in the engine and models power and efficiency losses on a cycle-averaged basis. Convergence to a solution is rapid, about 0.5 s per case. Since SEAM1 can fully account for energy flows in the engine while imposing a low computational burden, SEAM1 was selected as the simulation to be integrated into SEAMOPT.

SEAM2 is based on work done at NASA/LeRC by Tew [8], who also provided the original version of the code. SEAM2 can calculate mass and energy flows throughout the engine as a function of time and can be used to model experimental test rigs as well as engine geometries. The geometry to be analyzed can be divided into several control volumes as specified by the user and the governing equations are integrated for each control volume. SEAM2 provides considerably more computational information than SEAM1 since conditions throughout the engine and during the cycle are computed, but at the expense of computer time. The two codes agree closely on values of performance variables such as power and efficiency.

Table 1. Summary of Argonne Stirling Engine Analysis Capability--
SEAM (Stirling Engine Analysis Module) Structure

Description	Approaches/Features	Use
SEAM Library	Temperature dependent properties of working fluids and structural materials Heat transfer and fluid flow correlations All mechanical drives Standard input and output processors	Available to all SEAM codes Implements material and con- changes
SEAM1	Divides engine into five basic zones (heater, cooler, expansion space, compression space, and regenerator) and calculates average temperature temperature in each zone Calculates power, efficiency, and cycle average losses (conduction, friction leakage, etc.) Theoretical heat input and power corrected by calculated losses until convergence is obtained Fast convergence Precise accounting for separate phenomena that affect power and efficiency	Performance predictions Preliminary engine design Used with SEAMOPT
SEAM2	Finite difference analysis--engine divided into as many cells as desired Simultaneous integration of mass, energy, and state equations with small time steps Calculates detailed physical phenomena within each cell Very long time to converge (many cycles needed to reach steady state Uses SEAM1 input data	Detailed engine analysis and design Experimental data analysis Verification of SEAM1
SEAMOPT	Combines SEAM1 (above) with general optimization code All important influences on engine performance considered simultaneously during design Maximum stress levels and other physical constraints can be specified	Engine design studies Performance improvement

Stirling engine simulation is at a stage where modeling capability has outdistanced experimental validation of analytical models. Special effects experiments are needed to provide basic data on heat transfer and fluid friction losses under the high frequency reversing flow condition of the working fluid. Data on conditions in the regenerator during the cycle are needed such as: the temperature profile along the regenerator, mixing effects in the regenerator and heat transfer due to pressure changes during the cycle. Engine data are needed on energy flows during the cycle. Present simulations, including SEAM1 and SEAM2, contain models of effects known to occur in Stirling engines, but empiricism exists in the treatment of these effects and validation has usually only been made against engine performance data where the relative importance of the various contributing effects cannot be ascertained. Enough performance validation has been done to give confidence that existing simulations will predict trends cofrrectly, but a sounder experimental basis is needed before designs that are significantly different from current engines can be built with the confidence they will perform as expected. Since SEAMOPT always starts with a base design that has known performance characteristics the optimized design is expected to improve performance since trends are feft to be predicted correctly. An improved experimental basis for Stirling simulation would, however, also improve SEAMOPT.

3. WHAT THE DESIGNER DOES

This part of the report corresponds to the upper half of Fig. 1, the action performed by the designer. The first section discusses the use of the objective function and details the six objective functions presently available in the code, as well as alternatives a user might want to develop. The second section discusses constraint functions and details the 29 constraints currently in use as well as how to modify or add to the present constraints. The third section describes the input necessary to run SEAMOPT. The fourth section discusses the procedures followed to model the GPU-3 Engine as the base engine in SEAMOPT. The final section discusses how to implement a new engine design.

3.1 OBJECTIVE FUNCTION

The designer has the greatest opportunity to exercise creativity when devising an objective function which accomplishes the many diverse and often conflicting requirements of an engine design. The designer can choose to maximize power and find this leads to other desirable results such as a reduction in regenerator wire mass. Similarly, the objective can be minimum regenerator wire mass and a gain in power may result. The objective function that simultaneously increases power and reduces wire mass may provide a solution that is even more favorable when all implications of the solution are examined. The designer must determine the best approach; SEAMOPT provides the tool.

Each optimization problem has an objective function (or criterion function), f(x), that provides a measure of design quality and varies as the design variables (x) change. This analytical relationship must have a minimum value for some combination of design variables. For example,

$$\text{Minimize } f(x_1,x_2) = (x_1-2)^2 + (x_2-1)^2 .$$

This function has a minimum when the design variables are $x_1 = 2$ and $x_2 = 1$. A type of relationship of use for Stirling engine optimization is

$$\text{Minimize } f(\text{bore},\text{stroke}) = 1 - \text{efficiency}.$$

In this case the relationship between the design variables (bore and stroke) and the objective function (maximum efficiency) involves highly nonlinear relationships between mechanical and thermodynamic effects. In both cases, the optimum solution is found when the objective function is minimal. In both cases, the objective function varies with changes in the design variables. This is the only requirement for a valid objective function.

3.1.1 Models

The SEAMOPT user is limited to objective functions involving variables occurring in the SEAM common blocks. Many of these common block variables do not change with design variable changes and therefore should not be used in any objective function. This still leaves a significant variety of potential solution-related parameters that could be of interest. Most of the common block variables are defined in Appendix D. Many functions have been examined, and those used extensively are programmed in Subroutine LIMITS.

Table 2 summarizes the available objective functions in terms of which design characteristic is emphasized. A full description of each function and some of the consequences of its use are given in Appendix A. A satisfactory design is unlikely to be achieved solely through use of an objective function since design requirements are multi-faceted and often conflicting. The range of design requirements can be satisfied by use of an appropriate objective function such as those described in this section in combination with constraint functions such as those described in Section 3.2.

Table 2. SEAMOPT Objective Function Summary

Designation	Description
1	Efficiency
2	Minimum entropy production
3	Power
4	Power per total engine volume
5	Minimum regenerator wire mass
6	Power per engine swept volume

Performance improvement and power scaling can be done simultaneously. In the LIMITS routine this simultaneous capability is obtained by modifying the above objectives to include a power or efficiency ratio term. This is shown in the second sample problem.

Many other functions can be implemented, including those which minimize heater to cooler temperature ratio, increase pressure ratio, reduce engine weight, and improve part power performance. To implement a new objective function, the Fortran coding representing the new objective is inserted in the LIMITS routine just before the objective function normalization, about 435 card images from the start. Starting the objective and constraint values near unity is recommended. An input variable, OBJNRM, can be used to force the code to emphasize either the objective or constraint functions. When OBJNRM is greater than one, the resulting design will be affected by the objective function to a greater extent than when OBJNRM is less than one where the constraint functions will have a greater influence on the final design. Different designs are likely to result from use of this variable.

3.2 CONSTRAINT FUNCTIONS

Some optimization algorithms, including the one incorporated in SEAMOPT, have the ability to direct the search away from solutions which minimize the objective function but are unrealistic or do not meet other specified requirements. Use of such "constraint functions" make optimization useful as a design tool since designs that are undesirable from a practical standpoint can be eliminated. Without this feature, an optimization algorithm has little value in design of mechanical systems. It is important to note that the constraints need not be on the design variables but can be on the effect of using them. For example, previously, an objective function was given as

Minimize $f(x_1,x_2) = (x_1-2)^2 + (x_2-1)^2$.

Corresponding constraint functions could be

$$c_1(x_1,x_2) = x_1 - 2x_2 + 1 = 0$$

and $c_2(x_1,x_2) = -x_1^2/4 - x_2^2 + 1 \geq 0.$

In this case the final values for the design variables x_1 and x_2 must be such that constraint function c_1 is equal to 0 (an equality constraint) and constraint function c_2 must be greater than or equal to 0 (an inequality constraint).

Solving the unconstrained problem gave $x_2 = 1.0$ and $x_1 = 2.0$ at the objective function minimum ($f(x_1,x_2) = 0.0$). With the constraints added, the problem minimum occurs at $x_1 = 0.82$ and $x_2 = 0.91$. These values yield an objective function value $f(x_1,x_2) = 1.40$ and constraint values c_1,c_2 of 0.0. The constraint functions thus have a strong influence on the problem solution. As a further example, just changing the first constraint to the inequality condition,

$$c_1(x_1,x_2) = x_1 - 2x_2 + 1 \geq 0 ,$$

causes the solution to move to

$$x_1 = 1.66,\ x_2 = 0.55,\ \text{at}\ f(x_1,x_2) = 0.32 ,$$

a considerable change caused only by changing the type of constraint. Development of useful constraint functions (and combinations of objective and constraint functions) is an iterative process which allows the designer to evolve a desirable design. Intuition is of little use in this process due to the nonlinear nature of both the Stirling engine simulation and type of constraint functions needed.

Evaluation of the constraints and their gradients in SEAMOPT only slightly affects execution time so use of many constraints can be accommodated. The numerical values of the constraint functions should be within an order of magnitude of the objective function value for best results. For this reason constraint functions, $c(x_1,x_2,...)$ are normalized, relative to their limiting value.

3.2.1 Considerations in Use of Constraints

Constraint functions are typically related to performance, dimensional compatibility, material stress, or to SEAM model assumptions. Equality constraints (typically related to performance) are met before the objective function is minimized or inequality constraints are checked. The twenty-nine constraints shown in Table 3 have been found useful. The symbolic names in Table 3 correspond to those used in the output listing. They are all treated as inequality constraints during a problem unless equality constraints are specified as described in Sec. 3.2.1.1. A full description of these constraints is provided in Appendix A.

The input system was written for low-cost batch processing on the IBM 3033 available at Argonne. Input and output files are saved on disk to be accessed after execution. Operating SEAMOPT interactively would allow the user to monitor the optimization process, but at higher cost and longer execution time. A restart capability, described in Sec. 3.3.3, allows the user to continue the optimization from the last set of values rather than starting over each time. This allows the user to watch the optimization develop in steps if desired.

The SEAMOPT input uses the NAMELIST input specification used in SEAM1. NAMELIST is an input system used on minicomputers and mainframes to read data without specifying a list or format. With this system both the symbolic name and its value are input; this allows comments concerning the input to be included with the input file, along with explanations of other features. The input is subdivided into control variables and design variables. Although the remainder of the input discussion is concerned with the second block, or optimization input, a successful execution of SEAMOPT also requires the first block of engine simulation information detailed in Ref. 6.

3.3.1 Control Variables

The optimization process consists of calculating improved values for the design variables that decrease the objective function and satisfy constraints. The control variables tell the code how many times to calculate new values, how little change there must be between the current set of variables and the previous set before the solution is acceptable, how many design variables there are, and how large a change is necessary to determine gradients. A list of the control variables and their definitions is given in Table 5. Typical values are shown in the input listings in Appendix E.

3.3.2 Design Variables

The user must specify at least one parameter as being a design variable available to modification by SEAMOPT. At the finish of an optimization run design variables have new values that depend on the optimization goal (NOBJTV) selected and whether equality constraints are used (NEQULC). The number of variables to be changed is given by (NVAR). The user can input more than this number, but only the last NVAR variables will be optimized. This technique can be used to optimize components in sets rather than all together, as is shown in optimization case 4 in Appendix E. Table 6 lists the symbolic names used to initiate a variable, assign it a value, limit it, and assign a normalization value. The table also lists the 30 parameters available as design variables. Although the user can select them in any order, it has been found easier to give them in ascending order. The last card of the optimization input is an assignment of -1 for a design variable. This initiates the start of the optimization calculation.

Table 6 describes the input data for the 30 design variables shown. Internally modified versions of variables 4 and 5 are used to provide more nearly sinusoidal motion than resulted from the the original variables. Modified design variable 4 is the ratio of variable 4 to variable 3 in Table 6. Modified design variable 5 is the difference between variable 5 and variable 3 in Table 6. Variable 30 is presently always constant, since the

Table 5. SEAMOPT Control Variables Input

NVAR	-	Number of active design variables
MXEVAL	-	Computation terminates after MXEVAL new sets of design variables (default=6)
IPRINT	-	OPTSEAM print control
	0	basic output (default)
	1-4	additional output; 1 and 2 edit controls, inputs, and line search values; 3 and 4 give details of calculation. Cumulative print control, 3 gives output from 2, 1, and 0 also.
NOBJTV	-	Specified objective function
	1 =	maximum efficiency (default)
	2 =	minimum entropy
	3 =	maximum power
	4 =	minimum engine volume (includes crankcase)
	5 =	minimum regenerator wire volume
	6 =	maximum power per displacer volume
NEQULC	-	Specified equality constraints
	0 =	only inequality constraints are used (default)
	1 =	final power will equal DPOWER
	2 =	final efficiency will equal DEFFIC or final heat in will equal QSORCE (if positive value given to QSORCE in input)
	3 =	both power and efficiency are specified
DPOWER	-	Desired power level, default is initial calculated value
DEFFIC	-	Desired efficiency level, default is initial calculated value
QSORCE	-	Desired heat input level, default=0.0, (no source term specification is needed)
OBJNRM	-	Initial value of objective function (default=1.0) used to control the relative importance of the objective function to the constraint functions. This parameter has most significance when equality constraints are used. A value greater than 10 can cause the equality constraints to be unsatisfied, similarly a value less than 0.1 can give them such importance that a true objective function minimum is not found.
TOL	-	Computation is terminated when the change in the normalized value for f(x) is less than TOL (default=0.0001).
XDEL	-	Fractional change in a design variable used to construct gradients of the objective and constraint functions (default=.01)

Table 6. SEAMOPT Design Variables Input

IX - Design variable number
X - Design variable initial value (default = original engine design)
XDN - Design variable lower limit (default=0.1X)
XUP - Design variable upper limit (default=10.0X)
XNORM - Design variable normalization reference value (default=X)

IX	Definition (comments)
1	Engine mean pressure, Pa
2	Engine frequency, Hz
3	Crank radius, m
4	Connecting rod length, m (must be greater than 2.25 times crank radius)
5	Crank eccentricity, m
6	Expansion piston rod diameter, m
7	Volume phase angle, degrees
8	Combustor temperature, K
9	Cooler water temperature, K
10	Expansion cylinder bore, m
11	Expansion cylinder wall thickness, m
12	Displacer length, m (see SLNGTH in SEAM1)
13	Displacer wall thickness, m
14	Active heater tube length, m
15	Heater tube diameter, m
16	Heater tube wall thickness, m
17	No. of heater tubes per regenerator
18	Regenerator length, m
19	Regenerator diameter, m
20	Regenerator wall thickness, m
21	Regenerator minimum flow area to frontal area ratio (see SIGMA in SEAM1) (must be less than 0.95 and greater than 0.05)
22	Regenerator wire diameter, m (must be greater than 0.02 mm)
23	No. of regenerators per cylinder
24	Active cooler length, m
25	Cooler tube diameter, m
26	Cooler tube wall thickness, m
27	No. of cooler tubes per regenerator
28	Cooler-compression connecting duct diameter, m
29	No. of connecting ducts per regenerator
30	Ratio of the compression zone bore to expansion zone bore
-1	No additional optimization input

logic and input necessary for its use have not yet been implemented. Variables 17, 23, 27, and 29 all refer to integral numbers of tubes or ducts. Gale [9] examined this area and concluded that optimization with integers would not give good results. In addition, the lack of practical upper limits on the number of tubes prevented realistic solutions. Until better upper limits are available (chiefly a cost consideration), the user should specify these variables and not use them as optimization variables.

3.3.3 Restarts

A restart capability was added to SEAMOPT to allow the user to either perform series optimization or to do the optimization in steps without having to start from the beginning each time. To use the latter system the user can select three iterations (MXEVAL=3), interrogate the output file, and use the restart file to continue. The restart file is placed on logical unit 7 by the code and can then be merged to the end of the optimization input and used as the new input file.

Different solutions can occur if the code is run continuously for 15 iterations rather than 3 times for 5 iterations each with the restart capability. The final values of design variables would typically differ in the second significant figure under these conditions. The reason for the differences can be traced to the initialization of the optimization technique, where an initial directional step is needed. Thus a slightly different search direction is taken when restart is used, which can result in a local optimum being found that is different from an optimum found without restart. The more nonlinear the problem and its constraints, the more likelihood of several multiple solutions. The restart capability gives the user an easy way to determine the sensitivity of a solution to changes in design variables, make best use of control variables, and limit computation time.

3.4 ENGINE MODEL (GPU-3 DESIGN)

The engine model in SEAMOPT requires that physical connections between all structural components be given as well as information needed for thermodynamic simulation. For example, the simulation code is concerned with flow areas, wetted perimeters, and volumes. In addition to these obvious functions of length and diameter, other variables are needed to determine the engine structure that are uniquely design-dependent. For example, the shape of the heater head is related to the heating technique used, such as heat pipes, electrical resistance, or direct combustion. When the design is altered by the optimization process, reference planes are maintained either vertically or horizontally. For example, a cylinder (see R10 in Fig. 2) is maintained that runs through the cooler-regenerator-heater assemblies. This allows calculation of the effect of increasing the number of regenerators and heater or cooler tubes on overall dimensions. With non-opposed piston designs, a flat area is maintained that separates the heater assembly from the regenerator and expansion space. When combined with the assumption that the displacer piston cannot cover the entrance to the cooler connecting ducts, it forces the displacer piston length and stroke to maintain a relationship with the regenerator-cooler assembly (see Z10 in Fig. 2). With these assumptions, changes in the individual components can be related to the entire engine design.

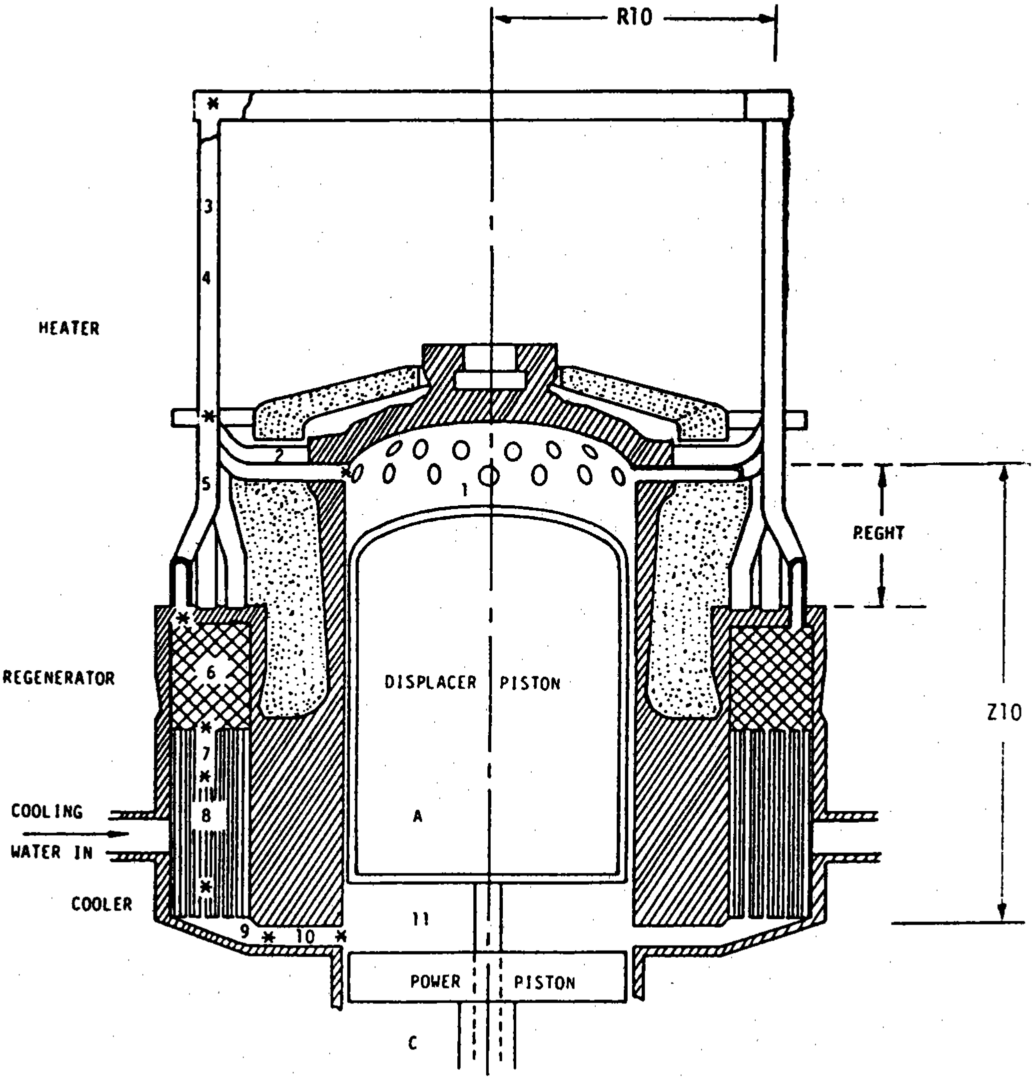


Fig. 2. GPU-3 SEAM1 Cell Location

SEAMOPT presently includes the GPU-3 design, shown in Fig. 2, as the reference engine. The surfaces described in previous discussions are shown as the cylinder whose radius would be R10 and the two horizontal planes separated by the distance Z10. The GPU-3 is the Ground Power Unit developed by General Motors for the U.S. Army in the 1960s [10]. The engine was subsequently tested at NASA/LeRC in the late 1970s and was extensively documented at that time [11]. Typical dimensions and experimental data are given in [11]; comparisons with SEAM1 calculations are documented in [6].

The numbers in Fig. 2 refer to the cell structure used in the SEAM analysis. The SEAM1 analysis uses five zones for mass flow and pressure variation calculations and uses all the cells for energy distribution analysis. Cells 2 through 5 and cells 7 through 10 are lumped together to form the heater and cooler zones for the mass flow and pressure calculations. The lumping calculation is done within SEAM1, and SEAMOPT must pass correct values for all cells to SEAM1; this is the task of subroutine OPTSEM.

On the first call to OPTSEM the original input values for flow area, surface area, hydraulic diameter, length, volume, and wall volume for each cell are saved. Values are then calculated for each cell based on theoretical relationships. For example, the original input volume of cell 3 is saved and then a volume is calculated based on the length and diameter of cell 3. The ratio of these two values is saved so that changes in the theoretical volume are transferred to the actual volume. It should be noted that this ratio is equal to unity for most cell values.

The GPU-3 heater head is composed of 80 tubes connected to a common ring; 40 connect to the expansion cylinder and 40 to the regenerator. The radius of this ring, RHTR, is calculated on the assumption that all 80 tubes have a constant pitch to diameter ratio. This is the minimum value for the R10 cooler-regenerator-heater axis. As the diameter, wall thickness, or number of heater tubes change, this minimum must also change. There are also eight regenerators arranged symmetrically around the cylinder; a similar radius calculation, RREGN, is performed to ensure compatibility. The current radial distance from the cylinder centerline to the regenerator centerline is compared with the previous radii to find the maximum value for the R10 plane. As this radius R10 changes, the lengths of the connecting ducts, cells 2 and 10, are adjusted.

Z10, the distance separating the reference planes, is calculated in two ways, both as the sum of the cooler and regenerator lengths plus that part of cell 5 shown as REGHT, and as the sum of the stroke and displacer piston height. The discrepancy between these values is saved and as the geometry changes it is taken into account to correctly modify the lengths of cell 5 and REGHT and therefore maintain the parallel nature of the two planes.

OPTSEM also adjusts the dead volumes associated with the variable volumes. The code assumes that they primarily increase with flow area; therefore, the compression space dead volume is proportional to the bore squared. The expansion space dead volume includes the displacer gap; this volume is removed from the total and the area proportion is increased relative to the displacer length and diameter only.

3.5 IMPLEMENTATION OF A NEW ENGINE

The GPU-3 engine is presently embedded in the computer routines which interface the engine simulation with the optimization code. SEAMOPT was written in as general a way as possible, since a long range goal is to have a design optimization code which is not engine-specific, but the immediate objective was to have a code operating so its capabilities and potential could be ascertained. This objective was met and, since the availability of SEAMOPT would provide a powerful new tool to most Stirling engine researchers and designers, future plans are to generalize the version described in this report.

SEAMOPT optimizes the engine structure as well as the thermodynamic section of the engine. It follows then, that engine-specific relationships will need to be provided for new engine designs, particularly since an expected use of the code is for power scaling where large changes in dimensions of structural components are to be expected. The approach expected to be followed will be to formalize standard procedures for writing the Fortran needed for a new engine rather than trying to accomplish this completely by input data as was done for SEAM1.

The following sections discuss issues involved in analyzing the thermodynamic performance of an engine in any of the SEAM codes as well as modeling the engine structure needed in SEAMOPT.

3.5.1 Simulation Validation for a New Engine

Stirling engine analysis codes, including those used at Argonne, contain empiricism in heat transfer correlations and fluid friction drop correlations. Also, judgement calls are needed by analysts as to what value should be used to represent heater temperature, whether individual volumes are isothermal or adiabatic and even the actual value to assign to volumes occupied by the working fluid in different components of the engine. Simulations are therefore "calibrated" to match performance data (power and efficiency) at one point and then checked to verify that agreement is satisfactory over the range of available data. An iterative process is followed if necessary to obtain reasonable agreement. This empiricism is needed because available correlations are based on steady flow experiments rather than the reversing flow, transient conditions experienced by the Stirling engine working fluid. Measured engine data do not exist which establish which individual effects are significant during a cycle, either in a qualitative or quantitative way. The data typically available represent the simultaneous interaction of many separate phenomena and are either gross effects such as power and efficiency or incomplete when fundamental variables such as pressure and temperature are measured at a few locations in the engine. These measurements do not support a basic understanding of energy flows in an engine either on an instantaneous or cycle-averaged basis and are worthless to an analyst trying to validate a simulation that accounts for instantaneous energy flows and inadequate for validating a code that predicts cycle-averaged data. Careful measurements are needed of engine conditions during the cycle, special effects experienced are needed to isolate and study individual effects known or expected to occur, and data on reversing flow phenomena are needed to eliminate the present empiricism of Stirling engine simulations.

In spite of the above problems, a SEAM1 input deck must be generated for the new engine geometry and a performance comparison made as described in [6]. It has been found helpful to correlate the friction enhancement over a range of engine speeds, rather than at just the design point. Good agreement must be obtained with experimental measurements of power and efficiency over a wide range of operating conditions in order to proceed with design optimization with any confidence.

3.5.2 Structural Relationships for a New Engine

Correct geometric relationships in subroutines OPTSEM and LIMITS must be provided for structures that will change during optimization. Presently in OPTSEM a screen regenerator is specified, the heater is formed in a ring, and the regenerator-cooler units form a similar ring around the cylinder. In the discussion of the GPU-3 engine model, these effects as well as the height effects were explained. A new design would probably have different relationships. In LIMITS a crank-type drive is specified, the cooler tubes are put on an equilateral pitch, and a relationship is given between engine, cylinder bore and other variables. In addition to the geometric relationships implied in several constraints, LIMITS contains several performance inputs. These are initialized on the first pass into LIMITS and include the maximum allowable heater head heat flux, the efficiency of the engine under maximum heat flux conditions, the ratio of maximum working pressure to design point working pressure, and the safety factor for stress relationships.

Although other designs have not been implemented in SEAMOPT, structuring the code for the GPU-3 provides the necessary framework and there is no obvious reason why additional designs would not be as adaptable to the optimization process as the GPU-3 engine has been.

4. WHAT THE CODE DOES

After the designer has chosen a design objective with appropriate constraints and design variables, the code system is ready to be used. The code will initialize the system and then run a performance simulation of the design at starting point conditions. Each of the design variables will then be increased in value by a small fraction, XDEL, and the effect of each change on the objective function and constraint functions will be stored. These values are then used to form numerical gradients that the optimization code uses to calculate improved values of the design variables. The code then runs a simulation with these new design variable values and determines new gradients. This loop is continued until an iteration limit, MXEVAL, is reached, or until the objective is satisfied to within a tolerance, TOL, with all constraints satisfied. The code then runs a map engine performance across the range of operating frequency and the designer must decide whether the design is satisfactory. This part of the report corresponds to the lower half of Fig. 1.

To perform all of the above processes requires an internal structure to control the logic flow, as shown in Fig. 3. The solid modules shown in Fig. 3 are the same used in SEAM1 and SEAM2 with the exception that additions have been made to the "executive" module needed for optimization. The other modules are discussed below and include an analysis module (the technique used to simulate the Stirling engine design), optimization module (the technique used to determine improved values for the design variables), and executive module (control of program flow).

4.1 ANALYSIS MODULE

The analysis module is the major component of the engine simulation system. It contains the technique used to translate the motion of the pistons into pressure, temperature, and velocity distributions. With this information the heat in, power out, and energy distributions are determined. The current model being used is discussed in Section 2.1. The engine is modeled as two variable volumes, the expansion and compression zones, that are polytropic (the default being adiabatic) connected by a series of fixed volume, constant temperature zones. With these assumptions an ordinary differential equation is formed that is integrated by a low-order Runge-Kutta scheme. By storing the mass distribution and pressure in time, a series of mass flowrates and their averages can be found. These yield basic work-energy distributions that are corrected for various loss mechanisms such as wall conduction, friction, and finite heat flow. Appendix C details the modifications made to adapt the simulation to the optimization system. The most important modification was to approximate evaluation of exponentials by use of 2nd order polynomials. Although the latter is slightly less precise, the threefold increase in speed allowed simulations to be executed in less than one second with no significant effect upon final design variables.

4.2 OPTIMIZATION MODULE

The development of methods for solving the constrained parameter optimization or nonlinear programming problem has been an area of great interest in the numerical analysis and applied mathematics community. A variety of tools exist that have been successfully applied to many complex systems. The basic mathematical problem of interest is given by Eq. 1.

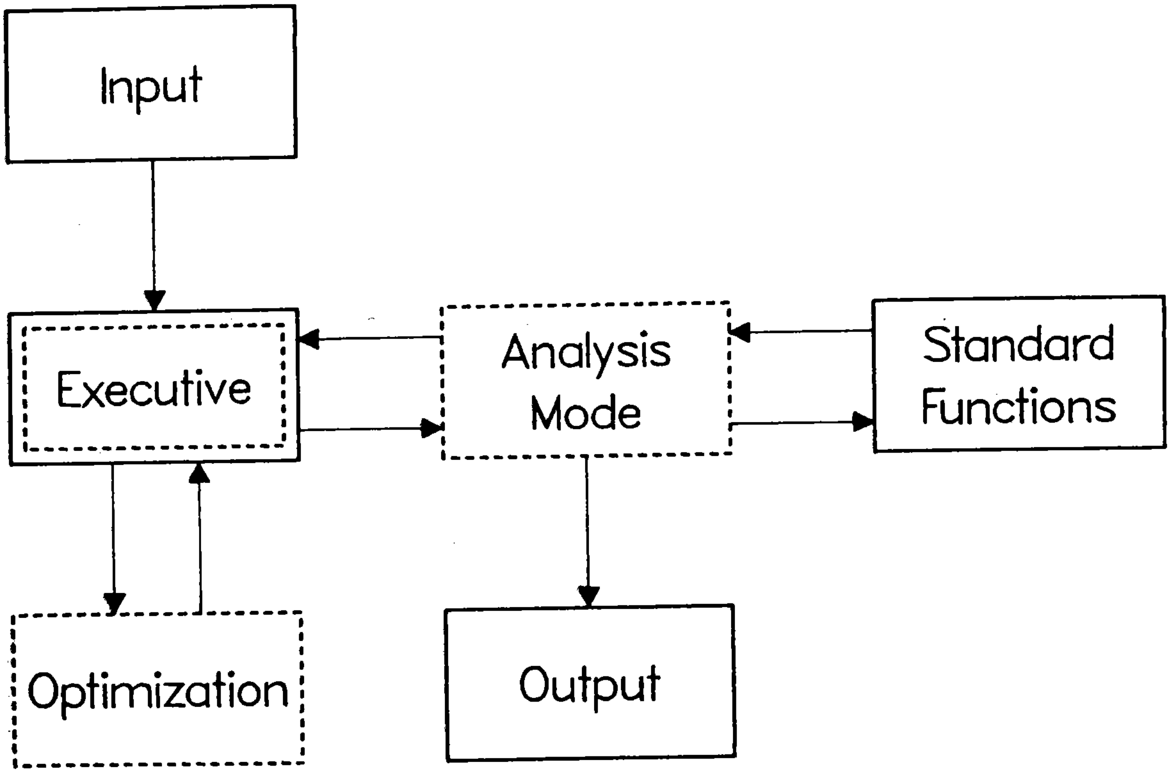


Fig. 3. SEAMOPT Framework

Heuristically constructed "direct search" techniques have generally been replaced by sophisticated techniques with strong theoretical foundations. Current state-of-the-art techniques can be grouped into three broad classes:

- Penalty function and augmented Lagrangian or multiplier techniques,
- Reduced or projected gradient methods, and
- Hybrid techniques and quadratic programming subproblem methods.

No one method or class of methods can be expected to solve all problems accurately and efficiently; each has strengths and weaknesses. Regardless of the method chosen, the optimization algorithm constructs a new estimate for the variables, x, that reduce the function f(x), subject to the constraints $c_i(x)$. The algorithm uses the new values for f(x) and $c_i(x)$ to construct another estimate for the variables x. This loop continues until an ideal solution is reached.

The method used to determine the design variables can be characterized mathematically. Let the Lagrangian function be defined as

$$L(x,\lambda) = f(x) - \sum_{i=1}^{m} \lambda_i c_i(x) \ , \qquad (2)$$

where the m parameters λ_i are called the Lagrangian multipliers. If x^* is to be a solution, then it is necessary that there exist an associated set of multipliers λ^*, such that

$$\nabla_x L(x^*,\lambda^*) = \nabla f(x^*) - \sum_{i=1}^{m} \lambda_i^* \nabla c_i(x^*) = 0 \ . \qquad (3)$$

$$\begin{aligned} &\lambda_i^* \geq 0 && i = k+1, \ldots m \\ &\lambda_i c_i(x^*) = 0 && \text{for all } i \\ &c_i(x^*) \geq 0 && i = k+1, \ldots m \\ \text{and} \quad &c_i(x^*) = 0 && i = 1, \ldots, k \end{aligned}$$

These necessary five conditions are known as the Kuhn-Tucker conditions, the first of which is a generalization of the totally unconstrained case where the minimum occurs when the gradient of the Lagrangian, $\nabla_x L(x^*,\lambda^*)$, vanishes. The second states that each inequality constraint has a Lagrange multiplier, λ_i^*, that is positive or equal to zero. The third condition, called the complementary condition, shows that either the Lagrange multiplier or the constraint is zero at the solution. The final two conditions ensure the feasibility of the constraints.

The optimization code presently used in SEAMOPT, VMCON [12], is a hybrid technique using an algorithm developed by Powell [13]. The code provides new values for x, by first obtaining the Lagrangian of Eq. 2 through linearizing the constraint functions over a small interval. The code then uses a minimization process that balances the two competing goals of reduction in the objective function while reducing the amount by which the constraints are violated. This method is designed to converge to a point that satisfies the Kuhn-Tucker, or necessary, conditions. There is no attempt by the code to satisfy sufficiency conditions. This method should work very well on problems that have reasonably continuous derivatives. Since Stirling cycle performance calculations are not involved with a working fluid that undergoes a phase change or other computational discontinuities, Powell's method should be faster than most others.

The version of VMCON used by SEAMOPT has been modified to include upper and lower bounds on the design variables as well as an additional return code to indicate whether gradient calculations would be needed. These modifications are explained in Appendix C.

4.3 EXECUTIVE MODULE

The purpose of the executive module is to control both optimization and simulation. Since this is not done with a single routine but rather with several, it will be described in terms of typical program flow and which routines are involved. There are two major control blocks of logic: MAIN, which sets up the grand scheme of things, and OBJCTV, which enables the optimization code to calculate objective functions, constraints, and their gradients with values from the simulation code.

4.3.1 MAIN

A logic flowchart for the MAIN program is shown in Fig. 4 starting with input and initialization, and proceeding through optimization of the design variables in terms of objective functions and constraints, and then displaying the results of using these new design values.

The input arrives from two sources. The call to INPDRV brings in data needed for simulation of the base engine. The call to OPTINP initializes the relationships between all 30 of the design variables with the various SEAM input variables (OPTSEM). It then reads the design variables to be used, normalizes them, and initializes the objective and constraint function coefficients (LIMITS). A further discussion of these various routines is given in Chapter 3.

The optimization module is a self-contained code. SEAMOPT presently uses VMCON, as described in the previous section. VMCON requires some initialization and calling sequence set up, which is provided by OPTMIZ. VMCON also requires values for objective functions, constraints, and gradients; these are provided by OBJCTV. After an optimized set of design variables has been determined, routine OPTMIZ saves the values on unit 7 for restart capabilities and edits the Kuhn-Tucker conditions referred to previously.

The map run of final values shows the user the performance of the new design over a range of engine speeds. Generally the users will be looking at part power performance or some other off design point information that will help them in their evaluation of the new design.

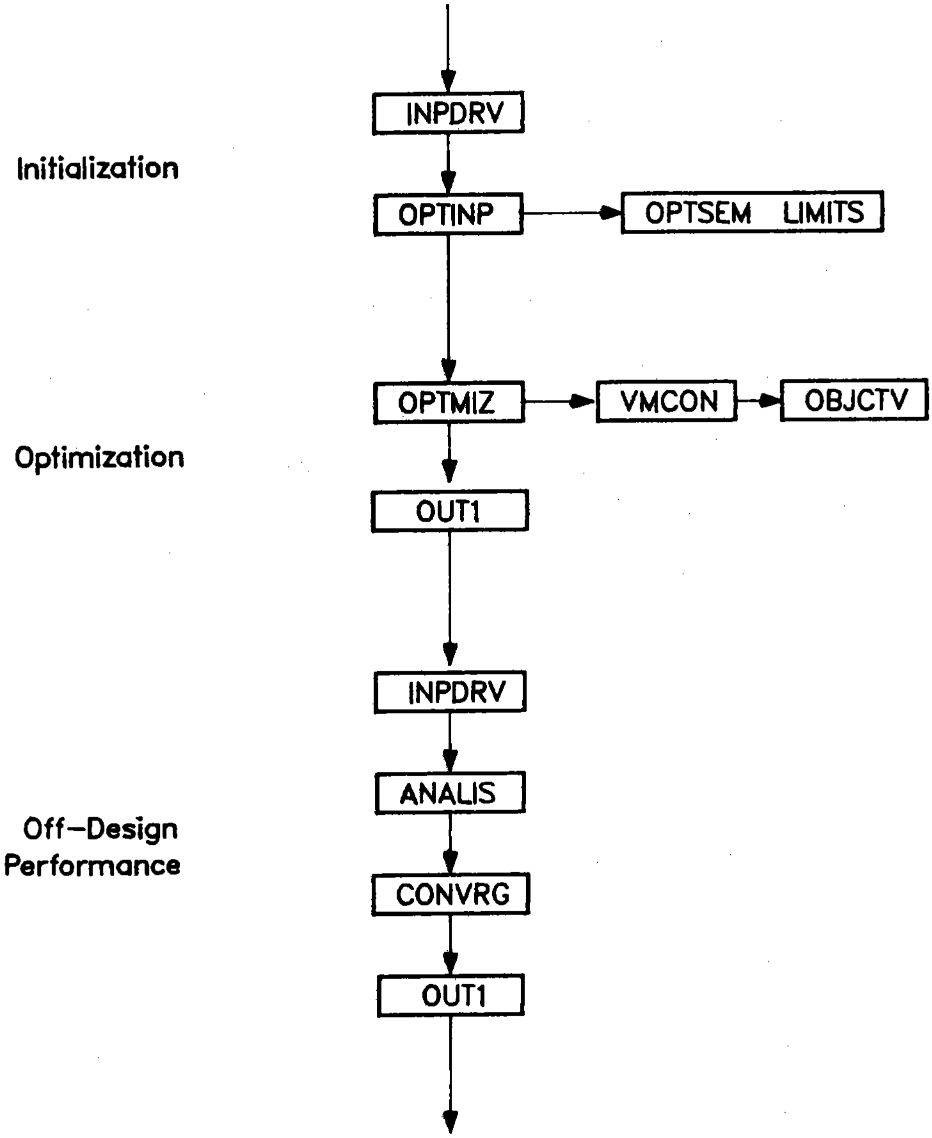


Fig. 4. Flowchart for MAIN Driver Program

4.3.2 OBJCTV

The OBJCTV subroutine supplies the optimization program with numerical values associated with the objective function, the constraint functions, and gradients of both objective and constraint functions with respect to the design variables. To do this the OBJCTV routine must

- Convert the design variables into SEAM variables as described in Section 3.3,
- Compute the performance with this set of data using the SEAM1 simulation system (Section 4.1),
- Evaluate the objective and constraint functions based on the performance simulation using the LIMITS routine (Sections 3.1 and 3.2), and
- Calculate the gradients of the objective function and constraint functions with respect to the design variables.

A flowchart of this process is shown in Fig. 5.

4.4 IMPLEMENTATION OF A NEW OPTIMIZATION CODE

Some users may prefer to use their own optimization systems in combination with our simulation system. Figures 4 and 5 indicate the primary areas of conversion. In terms of subprograms, they are OPTINP, OPTMIZ, VMCON, and OBJCTV.

OPTINP - Contains the read statements necessary to control the optimization program, as well as their default values and normalization variables.

OPTMIZ - Contains the actual calling sequence for the optimization code and edits optimization related parameters.

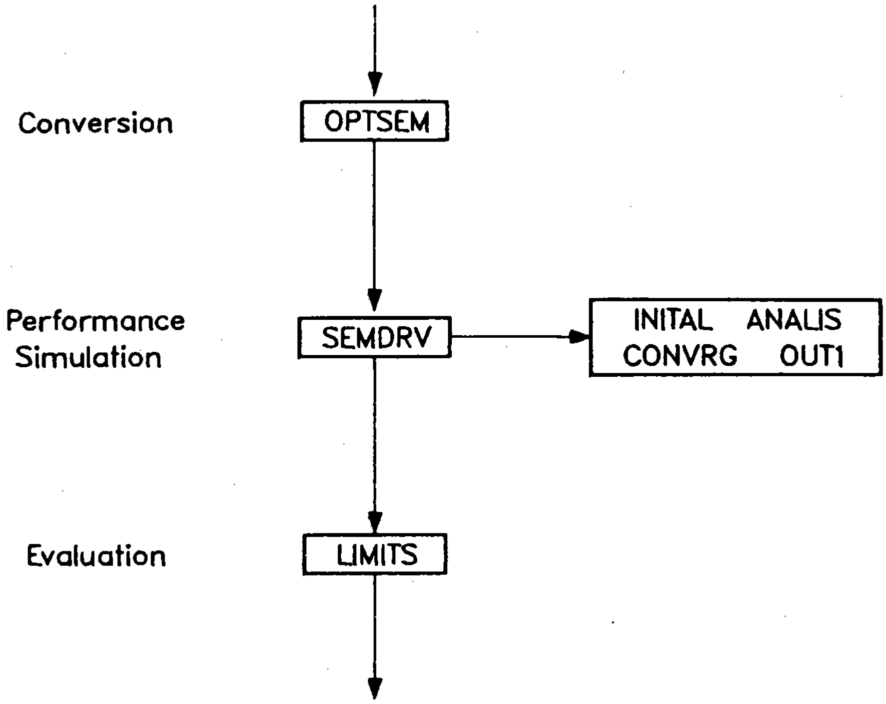
VMCON - Would be replaced by the new optimization code.

OBJCTV - This is called from VMCON to provide objectives, constraint, and gradient data. The return of these data is determined only by the value for JFLAG. Its storage and normalization are consistent with the other three routines.

Gale [9] has successfully replaced VMCON with a U.S. Navy optimization technique.

4.5 OUTPUT

Sample outputs from logical unit 6 are given in Appendices F and G for the two sample problems. All outputs start with a four-page detailed edit of the input values. This output section comes from the performance simulation



JFLAG = 0 Return objective and constraint values

JFLAG = 1 Return Gradients of Objective and Constraints for all Design Variables

Fig. 5. Flowchart of OBJCTV Subprogram

and can occur several times in a typical run. The frequency with which it occurs is controlled by the input value IPRINT. Its main use is to monitor changes in input variables such as porosity or stroke that are not design variables but are changed by design variables. This detailed input edit can be followed by a half-page RIOS consolidated edit where the values from the RIOS 5 volume analysis are given.

On page F2 of the sample problem output shown in Appendix F, an edit is given of the optimization input. In the input description a list of the design variables and names are given. Those chosen for the run are edited here and shown with their normalized values, upper and lower normalized bounds, and the normalization values. Typically, the normalized value is 1.0, the upper and lower bounds are 0.1 and 10.0, and the normalization value is the SEAM1 default value. If the user is doing a restart, the values shown reflect the restart values and not the initial problem values.

On pages F3 through F7 another detailed input list and RIOS input are given. In a restart these values will be different than the first detailed edit, as they will reflect the values from the previous run. This input is the actual input used for the first performance estimate and therefore corresponds to any default values needed by the optimization routines.

Page F8 starts a series of edits of the current values of the various parameters used in the optimization. If more detail is needed, increasing the value of IPRINT from 0 to an upper limit of 4 will yield the additional information. Page F8 output consists of the current values for all the constraints, the objective function, the weighted sum of the constraint violations, and the values of the design variables for the first two calculations. The optimization program is attempting to minimize the sum of the objective function and the weighted constraint violations by changing design variables. By observing how this output changes in a run the user can determine how well the optimization program is doing, which constraints are most important, and which design variables are most significant.

Every NPRT2 cases (10 in the sample problem) are compared on a performance basis. In sample problem 1 this occurs on page F11 for six of the nine calculations needed. The performance output can be used to determine which loss mechanisms or heat flows are changing most rapidly in a particular optimization run. In this problem the friction and power losses change significantly over the nine cases.

The optimization program typically stops because the limit of maximum gradient evaluations, MXEVAL has been exceeded. When the final case is run, enough additional information is provided to allow the user to decide if the design is satisfactory or whether to run the optimization to where the convergence criteria is met. Such an edit occurs on page F9 of sample problem 1. The objective and constraint values are displayed as before, but the design variables are given vertically. The ratio of each design variable to its initial value is given, so that at a glance one can tell, for instance, whether the regenerator is longer or shorter. Additionally, a series of derivatives or gradients are given for the objective function and for each of the constraint functions. With these gradients one can tell whether an increase in regenerator length will improve the objective function or change a constraint positively.

The effect of changes in individual design variables may be observed from the output. In Eq. 3 it was shown that the gradient of the objective function minus the sum of Lagrangian multipliers times the gradient of each constraint function is zero at the optimal point. In addition, each component of this equation is also zero at the optimal point, giving a result that provides a measure of the effect of each design variable. Accordingly, for each design variable, the output listing shows: the partial derivative of the objective function with respect to the variable (FGRD), the difference between FGRD and the sum of the Lagrangian multipliers times the partial derivative of each constraint function with respect to the variable (ERROR), the value (ERR.CNST) and constraint number (N-CNST) of the largest product of Lagrangian multiplier times the partial derivative of the constraint. The most important design variable (largest ERROR) and most important constraint (N-CNST) can then be found. In addition, the current objective function, constraint function, and Lagrangian multipliers are given to allow the user to compute the remaining Kuhn-Tucker conditions for each constraint. These conditions have been summed and are listed along with the weighted constraint error on the remaining lines.

At this point the optimization code has calculated new estimates for the design variables. The code system then reads the last few cards of input and performs an off-design performance calculation over a speed range specified by the input. This is given on page F12 where the engine geometry resulting from the design point optimization is used to generate predicted performance calculations over the specified speed range. It should be noted that optimization at a different design point would result in a different geometry.

5. EXAMPLE PROBLEMS

This section describes the results of using SEAMOPT to modify the GPU-3 engine to meet different design objectives. There are two example problems: the first modifies only the regenerator and determines three optimal designs; the second modifies most of the geometry and maximizes engine efficiency. Selected input and output have been listed for these problems in Appendices E, F, and G.

5.1 EXAMPLE 1 - GPU-3 REGENERATOR IMPROVEMENT

In this example design data from Ref. 11 are used to characterize the engine base design. Regenerator length, diameter, wall thickness, minimum flow area, and screen wire diameter are allowed to vary to meet different design objectives. Interactions between constraints, performance, and design variables are demonstrated in this example. Three objectives were chosen:

- Increase efficiency with no decrease in power,
- Reduce regenerator wire volume with no decrease in efficiency, and
- Increase power with no decrease in efficiency.

The optimization input for all three runs is given in Appendix E. The entire output for the increased efficiency case, and selected output from the other cases are given in Appendix F. The results of the three optimizations are given in Table 7 and compared with the initial design calculation. Experimental data from NASA tests using the GPU-3 [11] are given in Figs. 6 and 7 as well as the present calculated results for the three optimized designs. The design point used corresponds to the 3000 rpm 2.74 MPa case, designated as H242B in the NASA report. Performance is then optimized in each case at this design point by allowing regenerator characteristics to change. The final geometry is then used to predict the off-design performance characteristics shown in Figs. 6 and 7.

The values shown for measured data were obtained by assuming that the mechanical losses were equal to the sum of the heat gains in the cooler oil and in the buffer cooling water. This loss was added to the measured power out to obtain the indicated power. The indicated power was added to the measured cooling water heat gain to generate a net heat in. The indicated efficiency is the ratio of the power in to the net heat in.

The engine design data and variation of heater wall temperature with speed were according to Ref. 5. As Fig. 6 shows, the power predictions from the simulation are quite good and follow the data trends. The calculated efficiency shown in Fig. 7 is consistently higher than what was measured, although it follows the data trends.

5.1.1 Efficiency Maximized

The result of the first optimization problem (maximum efficiency) was a seven percentage point efficiency improvement and a slight power increase. When this new geometry was used to calculate part power performance, the power curve matched the base prediction at the design point and the efficiency curve

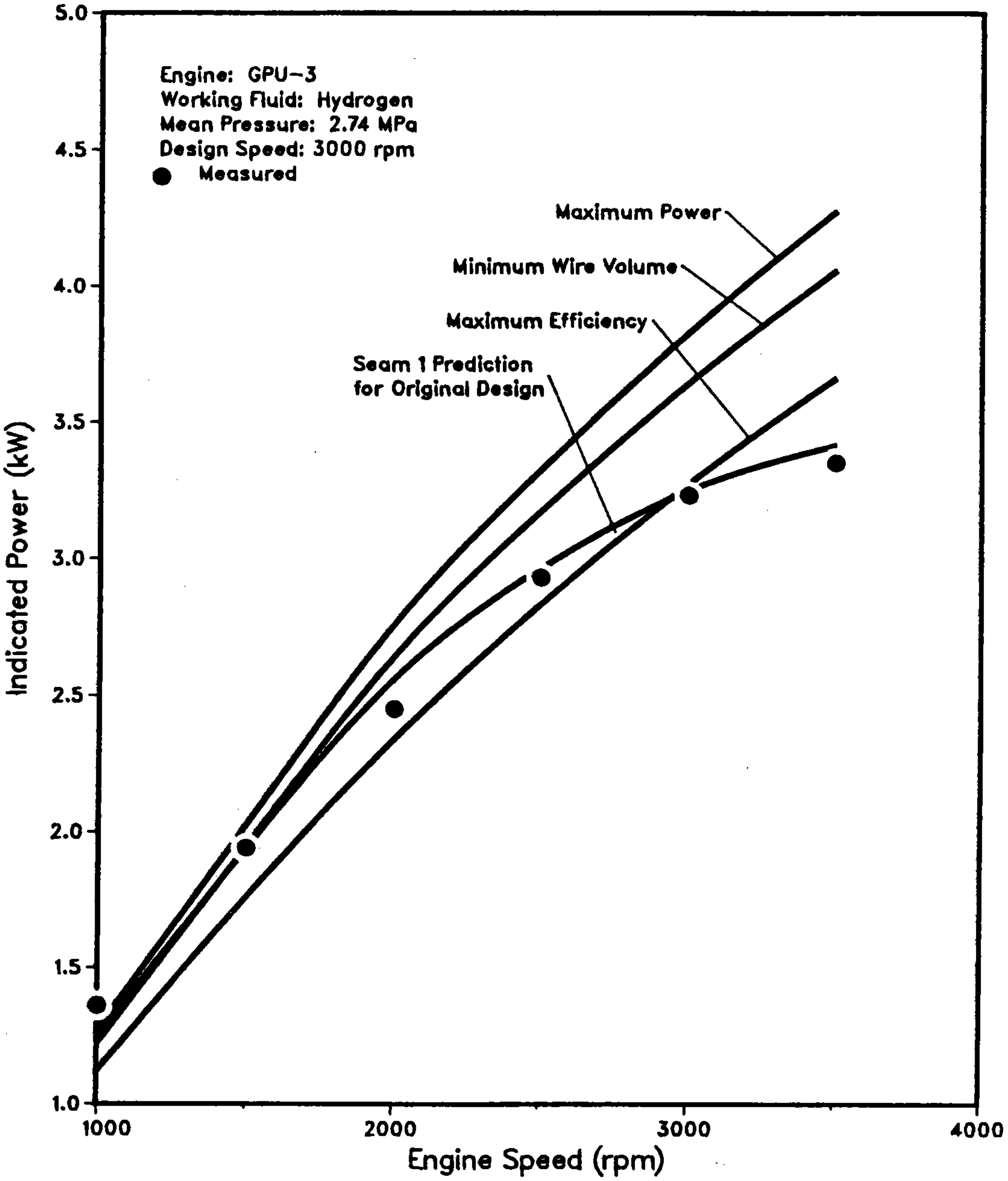


Fig. 6. Effect of Optimization Objective on Off-Design Power Performance

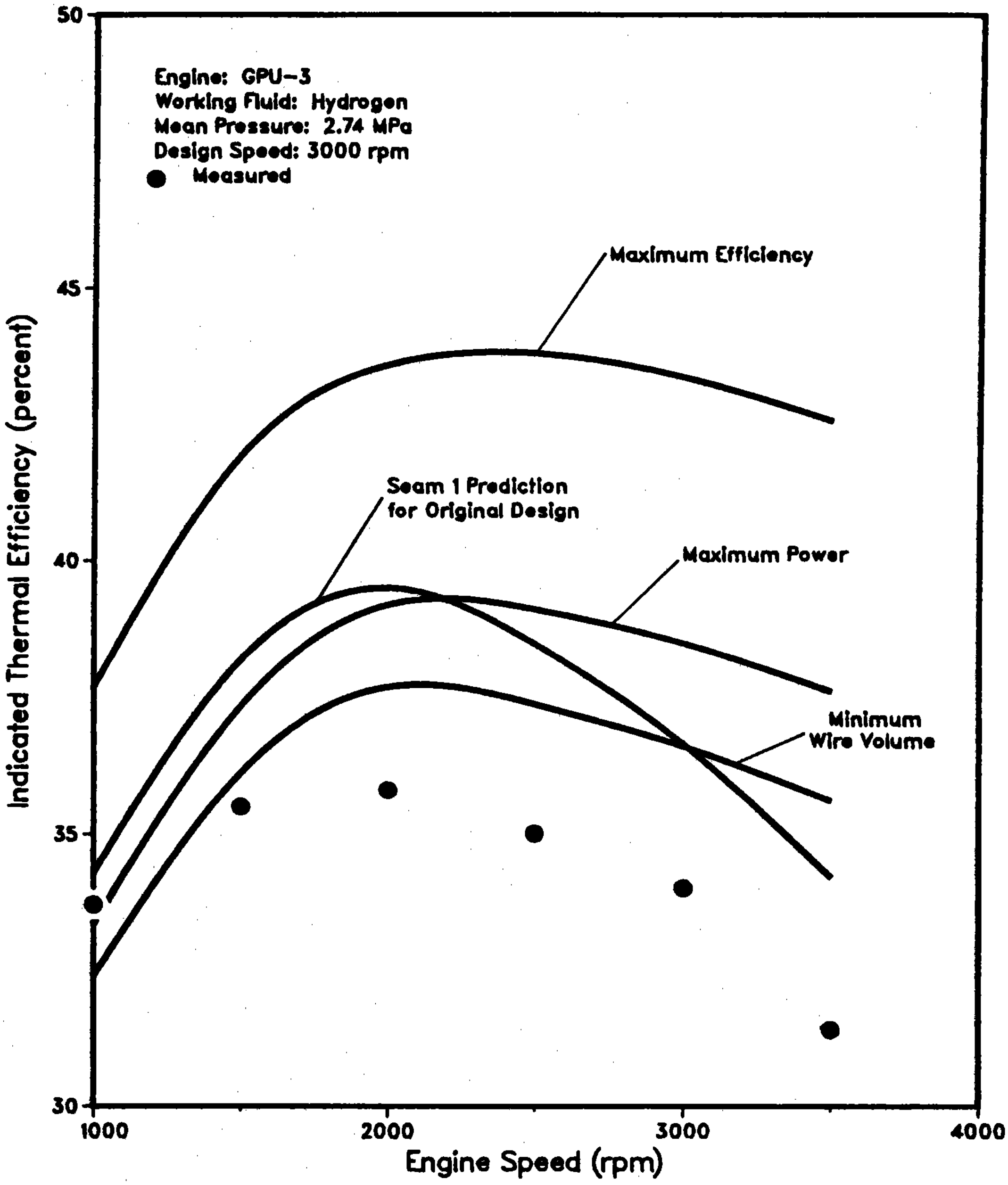


Fig. 7. Effect of Optimization Objective on Off-Design Efficiency Performance

heater tubes per regenerator and the number of regenerators. These are not varied due to the lack of published information that would allow an upper limit to be placed on their number (see Sec. 3.3.2). A detailed list of the input is included in Appendix D.

Figure 8 gives the history of several output variables for this problem. As shown, the equality constraint on power is the first variable to reach its design value of 10 kW. If efficiency had been plotted, it would have shown the same trend of reaching its final value within 10 iterations. Displaced volume (the objective function) is shown to converge within 15 iterations. Minimizing displaced volume, which SEAMOPT did, also maximizes power per displaced volume since power is fixed at 10 kW. Displacer length and heater length variation can be seen to be still changing after 30 iterations. A satisfactory design is often found before any of the criteria for stopping computation are met. The last five or so iterations do not achieve any significant performance change and should be considered computational noise in the problem. Stopping the calculation at 20 iterations would give an acceptable answer but, in terms of a design effort, some insight would be lost. It is in the middle range of calculations where the designer can see which parameters have little effect on the final solution. The entire run time for this 21-variable scaling problem was less than 10 minutes on our computer. With so short an execution time, the designer can include many design variables during the optimization process.

Table 8 gives some of the performance and dimensional results for this problem. The code balances average pressure and pressure ratio to increase both power and efficiency while achieving the specific power objective. As expected, the 10 kW engine is significantly larger, but the main engine envelope increase occurs in the crankcase. Both the height and the width increase were caused primarily by larger crank dimensions. The stroke was increased by over a factor of two, causing both the crank radius and eccentricity to be increased by more than a factor of two. This increased the crankcase width proportionally. The increase in crankcase height is caused by the increase in the connecting rod length, which appears to allow better performance due to smoother piston motion. Engine exterior envelope could have been minimized rather than displaced volume if this were desired.

Appendix G details much of the output from this problem. Particular attention should be given to the final values of design variables and constraint functions. With the criterion that values within ±0.1 indicate a constraint is heavily influencing the calculation, more than half of the constraints are involved. Likewise, if a criterion of more than a 10% change in a design variable is used as an indicator of involvement, then more than three-quarters of the design parameters are involved. This heavy involvement of design variables, constraint functions, objective functions, and bounds is a direct indication of the capabilities of the code system.

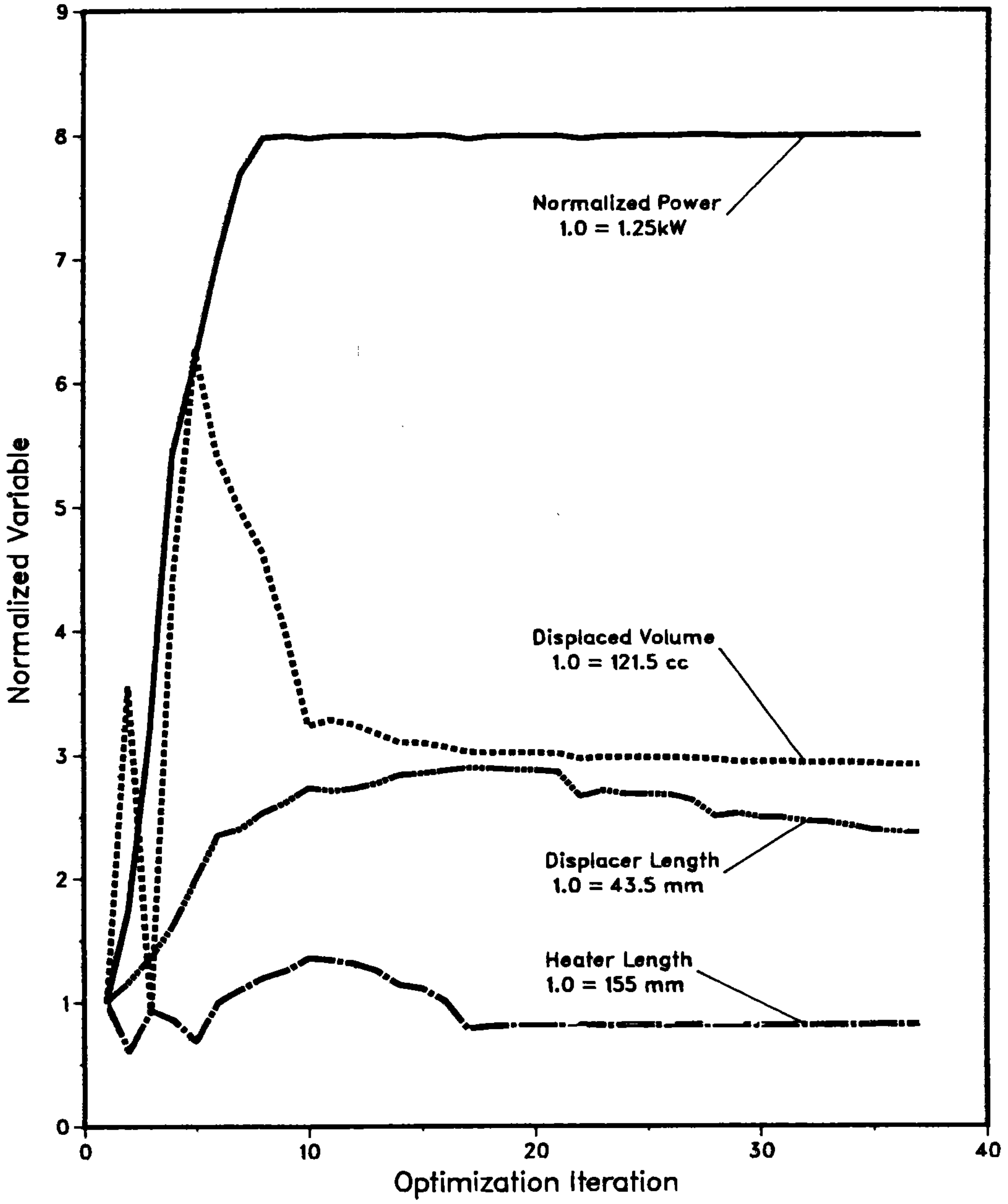


Fig. 8. Iteration History of Selected Variables in 10 KW Methane Design Study

Table 8. Engine Scaling Results

	Original Geometry	Optimized Design
Working fluid	Methane	Methane
Performance		
Power (kW)	1.25	10.0
Efficiency	0.32	0.4
Engine speed (rpm)	1000.0	845.0
Pressure (MPa)	2.74	11.35
Specific power (W/cc)	10.3	28.2
Pressure ratio	1.87	1.77
Dimensional		
Displaced volume (cc)	121.5	355.0
Total dead volume (cc)	190.4	387.0
Total engine height (m)	0.66	1.35
Crankcase height (m)	0.35	1.0
Preheater diameter (m)	0.3	0.33
Crankcase width (m)	0.35	0.7

6. CONCLUSIONS

Modern optimization theory provides a powerful tool for design of Stirling engines. A code system for optimization of an existing engine has been described and examples given of use of the code.

Extensive running of the code has established that: (1) the capability is inherent for satisfying the multiple (and often conflicting) requirements of a typical design, (2) solutions are obtained in very short computation times, making interactive use of the code possible, and (3) the code is quite "stable" in that a solution is usually found for a reasonably posed problem (an initial solution set "near" the final solution is needed with parameter values representing physically realizable conditions). The code gave good results when changing the power level of the engine, even when the change was an order of magnitude as shown in an example problem. In order that realistic solutions are found when scaling, spacial relationships between structural components must be correctly maintained as dimensions change. This requirement may mean that an engine-specific section of the code be provided for each new engine base design if scaling is to be done.

Future plans are to include more engine base designs in the code so that several types of engines are represented. Further work with constraints is needed to define useful constraint functions. While constraint functions have a strong influence on the final result, very little computation time is associated with treatment of constraints, making design refinement through use of constraints an important code capability.

ACKNOWLEDGMENTS

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APPENDIX A. DESCRIPTION OF OBJECTIVE AND CONSTRAINT FUNCTIONS

The following expressions have been formulated for use as objective and constraint functions during an Stirling engine design optimization problem. The user is able to provide his own expressions as explained earlier.

A.1 OBJECTIVE FUNCTIONS

One of the following functions will be assigned as the optimization function according to the value assigned to the integer variable NOBJTV in the input data (Appendix E).

1) **Efficiency Optimized**

Equation	Comments
obj = $1 - \eta$ where η = indicated efficiency	The design may result in large dead volumes, large flow areas and low power density.

2) **Minimum Entropy Production**

obj = $Q_h/T_h + Q_c/T_c$ where Q_h, T_h, Q_c, and T_c are the heat transferred and the temperature at the heater and cooler, respectively	The resulting design is similar to that obtained with the first objective function, but with improved power density.

3) **Power Optimized**

obj = P_1/P where P_1 is an initial value of power and P is the current value	The resulting design has little dead volume, small flow areas and low efficiency.

4) **Power per Total Engine Volume Optimized**

obj = Vol/P where Vol is the total engine volume (see App. D for a formulation for the GPU-3)	The resulting design emphasizes compactness and high power. Higher power density can be obtained by specifying power and efficiency through constraint functions (see below).

5) **Amount of Regenerator Wire Mass Minimized**

obj = VW/VW_1 where VW is the current volume of regenerator wire, and VW_1 is the initial volume	Use can greatly reduce the amount of wire in the regenerator matrix and hence cost. Efficiency must be constrained or a low efficiency design might result.

6) **Power per Engine Swept Volume Optimized**

obj = $(P_1/VO_1)/(P/VO)$ where P_1, VO_1, P, and VO are initial and present values of indicated power and engine swept volume, respectively

The resulting design emphasizes high specific power as does the third objective above, but usually results in low displaced volume at the expense of a large crankcase.

A.2 CONSTRAINT FUNCTIONS

Performance Constraints

1. Power

C(1) = CURRENT POWER/DESIGN POWER - 1.0; design power given by DPOWER

When used as an equality constraint, the current power becomes equal to the design power; in an inequality situation the final value will be greater than or equal to the design power. Design power is either an input value, DPOWER, or the first calculated value when a value of 0.0 is input for DPOWER.

2. Efficiency or Heat Input

C(2) = CURRENT EFFICIENCY/DESIGN EFFICIENCY - 1.0

or

CURRENT HEAT INPUT/DESIGN HEAT IN - 1.0

design efficiency given by DEFFIC;
design heat in by QSORCE

The choice of efficiency or heat input for this constraint is related to the input values. Generally speaking, heat input is the preferred constraint, since more variables contribute to its determination.

3. Piston Velocity

C(3) = 1.0 - CURRENT PISTON VELOCITY/5.0; design piston velocity = 5.0 m/s

A piston velocity of 5 m/s (1100 ft/min) is in the moderately high range for conventional engines and is chosen as an upper design limit. This constraint is a typical internal-combustion engine constraint that relates rings, bearings, loads, and performance criterion.

4. Engine Speed

C(4) = CURRENT ENGINE SPEED/60.0-1.0; minimum frequency = 1 Hz

Frequencies below 1 Hz (60 rpm) were arbitrarily considered to be too low for good design. This constraint can be replaced by design variable FREQ (frequency), and upper and lower limits directly applied. Only in very large engines might this constraint be active (having a calculated value near 0.0).

Dimensional Constraints

These constraints maintain realistic physical relationships. They are engine-dependent and new values of heat flux or tube to tube sheet arrangements may be necessary for engine designs other than the GPU-3.

5. Heater flux

   C(5) = 1 - CURRENT HEATER HEAT FLUX/DESIGN HEAT FLUX

   The design heat flux comes from the specified heater head heat input limit divided by its surface area. This constraint assumes that the GPU-3 heater head will not exceed this limit (400 kW/m^2). The code uses only 70% of the peak conditions for design point conditions, thereby lowering the maximum design flux to 280 kW/m^2 (35 hp/ft^2).

6. Cooler Flux

   C(6) = 1 - CURRENT COOLER HEAT FLUX/DESIGN HEAT FLUX

   The cooler design heat flux assumes a 40% efficient engine that yields a final design heat flux of 170 kW/m^2.

7. Regenerator Height

   C(7) = REGHT/REGENERATOR RADIUS - 1.0

   Heater tubes need room below the combustion zone to bend and link to the regenerator and expansion zone. In the original design eight regenerators are arranged in a ring about the cylinder and 40 heater tubes in a ring above the regenerators. To allow heater the room necessary for heater tubes to bend and connect smoothly to the regenerator, the minimum clearance in the region is held to the regenerator radius.

8. Heater Ring

   C(8) = 1 - (REGENERATOR RING RADIUS - HEATER RING RADIUS)/
   (REGENERATOR DIAMETER/3)

   The heater tube ring and the regenerator ring radii are kept close to the same size. The maximum separation is assumed to be ±1/3 of the regenerator diameter.

9. Cooler Size

   C(9) = REGENERATOR DIAMETER/(COOLER TUBE PITCH - NUMBER OF
   COOLER TUBE RINGS)

   The cooler tube bundle is assumed to have an equilateral pitch typical of a shell and tube heat exchanger. Fraas and Ozisik [A1] give the spacing requirements, assuming a central tube position. This yields the tube spacing shown in Appendix A to be related to the cooler tube wall.

10. Heater Size

    C(10) = HEATER TUBE LIGAMENT/6.0 WALL THICKNESS - 1.0

    A typical value for the tube spacing ligament to wall thickness ratio is shown in Appendix B to be 6.0. It is assumed that there are two rows of heater tubes surrounding the expansion cylinder and that the spacing between tubes cannot be less than six times the heater tube wall thickness.

11. Cold Connecting Duct

C(11) = (CONNECTING DUCT FLOW AREA/(1/2 COOLER TUBE BUNDLE
FLOW AREA)) - 1.0

The cooler to compression zone connecting duct must have a reasonable flow area if the assumption of zero pressure drop in the duct is to be justified. A value greater than half the cooler flow area is assured with this constraint.

12. Drive Mechanism

C(12) = CONNECTING ROD LENGTH/(ECCENTRICITY + CRANK RADIUS) - 1.0

Links in the drive mechanism must allow a crank device to operate smoothly and deliver near sinusoidal motion to the piston. It is shown [A2] that the piston position versus crank angle can be described as

$$x(\alpha) = r \cos \alpha + (\ell^2 - (e - r \sin \alpha)^2)^{1/2}.$$

This form not only shows that constraint 12 must be greater than zero but also that the eccentricity must be either 0.0 or greater than the crank radius to preserve the need for fewer harmonics. This latter constraint is handled by choosing an input design variable to be the eccentricity minus the crank radius.

Stress Constraints

As the components change in shape, the stresses in the materials change. As the temperature changes, the strength changes as well. The derivations of the stress and strength formulas used are given in Appendix B. The combined stresses, pressure plus thermal, are limited by the 2% material yield strength at temperature. All the pressure terms are multiplied by PRATIO (1.4) to allow the maximum design point conditions to have an effect. The stresses were all multiplied by a safety factor that has been set to 2.0 in the initialization of the limits routine.

13. Regenerator Axial Stress

C(13) = 1 - (SAFETY FACTOR)$^{1/2}$ (AXIAL THERMAL + PRESSURE STRESS)/
2% YIELD STRENGTH

This stress is due to the temperature profile along the wall and the effect of pressure in the regenerator causing the ends to change size differently than the cylindrical walls. Appendix B details the two stress terms and gives forms for designs other than the GPU-3.

14. Thermal Stress and Hoop Stress in Heater Tubes

C(14) = 1 - (SAFETY FACTOR)$^{1/2}$ (RADIAL THERMAL + PRESSURE STRESS/
2% YIELD STRENGTH

This stress is due to the radial temperature gradient in the heater tube walls and the thick-walled hoop stress. Appendix B details these two stress terms.

15. Displacer Dome Buckling

C(15) = 1 - SAFETY FACTOR (MAX. PRESSURE - MIN. PRESSURE)/CRITICAL PRESSURE

The displacer dome should be able to withstand the maximum foreseeable pressure change and not buckle. The critical pressure correlation given by Roark and Young [A3] is given in Appendix B.

16. Displacer Cylinder Buckling

C(16) = 1 - SAFETY FACTOR (MAX. PRESSURE - MIN. PRESSURE)/CRITICAL CRITICAL PRESSURE

The displacer cylinder also sees the same pressure effects as the dome and must not buckle. It is felt that this constraint on buckling is more stringent than that due to an axial temperature gradient or the pressure-induced end condition deflection situation. Appendix B gives the critical pressure formulation as well as a formulation for the pressure-temperature stress.

17. Cooler Tube Buckling

C(17) = 1 - SAFETY FACTOR (MAXIMUM PRESSURE/CRITICAL PRESSURE)

This constraint recognizes that the cooling water is at ambient conditions and that the only thing supporting the cooler tube sheets are the tube walls and can sides. An excessive gas pressure would cause the tubes to buckle, as is shown in Appendix B.

18. Displacer Piston Rod Buckling

C(18) = 1 - SAFETY FACTOR*(MAX. PRESSURE - MIN. PRESSURE)/CRITICAL PRESSURE

The displacer piston rod must support a load similar to a building column. The formulas are presented in Appendix B and the critical length for this drive system explained there. A fatigue formulation for this constraint might be an alternative limitation.

19. Cylinder Hoop Stress

C(19) = 1 - SAFETY FACTOR*STRESS/RUPTURE STRENGTH

Hoop stress occurs in the displacer cylinder due to the maximum engine pressure in the system. The rupture strength used is according to the 100 hour strength limit given in Appendix B.

20. Regenerator Wall Hoop Stress

C(20) = 1 - SAFETY FACTOR*STRESS/RUPTURE STRENGTH

The regenerator walls also experience hoop stress due to the maximum engine pressure.

Model Assumptions

As the component dimensions change, so too must the pressure drops, velocities, and temperatures in the engine thermodynamic section. In the development of the analytical models used in SEAM1, assumptions are made to allow solution of the governing equations. The following constraints ensure that these assumptions are valid.

21. Regenerator Profile

$$C(21) = 1 - 50*\frac{\partial T_{wire}}{\partial t} \quad U\frac{\Delta T}{\Delta X}$$

The regenerator wire temperature profile is assumed to be constant over the cycle with a correction then applied for small temperature variations. To ensure that the temperature variation is small, the wire temperature derivative must be substantially (a factor of 50) smaller than the gradient of the gas temperature times its velocity. This implies that the gas temperature derivative in time is also 50 times greater than the wire temperature derivative.

22. Inertial Effect in Heater Tube

C(22) = 1 - 20.0*HEATER GAS MACH NUMBER

This constraint limits the gas velocity to a value less than 5% of the acoustic velocity as calculated from the heater temperature and minimum gas pressure. Any inertial effects are thus minimized.

23. Inertial Effect in Cooler Tube

C(23) = 1 - 20.0*COOLER GAS MACH NUMBER

The comments above apply here.

24. Regenerator Reynolds Number

C(24) = 1 - REGENERATOR REYNOLDS NUMBER/750.0

The current steady flow correlations for heat transfer and fluid friction are typically derived from experimental data where the Reynolds number is less than 750. This limit prevents the actual Reynolds number exceeding the regime where the present correlations are valid.

25. Maximum Friction Pressure Correction

C(25) = 1 - 5*FRICTIONAL WORK/COMPRESSION SPACE WORK

In the SEAM1 analysis the expansion and compression work integrals are evaluated at the same pressure. Because there are frictional pressure drops, at least one of the integrals needs to be corrected. In the current version the expansion space is assumed to be correct and a correction is applied to the compression space work integral. This constraint ensures that the correction remain small by requiring that the frictional work be no more than 20% of the compression work. A derivation of this term is given in [A2].

26. Minimum Friction Pressure Correction

    C(26) = 20.0*FRICTION WORK/COMPRESSION WORK - 1.0

    This constraint forces a finite correction to exist and thereby insures the validity of the compression work integral assumption discussed previously.

27. Spatial Pressure Variation

    C(27) = 1.0 - 10*TOTAL PRESSURE DROP/MINIMUM GAS PRESSURE

    In the SEAM1 analysis there is an assumption that, at any time during the cycle, the spatial variation in pressure is small relative to the absolute value. This constraint maintains the validity of the assumption.

28. Stroke to Displacer Length Ratio

    C(28) = 0.9 - STROKE/DISPLACER LENGTH

    In the derivation of the displacer-cylinder wall gap loss equations, an assumption is made that the stroke is smaller than the displacer length. This constraint retains the validity of the assumption.

29. Regenerator Wall

    C(29) = 1 - 5*REGENERATOR WALL THICKNESS/REGENERATOR DIAMETER

    In the thermal stress analysis an assumption is made that the wall thickness was small. This constraint preserves the assumption.

REFERENCES—Appendix A

A1. Fraas, A. P. and M. N. Ozisik, Heat Exchanger Design, Wiley and Sons, 1965.

A2. Heames, T. J. et al., "Stirling Engine Thermodynamic Analysis: A Users Guide to SEAM1," ANL-82-59, September 1982.

A3. Roark, R. J. and W. C. Young, Formulas for Stress and Strain, 5th Ed. McGraw-Hill, 1975.

APPENDIX B. ANALYSIS OF ENGINE STRESSES

The high internal pressure and temperature necessary for Stirling engine compactness, power, and efficiency can cause excessive material stresses. In addition, temperature and lifetime ratings lead to low allowable design strength. This appendix details the strength and stress formulations used for constraint calculations. Three types of stress situations are described:

- Combined stresses due to both temperature and pressure and limited by a short-term strength criterion,
- Pressure-induced hoop stresses that are limited by a rupture strength that is affected by the duty cycle, and
- Buckling limitations due to pressure causing compressive yielding to occur.

B.1 COMBINED STRESSES

Combined stresses are due to both pressure and thermal gradients. Typical locations are the regenerator, displacer, and expansion zone walls, where an axial gradient can be found, and in the heater tubes, where a radial gradient is found. The pressure and thermally induced stresses are calculated separately and then added for a worst-case scenario.

B1.1 Axial Pressure Stresses

Bending stress in a sealed cylindrical tube under pressure is created by the end caps and its magnitude is determined by the method of attachment and shape of the cap. This series is referred to as a discontinuity stress by most authors, as it is related to the difference in expansion of the two shapes [B1]. The radius of the cylinder will dilate due to internal pressure P, as

$$d_c = Pr^2(2-\nu)/2Eh \ ,$$

where

r = radius,

ν = Poissons ratio,

E = modulus of elasticity, and

h = thickness of the wall.

When the dome is spherically shaped, the radius will dilate as

$$d_s = Pr^2(1 - \nu)/2Eh \ ,$$

and in the case of a rigid support plate or casting,

$$d_p = 0 \ .$$

where C equals 2.1 in the first case and 2.4 in the second. Since both cases are felt to give reasonable approximations to actual temperature profiles, an average value of 2.5 is used.

A slightly different solution is found for the case of a linear temperature profile occurring along the axis of part of cylinder and constant temperature for the remainder, or

$$T = T_0 + (T_L - T_0)\, x/L \qquad 0 < x \leq L$$

$$T = T_L \qquad x > L$$

The corresponding expression for maximum thermal stress in the cylinder is

$$\sigma_T = \frac{.329}{(1 - L^2)^{.75}} \frac{E\,\alpha}{L} \sqrt{rh}\,(T_L - T_0)\ .$$

Both of the above correlations are included in SEAMOPT. The latter expression was used to calculate thermal stress in the GPU-3 regenerator.

B1.4 Circumferential Thermal Stress

The stresses due to the radial temperature gradient in a heater or cooler tube can be calculated from [3]

$$\sigma_{TH} = \frac{E\alpha(T_{in} - T_{out})}{2(1 - \nu)\,\ell n\,\frac{r + h}{r}} \left[1 - \frac{2(r + H)^2}{(r + h)^2 - r^2}\,\ell n\left(\frac{r + h}{r}\right)\right].$$

This is the form used for the GPU-3 heater tube wall analysis.

B1.5 Short Term Yield Strength

Figure B-1 and Table B-1 show the short-term yield or 2% offset strength for Type 310 stainless steel. The code uses a linearized form to approximate the short term yield strength.

$$Y_s = 307 - 0.163T\ ,$$

where Y_s = yield strength (MN/m^2) and

T = wall temperature (K) .

This expression is shown as the solid line in Fig. B-1. It should be remembered that these combined stresses will be relieved due to creep in a short time. For this reason a reduced factor of safety, 1.4 is used for these calculations.

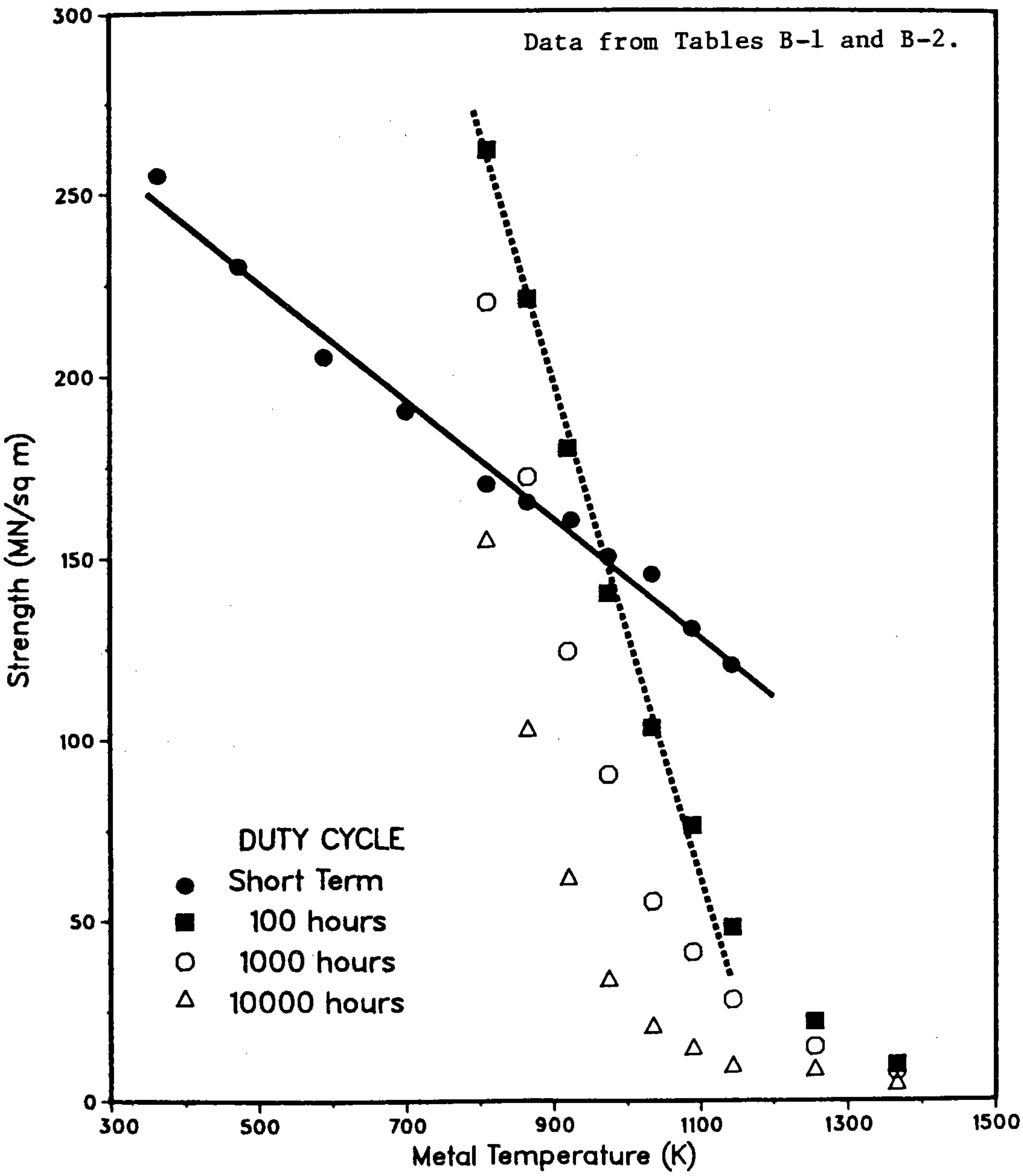

Fig. B-1. Type 310 Stainless Steel Strength vs. Temperature

B.2 HOOP STRESSES AND RUPTURE STRENGTH

The formulation for thick-wall pressure-induced hoop stress is given in the previous section. Because this stress acts through the entire duty cycle, it should be compared with the rupture strength. Table B-2 and Fig. B-1 show the rupture strength at 100, 1000, and 10,000 hours for a range of temperatures. These data have been linearized for use in the LIMITS routine as:

$R_s = 810.0 - 0.68T$ for 100 hr life ,

where R_s = rupture strength, MN/m^2,

T = wall temperature, K

$T < 1140$ K (1600 F),

$R_s = 810.0 - 0.74T$ for 1000 hr life ,

$T < 1060$ K (1450 F), and

$R_s = 810.0 - 0.81T$ for 10,000 hr life

$T < 980$ K (1300 F).

The code uses the 100 hour life equation shown by the dotted line in Fig. B-1, as this was the original design condition. The full safety factor is used for the data in Fig. B-1 and in the constraint equations.

B.3 BUCKLING STRESSES

Buckling stresses occur when external gas pressure is applied to structures such as the displacer and power pistons. This section discusses the buckling stresses in the displacer dome, cylinder, and piston rod and on the cooler tube walls and tube bundle sheets.

Displacer buckling is considered an elastic stability problem. Roark and Young [B5] give the following formulas:

For a thin walled sphere, like the <u>displacer dome</u>, under uniform external pressure the critical buckling pressure P_c is

$$P_c = 0.365\ Eh^2/r^2 \ .$$

This form yields the probable minimum pressure and is valid for $r > 10h$. The code uses it with a safety factor of two.

For a thin walled cylinder under uniform external pressure the critical buckling pressure is

$$P_c = 0.807\ \frac{Eh^2}{lr^2} \left[\left(\frac{1}{1 - r^2} \right)^3 \frac{h^2}{r^2} \right]^{1/4} ,$$

Table B-1. Yield Strength vs. Temperature for Type 310 Stainless Steel [B3]

Temperature K	Strength MN/m^2
365	255
475	230
590	205
700	190
810	170
865	165
925	160
975	150
1035	145
1090	130
1144	120

Table B-2. Rupture Strength vs. Temperature for Type 310 Stainless Steel

Temperature	Rupture Strength, MN/m^2		
K	100 hr [B4]	1000 hr [B4]	10,000 hr [B3]
810	262	220	155
865	221	172	103
920	180	124	62
975	140	90	34
1035	103	55	21
1090	76	41	15
1144	48	28	10
1255	22	15	9
1366	10	8	5

where l = length of the cylinder.

This form is valid for $r > 10h$ and is used by the code with a safety factor of two.

For a column, such as a displacer rod, that experiences continuous compressive loading, failure is due to buckling. The buckling pressure can be found from [6]

$$P_c = S_{cm}\left[1 - \frac{S_{cm}}{113\ E}\left(\frac{l}{r}\right)\right]^2 ,$$

where S_{cm} = allowable compressive stress, MN/m^2,

l = unsupported length, m, and

r = piston rod radius, m.

For steels typical of the GPU-3, this is

$$P_c = 117.2 - 0.003343\left(\frac{l}{r}\right)^2 .$$

This is the form used in the constraint equations with a safety factor of two. The critical dimension is the length; the code uses the maximum of either the bore plus twice the stroke to represent the rod in the engine or the eccentricity plus twice the connecting rod length to represent the rod in the crankcase. This formulation does not have a fatigue limit and is not entirely satisfactory, but it is all that was currently available.

The cooler tube bundle experiences buckling stress due to the pressure differential across the tube sheets. It is assumed that the cooler water pressure is small compared with the gas pressure and that the tube walls will support 1/3 of the load. The remaining 2/3 is assumed to be supported by the bundle enclosure walls. The load on the tube sheet is

$$\text{Load} = P(A_R - A_C) ,$$

where P = maximum pressure, MPa,

A_R = frontal area of regenerator, m^2, and

A_C = flow area of cooler, m^2.

The Euler formula for the maximum strength is

$$S_T = \frac{4\pi E A_w}{(l/r)^2} ,$$

where E = modulus of elasticity, MPa),

A_w = cross-sectional area of cooler tube walls, m^2,

l = cooler length, m, and

r = radius of gyration of the cooler, m.

These can be combined to yield the critical buckling pressure, P_c:

$$P_c = \frac{4\pi E A_w}{(l/r)^2 (A_R - A_C)} .$$

This is the form used in the constraint equations with a safety factor of two and a factor of 1/3 to represent the load carried by the tube walls.

The tube to tube sheet connection is typically made by expanding the tube to fill the hole in the sheet. This implies that the tube material yield strength must be less than or equal to the sheet material. The minimum ligament between tubes can be determined from stress considerations [1]. The stress at the ligament midpoint is

$$\sigma = \frac{S_s}{2} + 3S_t \frac{h}{w} ,$$

where S_s = yield strength of the sheet material, MN/m^2,

S_t = yield strength of the tube material, MN/m^2,

h = tube wall thickness, m, and

w = ligament width, m.

For the case of $S_s = S_t = \sigma$, the minimum ligament will be $w = 6h$. If the joint has a weld or additional connector, the minimum width will be lower. The code computes the width to thickness ratio for the cooler tube bundle and maintains the relationship. In the case of the the heater to expansion space joints the factor of six is used as a limit.

REFERENCES —Appendix B

B1. J. F. Harvey, "Theory and Design of Modern Pressure Vessels," 2nd Edition, Van Nostrand, 1974.

B2. D. T. Johns, "Thermal Stress Analysis," Pergamon Press, 1965.

B3. General Motors Research, "A Collection of Stirling Engine Reports from General Motors Research - 1958 to 1970," GMR-2690, 1978.

B4. D. Peckner and I. M. Bernstein, "Handbook of Stainless Steels," McGraw-Hill, 1978.

B5. R. J. Roark and W. C. Young, "Formulas for Stress and Strain," 5th Edition, McGraw-Hill, 1975.

B6. T. Baumeister (Ed.), "Marks Standard Handbook for Mechanical Engineers," 8th Edition, McGraw-Hill, 1978.

APPENDIX C. MODIFICATIONS TO SEAM1 AND VMCON

This appendix concers modifications to the two codes as they were available from the National Energy Software Center in January 1984. Those who received a copy of SEAMOPT also have the changes discussed here.

C.1 MODIFICATIONS TO SEAM1

There were many modifications to the basic code that are related to pointers, counters, and general input/output. This section concerns only modifications that produced a notable change in the output. Changes can be loosely grouped into three areas--those affecting the speed of computations, those allowing an extension to new models or designs, and those affecting accuracy.

Two changes enhanced the speed of calculation. The first deals with initialization. When SEAM1 starts a calculation the volume variation with crank angle is stored. The code was modified so that this initialization is skipped if the crank mechanism is unchanged. The second modification involved replacing the evaluation of x^a with a Taylor series expansion around the current value. The computation time per case dropped from 2 seconds to 0.5 seconds. The time per step subsequently increased to 0.8 seconds, as an additional inner iteration was added to maintain accuracy.

Calculation has been extended to include two additional types of effects. In the expansion and compression zone the gas was assumed to go through an adiabatic change. The change is polytropic probably, and not adiabatic. The difference between the adiabatic and polytropic effects has been called a hysteresis loss. Because the value of the polytropic exponent is somewhere between the specific heat ratio and a nominal value of 1, an input was added to allow the user to choose a degree of imperfection. A value of 0 yields the adiabatic result and a value of 1, an isothermal result. A value between 0 and 1 would yield a typical polytropic case. The code will also use the average zone temperature to compute the specific heat. This effect is noticeable only for gas with a significant temperature variation of specific heat, like methane. The second area of extension concerns the connecting ducts from variable volumes to heat exchangers. A modification of the input option, MQFLOW = 4, was added to allow for a half adiabatic and half isothermal zone to correspond to results at General Motors [C1]. Of potential importance was a modification to the friction and heat transfer library that was added to allow a user-dependent set of correlations to be included. We currently are using the friction formulation from H. Miyabe [C2].

There were several changes to the code to improve accuracy. In the energy deposition routine both the frictional pressure drop and regenerator heat transfer coefficient were related to interface conditions rather than middle cell conditions. A similar low-level improvement was the renormalization of the dead volumes to reflect the current rather than the last iterative temperatures. The final change dealt with the volume variation with crank angle routine. It can now use different piston areas for all designs except those within one cylinder where the piston positions overlap as in rhombic drives.

All other modifications related to linking problems found in attaching VMCON to the SEAM system were concerned with the correct order of calculation.

C.2 VMCON MODIFICATIONS

VMCON was modified for this implementation. Because the calculation of the performance by SEAM1 uses about 1 second per case, it was decided that the calculation of the gradients should be made only when needed to reduce calls to SEAM1. At the same time, the optimization solution was modified to include an input upper and lower limit for each design variable.

A limit was imposed upon the magnitude of change in each variable during a optimization step since large changes often lead to physically unrealizable sets of variables in SEAM1. The primary method found to control the problem was to limit the change in any design variable to a factor of two in any single step. Although this requires a few more steps, the resulting stability of solution was worth the cost.

When VMCON passes a solution to SEAM1 that does not result in an improvement of the situation, a line search is begun. The assumption is that somewhere between the last accepted set of design variables and the current set there must exist a better set. VMCON used several techniques to estimate where that set is. It assumes that the line must lie on the linear plane connecting the two sets, for example, halfway between the two pressures, speeds, etc. The code originally had a limit of 10% as the minimum change and 10 changes before failure. The code currently uses a 20% minimum and 13 changes over two steps before failure. This latter change allows the Hessian to be set to a singular array and a unique step to be taken before failure is accepted.

Several cosmetic changes were also made to reduce the volume of output and to allow the current convergence errors to be passed to the main program for editing.

REFERENCES—Appendix C

C1. General Motors Research, "A Collection of Stirling Engine Reports from General Motors Research - 1958 to 1970," GMR-2690, 1978.

C2. H. Miyabe et al., "An Approach to the Design of Stirling Engine Regenerator Matrix Using Packs of Wire Gauzes," Proc. of 17th IECEC, Paper No. 829306, 1982.

APPENDIX D. DEFINITION OF VARIABLES

Name	Common	Defined	Definition
AFLOW	(FLCELL)	INPUT	Gas flow area per tube for a cell, m^2
AWET	(FLCELL)	INPUT	Wetted area per tube for a cell, m^2 Hydraulic dia = 4.0•volume/AWET
AXWALL	(FLCELL)	INPUT	Cross-sectional area of wall for heat conduction for one tube, m^2
BASE		OPTSEM	Difference between R10 as measured along the centerline or along R10 (see Fig. 2)
BNDL		OPTMIZ	Array of normalized lower limits for design variables
BNDU		OPTMIZ	Array of normalized upper limits for design variables
CCC	(FIOPT)	OBJCTV	Array of design variables (see Table 6)
CCHI		OPTINP	Array of upper limits per design variables
CCLOW		OPTINP	Array of lower limits per design variables
CCNORM	(FIOPT)	OPTINP	Array of normalizing value for design variables
CMPAP	(VARVOL)	INITAL	Compression piston area, m^2
CMPBOR	(VARVOL)	INITAL	Compression cylinder bore, m
CMPCR	(FLTGEN)	INPUT	Compression piston crank radius, m
CMPCRL	(FLTGEN)	INPUT	Compression piston connecting rod length. m should be less than 1000*CMPCR but greater than CMPCR+CMPECC
CMPDEL	(VARVOL)	VOLUME	Overlap in compression zone piston motion typical of displacer machines, m
CMPDV	(FLTGEN)	INPUT INITAL	Compression zone dead volume, m^3 will be treated the same as active volume (i.e., will be adiabatic in SEAM1)
CMPECC	(FLTGEN)	INPUT	Compression piston crank radius eccentricity, m should be greater than CMPCR for a rhombic drive
CMPGAP	(FLTGEN)	INPUT	Compression piston cylinder wall gap, m
CMPRD	(FLTGEN)	INPUT	Compression piston rod diameter, m
CMPSRK	(VARVOL)	VOLUME	Compression piston net stroke, m
CNORM		OBJCTV	Derivative of constraint functions with respect to the design variables
CNSTR	(FIOPT)	LIMITS	Current value of constraint functions, Section 3.2
COLDLN	OPTSEM		Distance from regenerator top to cooler base, m
CONF		OBJCTV	Value of constraints at start of gradient calculation

DEL	(FIOPT)	OPTINP	Fractional change in design variable used to determine gradient
DEFFIC		INPUT	Design efficiency
DESGNE	(FIOPT)	OPTINP LIMITS	Desired efficiency, uses DEFFIC or initial calculated value when DEFFIC < 0
DESGNP	(FIOPT)	OPTINP LIMITS	Desired power, uses DPOWER or initial calculated value when DPOWER < 0, W
DESGNQ	(FIOPT)	OPTINP LIMITS	Desired heat input, uses QSORCE or ratio of DESGNP to DESGNE when QSORCE < 0, W
DESGNS		LIMITS	Current yield strength, MN/m^2
DH	(FLCELL)	INPUT	Hydraulic diameter of a SEAM cell, m^2
DIABOT		LIMITS	Current diameter of GPU-3 crankcase, m See Fig. D-1,, set equal to 5 times the distance from the centerline to the crank DIABOT = 5 (EXPBOR/2 + EXPECC + EXPCR)
DIAOU		LIMITS	Cooler tube outer diameter, m
DIATOP		LIMITS	Current diameter of the GPU-3 burner + preheater, m^2. See Fig. D-1, set equal to 4 times the ring radius, R10, used in the constraint equations
EA		LIMITS	Young's modulus times the coefficient of linear expansion, Pa/K
ENGVOL		LIMITS	Current volume of GPU-3 engine, m^2; calculated from DIABOT, HTBOT, DIATOP, and HTTOP
EXPAP	(VARVOL)	INITAL	Expansion piston area, m^2
EXPBOR	(VARVOL)	INITAL	Expansion cylinder bore, m
EXPCR	(FLTGEN)	INPUT	Expansion piston crank radius, m
EXPCRL	(FLTGEN)	INPUT	Expansion piston connecting rod length. m
EXPDSP		LIMITS	Initial expansion space displacement, m^3
EXPDV	(FLTGEN)	INPUT INITAL	Expansion zone dead volume, m^3 will be treated the same as an active volume (i.e., will be adiabatic in SEAM1)
EXPECC	(FLTGEN)	INPUT	Expansion piston crank radius eccentricity, m should be greater than EXPCR for a rhombic drive
EXPGAP	(FLTGEN)	INPUT	Expansion piston cylinder wall gap, m
EXPRD	(FLTGEN)	INPUT	Expansion piston rod diameter, m
EXPSRK	(VARVOL)	VOLUME	Expansion piston net stroke, m
FGRD		OBJCTV	Derivative of objective function with respect to design variables
FREQ	(FLTGEN)	INPUT INPDRV	Frequency, Hz
FRICML	(FLTGEN)	INPUT	Friction multiplier for a cell
HAF	(AMISC)	INITAL	0.5

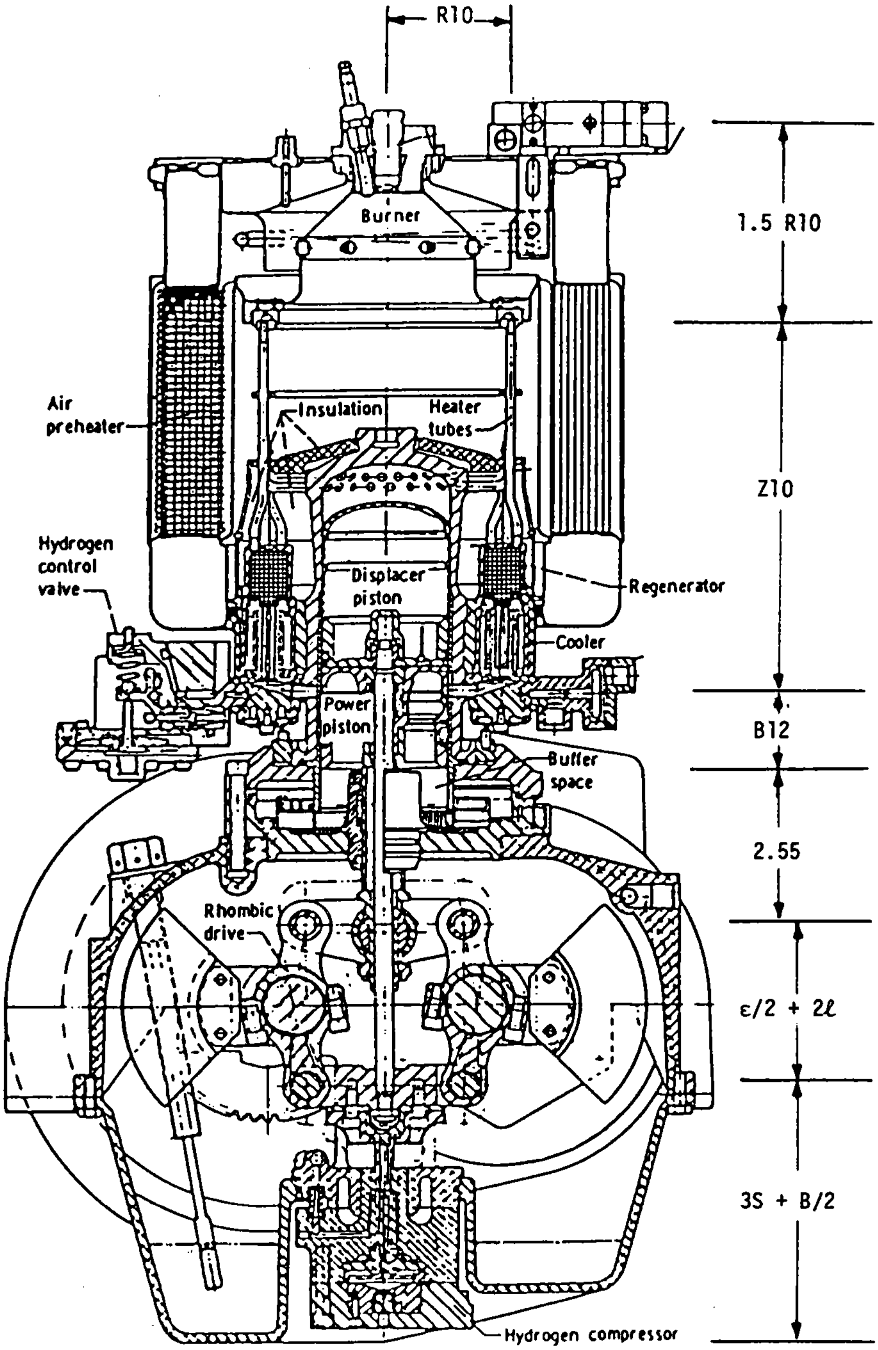

Fig. D-1. Cross Section of GPU-3 Engine

HEIGHT		LIMITS	Total height of GPU-3 engine, m
HTBOT		LIMITS	Height of crankcase of GPU-3 engine, m; see Fig. D-1; the height is approximated by summing the distances from the buffer space to the hydrogen compressor
HTRPWR		LIMITS	Maximum GPU-3 design heater head power (37.25 kW)
HTTOP		LIMITS	Height of engine + burner, m; see Fig. D-1, the height is approximated by summing the distances from the spark plug through the power piston
IERROR	(JMISC)	MAIN SEAM	General SEAM1 error key, if = 0 then valid calculation
INFO	(IIOPT)	VMCON OBJCTV	Optimization status variable -1 Error in SEAM1 or last calculation 0 Error in optimization input 1 Normal ending 2 MXEVAL exceeded 3 Line search failure 4 Uphill search found 5 No feasible answer 6 Singular matrix or quadratic subproblem failure
IOPT	(IIOPT)	OPTINP	SEAMOPT print control
IPRINT	(IIOPT)	INPUT	Optimization input print control
IRUN		MAIN	Control to initialize SEAM1 data SEMDRV
IVARY	(IIOPT)	OPTINP	Array of active design variable locations
JCELL	(JMISC)	INPUT OUT1	Cell number being input, after input completed JCELL=total number of cells in the engine; on last step JCELL is set to 0 to key timers
JCOLN	(JMISC)	INITAL	Last cooler cell
JCOL1	(JMISC)	INITAL	First cooler cell
JCOOL	(JMISC)	INITAL	Middle cooler cell
JFIXN	(JMISC)	INITAL	Last constant volume cell
JFIX1	(JMISC)	INITAL	First constant volume cell
JFLAG	(FLAGS)	VMCON	Optimization status JFLAG = 0, objective and constraint function values needed = 1, gradients of objective and constraint functions needed
JHOT	(JMISC)	INITAL	Middle heater cell
JHTRN	(JMISC)	INITAL	Last heater cell

JHTR1	(JMISC)	INITAL	First heater cell
JREG	(JMISC)	INITAL	Middle regenerator cell
JREGN	(JMISC)	INITAL	Last regenerator cell
JREG1	(JMISC)	INITAL	First regenerator cell
MHTRF	(INCELL)	INPUT	Nusselt number correlation desired
MCNSTR	(IIOPT)	LIMITS	Number of active constraints
MEQULC	(IIOPT)	OPTINP	Number of equality constraints
MKEY1	(IIOPT)	OPTINP	Pointer to indicate variable volume initialization necessary
MTOT		LIMITS	Number of tubes within ring (n); see Ref. D1
MXEVAL	(IIOPT)	INPUT	Maximum number of optimization iterations
NCASE	(IIOPT)	OBJTV	Total number of SEAM1 cases MAIN
NCODE	(IGEN)	MAIN	Pointer to control SEAM1 initialization SEMDRV OPTMIZ
NCYC	(IIOPT)	SEMDRV	Total number of SEAM1 inner iterations
NCYL	(IGEN)	INPUT	Number of cylinders in engine cylinder=expan+htr+regen+cool+cmprs zones
NEQULC	(IIOPT)	INPUT	Input control of equality constraints
NPRT2	(IGEN)	INPUT	Number of cases to be edited together
NIHX	(INCELL)	INPUT	Number of tubes per cylinder
NVAR	(IIOPT)	INPUT	Number of design variables in use
OBJ	(FIOPT)	LIMITS	Current value of objective function, Section 3.1
OBJNRM	(FIOPT)	INPUT	Objective function multiplier, code sets OBJ to OBJNRM at start of calculation
ONE	(AMISC)	INITAL	1.0
ORIFCE	(FLCELL)	INPUT	Cell orifice pressure drop coefficient equivalent to constant Fanning friction
OUTDV	(FIOPT)	OPTINP	Design variable mneumonic array
OUTCON	(FIOPT)	LIMITS	Constraint function mneumonic array
PCRIT		LIMITS	Current value of critical buckling pressure, Pa
PDROPC	(PDROP)	HTFLOW	Friction pressure drop in cooler, Pa
PDROPH	(PDROP)	HTFLOW	Friction pressure drop in heater, Pa
PDROPR	(PDROP)	HTFLOW	Friction pressure drop in regenerator, Pa
PGAS	(FLCELL)	INPUT INITAL	Gas pressure in cell, Pa
PHASED	(FLTGEN)	INPUT INITAL INPDRV	Crank angle between expansion volume minimum and compression volume minimum, deg
PI	(AMISC)	INITAL	3.14159...
PMAX	(ENFLOW)	HTFLOW RIOSIN	Current maximum pressure in cycle, Pa

PMEAN	(FLTGEN)	INPUT INITAL	Mean pressure in cycle, Pa
PMEAND	(DYNPRP)	INITAL RIOSIN HTFLOW	Current mean pressure in cycle, Pa
PMIN	(ENFLOW)	HTFLOW	Current minimum pressure in cycle, Pa
POROS	(FLCELL)	INPUT	Porosity (void fraction) of filler material
POWER	(ENFLOW)	ANALIS	Net indicated power in engine, W
PR	(RIOSD)	CYCLER	Dimensionless pressure at each step in cycle
PRATIO		LIMITS	Ratio of maximum mean working pressure to design mean working pressure
PTCHOD		OPTSEM	Pitch to diameter ratio of heater tubes
QACTC	(ENFLOW)	HTFLOW	Gas to cooler wall heat flow, W
QACTH	(ENFLOW)	HTFLOW	Gas to heater wall heat flow, W
QCYLWL	(ENFLOW)	LOSSES	Heat flow along cylinder wall, W
QDYNAM	(ENFLOW)	LOSSES	Sum of heat flow due to QEXPGP+QEXPSH, W
QFLOW	(FLCELL)	INPUT	Heat flow from cell wall to external environment. W
QFRIC	(ENFLOW)	HTFLOW	Total frictional heat flow, W
QKOLFR		HTFLOW	Heat flow due to cooler friction, W
QLEAK	(ENFLOW)	LOSSES	Heat flow due to gas leakage, W
QPISWL	(ENFLOW)	LOSSES	Heat flow along piston wall, W
QREGER	(ENFLOW)	HTFLOW	Total regenerator reheat loss, W
QREGFL	(ENFLOW)	LOSSES	Conduction heat flow thru filler material, W
QREGFR		HTFLOW	Heat flow due to regenerator friction, W
QREGWL	(ENFLOW)	LOSSES	Conduction heat flow along the regenerator wall, W
QSTATC	(ENFLOW)	LOSSES	Sum of all conduction heat flows, W QPISWL+QCYLWL+QREGWL+QREGFL
QSORCE		INPUT	Input design heater head heat flow, W
REGDIA	(FIOPT)	OPTSEM	Regenerator diameter, m
REGHT	(FLTGEN)	INPUT	Conduction height from the top of the regenerator to the top of the expansion cylinder, m
REYC	(PDROP)	HTFLOW	Reynolds number based on DH and XMDOTC, for the cooler
REYH	(PDROP)	HTFLOW	Reynolds number based on DH and XMDOTH, for the heater
REYR	(PDROP)	HTFLOW	Reynolds number based on DH and XMDOTR, for the regenerator
RGAS	(GASPRP)	INITAL	Reference gas constant J/kg-K
RHTR	(FIOPT)	OPTSEM	Radius of heater tube ring, m
RREGN	(FIOPT)	OPTSEM	Radius of regenerator ring, m

RUPTUR		LIMITS	Current value of 100 hour rupture strength
SAWC	(FIOPT)	OPTSEM	External surface area of active cooler, m^2
SAWH	(FIOPT)	OPTSEM	External surface area of active heater, m^2
SCALR	(OUTPT)	RIOSPR	Array containing 50 variables for output A full list of this container array for a single converged case is

Performance

1 engine pressure
2 speed
3 gas mass
4 total heat into engine
5 power out
6 efficiency
7
8

Expansion Zone

9 wall temperature
10 mean gas temperature
11 wall-gas heat flow
12 P-V work
13 connecting duct temperature

Heater Zone

14 combustor temperature
15 wall temperature
16 mean gas temperature
17 wall-gas heat flow
18 conduction heat flow
19 frictional pressure

Regenerator Zone

20
21 wall temp. @ JREG
22 mean gas temperature
23 wall-gas heat flow
24 conduction heat flow
25 frictional pressure

Cooler Zone

26 cooler water temp.
27 wall temperature
28 mean gas gemperature
29 wall-gas heat flow
30 conduction heat flow
31 frictional pressure
32

Compression Zone

33 wall temperature
34 mean gas temperature
35 wall-gas heat flow
36 P-V work
37 connecting duct temperature

Pressure Wave

38
39 maximum pressure
40 minimum pressure
41 initial pressure
42 P-exp. vol. phase angle

Total Heat Flows

43 conduction
44 piston gap + shuttle
45 frictional
46 regenerator reheat
47
48 cooler water
49 unaccounted
50 regenerator effectiveness

SFTYF		LIMITS	Safety factor used in stress calculations
SIGMA	(FLCELL)	INPUT	Ratio of minimum flow area to frontal area in a cell, SIGMA<=POROS

SLNGTH	(FADD)	INPUT	Length used for shuttle heat flow, m must be greater than the displacer stroke
SPACE		LIMITS	Design cooler bundle minimum ligament to tube wall thickness ratio
TCMBST	(FLTGN)	INPUT INPDRV INITAL	External heater (combustor) temperature, K
TGASD	(DYNPRP)	INITAL HTFLOW	Current gas temperature in a cell, K
TMASSD	(DYNPRP)	RIOSIN HTFLOW	Current total mass of working gas in engine, kg
TMAXC	(PDROP)	CYCLER	Maximum gas temperature during cycle in compression zone, K
TMAXE	(PDROP)	CYCLER	Maximum gas temperature during cycle in expansion zone, K
TOL		OPTINP	Convergence limit, 1.0D-4
TREGWD	(DYNPRP)	INITAL	Current regenerator wall temperature, K
TWALLD	(DYNPRP)	INITAL HTFLOW	Current wall temperature of cell, K
TWATER	(FLTGEN)	INPUT INITAL	Cooling water average temperature, K
TWENVR	(FLTGEN)	INPUT INITAL	Average environment temperature, K initalized to 300 K
TWO	(AMISC)	INITAL	2.0
VECTR	(OUTPT)	RIOSPR	Array of cyclic variables for each zone: 1 maximum gas temp. 2 average gas temp. 3 wall-gas avg. temp. 4 wall-gas max. temp. 5 wall-gas heat flow 6 gas mass flowrate 7 max. gas flowrate 8 avg. Reynolds number 9 pressure drop 10 avg. Nusselt number
VOL	(FLCELL)	INITAL	Cell volume, m^3
WALLTH	(FLCELL)	INPUT	Wall thickness of a cell, m
WIRED		OPTSEM	Regenerator wire diameter, m
WIRVOL		INITAL	Regenerator total wire volume, m^3
WORKC	(ENFLOW)	ANALIS	Compression space work, J
WORKE	(ENFLOW)	ANALIS	Expansion space work, J
XLNGTH	(FLCELL)	INPUT	Cell length, m
XMACH		LIMITS	Mach number of gas in a zone
XMDOTC	(PDROP)	HTFLOW	Average cycle mass flux in cooler, kg/s
XMDOTH	(PDROP)	HTFLOW	Average cycle mass flux in heater, kg/s
XMDOTR	(PDROP)	HTFLOW	Average cycle mass flux in regenerator, kg/s
XNU		LIMITS	Poissons ratio, 0.3
XNUSC	(PDROP)	HTFLOW	Nusselt number in cooler

XNUSH	(PDROP)	HTFLOW	Nusselt number in heater
XNUSR	(PDROP)	HTFLOW	Nusselt number in regenerator
ZERO	(AMISC)		0.0

REFERENCES—Appendix D

D1. Fraas, A. P. and M. N. Ozisik, Heat Exchanger Design, Wiley and Sons, 1965.

E.1 JOB CONTROL LANGUAGE NEEDED FOR IBM 3033 COMPUTER

```
//REGEFF   JOB ('11449862-0016399,B16399  ,  ,F16399  '),HEAMES,          *
// REGION=320K,CLASS=W,TIME=3,USER=B16399
//*MAIN ORG=ANLVM.B16399,LINES=25
//*
//*
//STEP1  EXEC PGM=IEBGENER
//SYSFRINT DD SYSOUT=A
//SYSUT1 DD *
//SYSUT2  DD DSN=&&DATA,DISP=(NEW,PASS,DELETE),UNIT=DISK,
//    SPACE=(TRK,(5,5)),DCB=(RECFM=FB,LRECL=80,BLKSIZE=2000)
//SYSIN  DD DUMMY
//*
//*            LIST INPUT FILE ON OUTPUT DEVICE
//*
//STEP2  EXEC SDSKLIST,INDSN='&&DATA',SYSOUT=A,PREFORM=A
//*
//*        LOAD SEAMOPT INTERFACE ROUTINES (OPT2)
//*             OPTIMIZATION CODE (VMCON)
//*             SEAM1 ANALYSIS MODULE (RIOS)
//*             SEAM LIBRARIES AND I/O (SEAM)
//*
//STEP  EXEC  PGM=LOADER,PARM='NOLET,RES,EP=MAIN,SIZE=320K,PRINT'
//SYSLIN    DD DSN=B16399.OPT2.OBJ,DISP=SHR
//          DD DSN=B16399.VMCON.OBJ,DISP=SHR
//          DD DSN=B16399.RIOS.OBJ,DISP=SHR
//          DD DSN=B16399.SEAM.OBJ,DISP=SHR
//SYSLIB    DD DSN=SYS1.AMDLIB,DISP=SHR
//          DD DSN=SYS1.FORTLIB,DISP=SHR
//SYSLOUT   DD SYSOUT=A
//*                LUN 5  STANDARD INPUT FILE
//*                LUN 6  STANDARD OUTPUT FILE
//*                LUN 7  RESTART FILE
//*                LUN 8  SEAM1 GRAPHICS FILE
//FT06F001  DD SYSOUT=A
//FT07F001  DD SYSOUT=A
//FT08F001  DD DUMMY
//FT05F001  DD DSN=&&DATA,DISP=(OLD,DELETE,DELETE)
```

E.2 INPUT DATA NECESSARY FOR CASE 1 OF EXAMPLE 1

```
GPU-3 REGENERATOR OPTIMIZATION      STIRLING ENGINE DATA: CASE 1
*
&GINPT          ICHNG=0,        IDESGN=3,         IDRIVE=1,
IGAS=1,         IPRT1=0,        IGRAPH=0,         NADD=2,
NCOMP=11,       NPRT2=10,       NCYL=1,           NOPT=1,
NPRT1=720,      NREVL=12,       NSTEP=360,
CMPCR=.0138,    CMPCRL=.0460,   CMPDV=1.093E-5,   CMPECC=.0208,
CMPGAP=.00015,  CMPRD=.0222,    EXPCR=.0138,      EXPCRL=.0460,
EXPDV=1.25E-5,  EXPECC=.0208,   EXPGAP=.00025,    EXPRD=.00952,
FREQ=50.000,    PMEAN=2.74E6,   STARTD=0.,        REGHT=0.0286,
TCMBST=927.0,   THATER=287.,    ZOPT1=0.5,
&END
*
EXPANSION SPACE
&CELL         II=0,        1,           12,          0,
              25,          1,           0,           1,
AA=3.86E-3,    0.0,        1.07E-3,     1.43E-2,     .0701,
   1.0,        7*0.,       927.,        0.,          3.54E-5,
   0.,         3.313E-2,   &END
*
EXPANSION SPACE TO HEATER
&CELL         II=0,        3,           12,          0,
              15,          40,          0,           1,
AA=7.16E-6,    0.0,        1.116E-5,    3.67E-4,     3.02E-3,
   1.0,        7*0.,       927.,        2.42E-7,     4.32E-7,
   0.,         .0397,      &END
*
HEATER (TUBES UP + 1/2 HEADER)
&CELL         II=0,        1,           12,          0,
              10,          40,          0,           1,
AA=7.16E-6,    0.0,        1.116E-5,    7.37E-4,     3.02E-3,
   1.0,        7*0.,       927.,        6.891E-7,    8.67E-7,
   0.0,        .0777,      &END
*
HEATER (1/2 HEADER + TUBES DOWN)
&CELL         II=0,        1,           12,          0,
              10,          40,          0,           1,
AA=7.16E-6,    0.0,        1.116E-5,    7.37E-4,     3.02E-3,
   1.0,        7*0.,       927.,        6.891E-7,    8.67E-7,
   0.0,        .0777,      &END
*
HEATER TO REGENERATOR
&CELL         II=0,        3,           12,          0,
              15,          40,          0,           1,
AA=7.16E-6,    0.0,        1.116E-5,    4.86E-4,     3.02E-3,
   1.0,        7*0.,       927.,        5.848E-7,    5.71E-7,
   0.0,        .0512,      &END
*
REGENERATOR         VOLUME LOWERED BY 5+% AS NASA TM79103
&CELL         II=0,        1,           12,          11,
              30,          8,           0,           5,
AA=1.877E-4,   0.0,        1.322E-4,    0.0,         9.35E-5,
   2.,         3*0.,       .697,        0.,          .468,
   0.,         634.,       6.325E-6,     2.99E-6,     0.,
   .0226,      &END
*
REGENERATOR TO COOLER TUBES
&CELL         II=0,        3,           12,          0,
              15,          312,         0,           1,
AA=9.16E-7,    0.0,        1.070E-6,    1.80E-5,     1.08E-3,
```

```
   1.0,       7*0.,       339.,       2.013E-8,  5.67E-9,
   2.55E-4,   .0053,      &END
*
COOLER
&CELL        II=0,       1,          12,        0,
             10,         312,        0,         1,
AA=9.16E-7,   0.,        1.070E-6,   1.20E-4,   1.08E-3,
   1.0,       3000.,      6*0.,       339.,      3.241E-8,
   3.80E-8,   2.55E-4,    .0355,      &END
*
COOLER TO CONNECTING DUCT TUBES
&CELL        II=0,       3,          12,        0,
             15,         312,        0,         1,
AA=9.16E-7,   0.0,        1.07E-6,    1.80E-5,   1.08E-3,
   1.0,       7*0.,       339.,       4.838E-9,  5.67E-9,
   2.55E-4,   .0053,      &END
*
CONNECTING DUCTS
&CELL        II=0,       3,          12,        0,
             15,         8,          0,         1,
AA=2.80E-5,   0.0,        3.8E-5,     0.0,       5.97E-3,
   1.0,       7*0.,       339.,       1.281E-6,  1.20E-6,
   0.,        3.18E-2,    &END
*
COMPRESSION SPACE
&CELL        II=0,       1,          12,        0,
             20,         1,          0,         1,
AA=3.77E-3,   0.0,        3.8E-5,     1.52E-2,   6.04E-2,
   1.0,       7*0.,       339.,       0.0,       3.54E-5,
   0.,        3.18E-2,    &END
*
  DISPLACER DOME
&ADDED       JJ=2,       1,          12,        10,
             0,          2,
BB=2*0.,      890.,       1.66E-4,    .04359,    .06642,
   .00159,    0.3,        4*0.,       .04359,    &END
*
POWER PISTON BUFFER SPACE
&ADDED       JJ=2*1,     12,         20,        2,
             0,
BB=3.23E+6,   0.,         315.,       5.21E-4,   0.,
   .0701,     .00381,     2*0.,       .15,       .002,
   &END

     END OF SEAM1 INPUT DESCRIBING ENGINE

    INSERT DESIRED OPTIMIZATION CASE INPUT HERE
     ***     ***     ***     ***     ***     ***     ***


        OPTIMIZER INPUT FOR CASE 1

  MAXIMIZE EFFICIENCY (NOBJTV=1) WHILE MAINTAINING POWER (NEQULC=0)

&OPTIN  NVAR=5, MXEVAL=15, IPRINT=0, XDEL=0.01, DPOWER=3250.0,
        NOBJTV=1,  NEQULC=0,  &END

    OPTIMIZE REGENERATOR VALUES

&OPTIN  IX=18,  &END
```

```
&OPTIN  IX=19,  &END
&OPTIN  IX=20,  &END
&OPTIN  IX=21,  &END
&OPTIN  IX=22,  &END

    NO ADDITIONAL INPUT

&OPTIN  IX=-1,  &END

      PERTURBATION ON SPEED USING FINAL RESULTS

&PERTIN ICHNG=0, NMODS=6, FREQ=16.667, DFREQ=8.334,
  TWLHI=890.0,    899.0,     909.0,     918.0,     927.0,
        937.0,    4*0.0,  ITWLCH=1,    &END

     END OF OPTIMIZATION CASE 1 DATA
*
```

E.3 INPUT DATA FOR CASE 2 AND CASE 3 OF EXAMPLE 1

```
*
*        OPTIMIZER INPUT FOR CASE 2
*
*    MINIMIZE WIRE VOLUME  (NOBJTV=5)
*
&OPTIN  NVAR=5, MXEVAL=15, IPRINT=0, XDEL=0.01,
        NOBJTV=5,  NEQULC=0,  &END
*
*    OPTIMIZE REGENERATOR VALUES
*
&OPTIN  IX=18,  &END
&OPTIN  IX=19,  &END
&OPTIN  IX=20,  &END
&OPTIN  IX=21,  &END
&OPTIN  IX=22,  &END
*
*    NO ADDITIONAL INPUT
*
&OPTIN  IX=-1,  &END
*
*      PERTURBATION ON SPEED USING FINAL RESULTS
*
&PERTIN ICHNG=0, NMODS=6, FREQ=16.667, DFREQ=8.334,
  TWLHI=890.0,    899.0,     909.0,    918.0,    927.0,
        937.0,    4*0.0,  ITWLCH=1,    &END
*
*     END OF OPTIMIZATION CASE 2 DATA
*
*
*        OPTIMIZER INPUT FOR CASE 3
*
*  MAXIMIZE POWER (NOBJTV=3) WHILE MAINTAINING EFFICIENCY (NEQULC=0)
*
&OPTIN  NVAR=5, MXEVAL=15, IPRINT=0, XDEL=0.01,
        NOBJTV=3,  NEQULC=0,  &END
*
*    OPTIMIZE REGENERATOR VALUES
*
&OPTIN  IX=18,  &END
&OPTIN  IX=19,  &END
&OPTIN  IX=20,  &END
&OPTIN  IX=21,  &END
&OPTIN  IX=22,  &END
*
*    NO ADDITIONAL INPUT
*
&OPTIN  IX=-1,  &END
*
*      PERTURBATION ON SPEED USING FINAL RESULTS
*
&PERTIN ICHNG=0, NMODS=6, FREQ=16.667, DFREQ=8.334,
  TWLHI=890.0,    899.0,     909.0,    918.0,    927.0,
        937.0,    4*0.0,  ITWLCH=1,    &END
*
*     END OF OPTIMIZATION CASE 3 DATA
*
```

E.4 INPUT DATA NECESSARY FOR CASE 4 OF EXAMPLE 1

Other required data supplied from restart file generated at completion of Case 1.

```
*
*
*        OPTIMIZER INPUT FOR CASE 4
*
*  MAXIMIZE EFFICIENCY (NOBJTV=1) WHILE MAINTAINING POWER (NEQULC=0)
*
&OPTIN  NVAR=5, MXEVAL=15, IPRINT=0, XDEL=0.01, DPOWER=3250.0,
        NOBJTV=1,  NEQULC=0,  &END
*
*    OPTIMIZE REGENERATOR VALUES
*
&OPTIN  IX=18,  &END
&OPTIN  IX=19,  &END
&OPTIN  IX=20,  &END
&OPTIN  IX=21,  &END
&OPTIN  IX=22,  &END
*
*   INSERT OPTIMIZED REGENERATOR VALUES
*
&OPTIN  DPOWER=  3250.0,  DEFFIC= 0.3661,  OBJNRM= 0.8937,  &END
&OPTIN  IX=18,  X= 3.5387D-02,   &END
&OPTIN  IX=19,  X= 2.7113D-02,   &END
&OPTIN  IX=20,  X= 1.2180D-03,   &END
&OPTIN  IX=21,  X= 6.1428D-01,   &END
&OPTIN  IX=22,' X= 4.9281D-05,   &END
*
*   OPTIMIZE COOLER VALUES NEXT
*
&OPTIN  NVAR=3, &END
&OPTIN  IX=24,  &END
&OPTIN  IX=25,  &END
&OPTIN  IX=26,  &END
*
*    NO ADDITIONAL INPUT
*
&OPTIN  IX=-1,  &END
*
*      PERTURBATION ON SPEED USING FINAL RESULTS
*
&PERTIN ICHNG=0, NMODS=6, FREQ=16.667, DFREQ=8.334,
  TWLHI=890.0,    899.0,     909.0,    918.0,    927.0,
        937.0,    4*0.0,  ITWLCH=1,    &END
*
*     END OF OPTIMIZATION CASE 4 DATA
*
```

E.5 INPUT DATA USED FOR EXAMPLE 2

```
GPU-3  METHANE  877K/287K        STIRLING ENGINE DATA:     (SAMPLE PROBLEM 2)
*
&GINPT           ICHNG=-1,        IDESGN=3,        IDRIVE=1,
IGAS=6,          IPRT1=0,         IGRAPH=0,        NADD=2,
NCOMP=11,        NPRT2=6,         NCYL=1,          NOPT=1,
NPRT1=720,       NREVL=12,        NSTEP=360,
CMPCR=.0138,     CMPCRL=.0460,    CMPDV=1.093E-5,  CMPECC=.0208,
CMPGAP=.00015,   CMPRD=.0222,     EXPCR=.0138,     EXPCRL=.0460,
EXPDV=1.25E-5,   EXPECC=.0208,    EXPGAP=.00025,   EXPRD=.00952,
FREQ=16.67,      PMEAN=2.74E6,    STARTD=0.,       REGHT=0.0286,
TCMBST=890.0,    TWATER=287.,     ZOPT1=0.5,
ICHNG=0,         TCMBST=877.0,
&END
*
EXPANSION SPACE
&CELL        II=0,       1,          12,         0,
             25,         1,          0,          1,
AA=3.86E-3,   0.0,       1.07E-3,    1.43E-2,    .0701,
   1.0,       7*0.,      877.,       0.,         3.54E-5,
   0.,        3.313E-2,  &END
*
EXPANSION SPACE TO HEATER
&CELL        II=0,       3,          12,         0,
             15,         40,         0,          1,
AA=7.16E-6,   0.0,       1.116E-5,   3.67E-4,    3.02E-3,
   1.0,       7*0.,      877.,       2.42E-7,    4.32E-7,
   0.,        .0397,     &END
*
HEATER (TUBES UP + 1/2 HEADER)
&CELL        II=0,       1,          12,         0,
             10,         40,         0,          1,
AA=7.16E-6,   0.0,       1.116E-5,   7.37E-4,    3.02E-3,
   1.0,       7*0.,      877.,       6.891E-7,   8.67E-7,
   0.0,       .0777,     &END
*
HEATER ( 1/2 HEADER + TUBES DOWN)
&CELL        II=0,       1,          12,         0,
             10,         40,         0,          1,
AA=7.16E-6,   0.0,       1.116E-5,   7.37E-4,    3.02E-3,
   1.0,       7*0.,      877.,       6.891E-7,   8.67E-7,
   0.0,       .0777,     &END
*
HEATER TO REGENERATOR
&CELL        II=0,       3,          12,         0,
             15,         40,         0,          1,
AA=7.16E-6,   0.0,       1.116E-5,   4.86E-4,    3.02E-3,
   1.0,       7*0.,      877.,       5.848E-7,   5.71E-7,
   0.0,       .0512,     &END
*
REGENERATOR       VOLUME LOWERED BY 5+% AS NASA TM79103
&CELL        II=0,       1,          12,         11,
             30,         8,          0,          5,
AA=1.877E-4,  0.0,       1.322E-4,   0.0,        9.35E-5,
   2.,        3*0.,      .697,       0.,         .468,
   0.,        634.,      6.325E-6,    2.99E-6,    0.,
   .0226,     &END
*
REGENERATOR TO COOLER TUBES
&CELL        II=0,       3,          12,         0,
             15,         312,        0,          1,
```

```
AA=9.16E-7,  0.0,      1.070E-6, 1.80E-5,  1.08E-3,
   1.0,      7*0.,     339.,     2.013E-8, 5.67E-9,
   2.55E-4,  .0053,    &END
*
COOLER
&CELL       II=0,      1,        12,       0,
            10,        312,      0,        1,
AA=9.16E-7,  0.,       1.070E-6, 1.20E-4,  1.08E-3,
   1.0,      3000.,    6*0.,     339.,     3.241E-8,
   3.80E-8,  2.55E-4,  .0355,    &END
*
COOLER TO CONNECTING DUCT TUBES
&CELL       II=0,      3,        12,       0,
            15,        312,      0,        1,
AA=9.16E-7,  0.0,      1.07E-6,  1.80E-5,  1.08E-3,
   1.0,      7*0.,     339.,     4.838E-9, 5.67E-9,
   2.55E-4,  .0053,    &END
*
CONNECTING DUCTS
&CELL       II=0,      3,        12,       0,
            15,        8,        0,        1,
AA=2.80E-5,  0.0,      3.8E-5,   0.0,      5.97E-3,
   1.0,      7*0.,     339.,     1.281E-6, 1.20E-6,
   0.,       3.18E-2,  &END
*
COMPRESSION SPACE
&CELL       II=0,      1,        12,       0,
            20,        1,        0,        1,
AA=3.77E-3,  0.0,      3.8E-5,   1.52E-2,  6.04E-2,
   1.0,      7*0.,     339.,     0.0,      3.54E-5,
   0.,       3.18E-2,  &END
*
  DISPLACER DOME
&ADDED      JJ=2,      1,        12,       10,
            0,         2,
BB=2*0.,     890.,     1.66E-4,  .04359,   .06642,
   .00159,   0.3,      4*0.,     .04359,   &END
*
POWER PISTON BUFFER SPACE
&ADDED      JJ=2*1,    12,       20,       2,
            0,
BB=3.23E+6,  0.,       315.,     5.21E-4,  0.,
   .0701,    .00381,   2*0.,     .15,      .002,
   &END
*   OPTIMIZE ALL MAJOR GEOMETRIC PARAMETERS

    MAXIMIZE POWER PER DISPLACED VOLUME (NOBJTV=6)
    WHILE SCALING TO A 10KW 40% EFFICIENT DEVICE (NEQULC=3)
    NOTE THAT THE SWITCH TO METHANE (IGAS=6) IS MADE IN THE BASE DATA


 &OPTIN NVAR=21,  MXEVAL=44,  IPRINT=0,  XDEL=0.02,  DPOWER=10000.0,  &END
 &OPTIN NEQULC=2,  QSORCE=25000.0,  NOBJTV=6,  &END
      PRESSURE AND FREQUENCY


 &OPTIN  IX=1,  &END
 &OPTIN  IX=2,  &END
         DRIVE MECHANISM
```

```
&OPTIN  IX=3,   &END
&OPTIN  IX=4,    &END
&OPTIN  IX=5,   &END
      EXPANSION CYLINDER


&OPTIN  IX=10,   &END
&OPTIN  IX=11,   &END
&OPTIN  IX=12,   &END
&OPTIN  IX=13,   &END
               HEATER


&OPTIN  IX=14,  &END
&OPTIN  IX=15,  &END
&OPTINN IX=16,, &ENDD

    CHANGE REGENERATOR VALUES AS BEFORE


&OPTIN  IX=18,  &END
&OPTIN  IX=19,  &END
&OPTIN  IX=20,  &END
&OPTIN  IX=21,  &END
&OPTINTIIX=22,23&ENDEND

             COOLER


&OPTIN  IX=24,  &END
&OPTIN  IX=25,  &END
&OPTINN IX=26,, &ENDD

             CONNECTING DUCT


&OPTINN IX=28,, &ENDD

      COMPRESSION CYLINDER

*:OPTIN  IX=30,   &END

     FIRST RESTART YIELDS A 10 KW DEVICE


     NO ADDITIONAL OPTIMIZER INPUT


&OPTIN  IX=-1,  &END
      PERTURBATION ON SPEED USING FINAL RESULTS


&PERTIN ICHNG=0, NMODS=6, FREQ=1.6667, DFREQ=3.33331,  NPRT2=6,
&END
```

APPENDIX F. EXAMPLE 1 - OUTPUT

The following output is only part of the output that would be generated from using the data of Appendix E. The page numbers on the title line are from the full output. The numbers to the left of this number are the date of execution and elapsed time in seconds. There were three cases in example 1: increase efficiency, minimize regenerator wire volume, and increase power. The output from these cases is presented in that order. The discussion in Section 4.5 refers to the first case and is the reason why more output from that case is presented.

F.1 SELECTED OUTPUT FROM EXAMPLE 1 - CASE 1

```
GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 1

    OPTIMIZATION INPUT:

    INTEGER INPUT -----------------------

NVAR  MXEVAL IPRINT NOBJTV NEQULC MCNSTR  MEQULC  MKEY1  NPRT2
 5     15      0      1      0     29       0       0     10

    FLOATING INPUT ----------------------

 TOLERANCE      DELTA     DESIGN EFFIC    DESIGN POWER    DESIGN SOURCE    OBJCTV NORM
1.0000E-04      0.0100        0.0             3250.0            0.0           1.0000

 NUMBER     KEY     LABEL     VALUE      MINIMUM    MAXIMUM   NORMALIZATION
   1         18    RGN LNG    1.000      0.1000      10.00     2.260D-02
   2         19    RGN DIA    1.000      0.1000      10.00     2.260D-02
   3         20    RGN WAL    1.000      0.1000      10.00     1.730D-03
   4         21    RGN SIG    1.000      0.1068       2.03     4.680D-01
   5         22    RGN WIR    1.000      0.4920      10.00     4.065D-05
```

```
GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 1

                 EDIT COMMON IGEN FOR RUN NUMBER   1

0-NO CHANGES TO CELL INFORMATION  1-CHANGES TO BE MADE             0
1-IN-LINE   2-DOUBLE ACTING   3-DISPLACER                          3
0-CONNECTING ROD  1-RHOMBIC DRIVE  2-DYNA. ANALYSIS(NA)            1
WORKING FLUID TO BE USED                                           1
1-H2  2-HE  3-N2  4-AIR  5-CO2  6-CH4
0=PMEAN IS CONSTANT, 1=TMASS IS CONSTANT                           0
0-FRICTION ONLY MOMEMTUUM EQUATION                                 0
1-FULL MOMENTUUM EQUATION
0-STANDARD PRINT  >0-CODE DEPENDENT ADDITIONAL PRINTS              0
0 NO GRAPHICS, 1=SAVE CYCLE FILE, 2=SAVE CASE FILE                 0
NUMBER OF ADDED COMPONENTS (LETTERED VOLUMES IN DESIGN)            2
0-RIOS  1-TEW  2-URIELI                                            0
NUMBER OF COMPONENTS (NUMBERED VOLUMES IN DESIGN)                 11
NUMBER OF CYLINDERS IN ENGINE                                      1
COOLER GAS HEAT FLOW OPTION                                        1
NUMBER OF STEPS/PRINT OF ZONE INFORMATION                        720
NUMBER OF CASES/PRINT OF ENERGY FLOW INFORMATION                  10
NUMBER OF FULL REVOLUTIONS TO BE CALCULATED(CYCLES)               12
NUMBER OF TIME STEPS IN 1 REVOLUTION                             360

                 EDIT COMMON FLTGEN

COMPRESSION PISTON CRANK RADIUS (M)                           0.01380
COMPRESSION PISTON CONNECTING ROD LENGTH (M)                  0.04600
COMPRESSION ZONE DEAD VOLUME (CU M)                           1.093E-05
COMPRESSION PISTON CRANK RADIUS ECCENTRICITY (M)              0.02080
COMPRESSION PISTON-CYLINDER WALL GAP (M)                      0.00015
COMPRESSION PISTON ROD DIAMETER (M)                           0.02220
EXPANSION PISTON CRANK RADIUS(M)                              0.01380
EXPANSION PISTON CONNECTING ROD LENGTH (M)                    0.04600
EXPANSION ZONE DEAD VOLUME (CU M)                             1.250E-05
EXPANSION PISTON CRANK RADIUS ECCENTRICITY (M)                0.02080
EXPANSION PISTON-CYLINDER WALL GAP (M)                        0.00025
EXPANSION PISTON ROD DIAMETER (M)                             0.00952
FREQUENCY OF SYSTEM (CYCLES/SEC)                                50.00
THERMAL HYSTERISIS FACTOR                                     1.00000
VOLUME PHASE ANGLE (DEGREES)                                   131.16
PISTON PHASE ANGLE (DEGREES)                                    60.59
AVERAGE PRESSURE OF SYSTEM IN CYCLE (PA)                      2.740E+06
CRANK ANGLE AT TIME = 0.0 (DEGREES)                               0.0
DISTANCE BETWEEN TOP OF REG. AND BOTTOM OF CYL. DOME          0.02860
EXTERNAL HEATER TEMPERATURE                                    927.00
EXTERNAL COOLER TEMPERATURE                                    287.00
TOTAL MASS IN CONTIGUOUS SYSTEM (KG)                          3.999E-04
EXTERNAL ENVIRONMENT TEMPERATURE                               300.00
CODE DEPENDENT OPTION ZOPT1                                      0.50
TIME STEP (SEC)                                               5.556E-05
```

```
  GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 1

                 EDIT MOST OF COMMON JMISC

FIRST FIXED VOLUME CELL                                            2
FIRST CELL IN HEATER                                               3
TYPICAL HEATER VALUES COME FROM CELL                               3
LAST CELL IN HEATER                                                4
FIRST CELL IN REGENERATOR                                          6
TYPICAL REGENERATOR VALUES COME FROM CELL                          8
LAST CELL IN REGENERATOR                                          10
FIRST CELL IN COOLER                                              12
TYPICAL COOLER VALUES COME FROM CELL                              12
LAST CELL IN COOLER                                               12
LAST FIXED VOLUME CELL                                            14
CELL NUMBER OF COMPRESSION ZONE                                   15
LAST CONTIGUOUS CELL IN SYSTEM                                    15

                 EDIT COMMON VARVOL

CURRENT CRANK ANGLE (RAD)                                   3.12776
COMPRESSION PISTON AREA (SQ M)                              3.770E-03
COMPRESSION CYLINDER BORE (M)                               0.06993
COMPRESSION ZONE PISTON OVERLAP (M)                         0.00008
COMPRESSION PISTON MAXIMUM STROKE POSITION (M)              0.05607
COMPRESSION PISTON MINIMUM STROKE POSITION (M)              0.02458
COMPRESSION PISTON CURRENT STROKE POSITION (M)              0.03034
COMPRESSION PISTON TOTAL STROKE (CMPMAX-CMPMIN) (M)         0.03149
COMPRESSION ZONE MAXIMUM DISPLACEMENT (CU M)                1.145E-04
CRANK ANGLE BETWEEN TDC OF EXP. & COM. PISTONS(RAD)         1.05753
VOLUME PHASE ANGLE (RADIANS)                                2.28909
ABSOLUTE ANGLE AT MINIMUM VOLUME (RAD)                      1.58681
ABSOLUTE EXPANSION CRANK ANGLE AT STARTD=0. (RAD)           1.58681
SCHMIDT ANALYSIS PRESSURE RATIO                             2.78697
SCHMIDT ANALYSIS PRESSURE PHASE ANGLE (RAD)                 1.14454
INCREMENTAL CRANK ANGLE (RAD/SEC)                           0.01745
EXPANSION PISTON AREA (SQ M)                                3.860E-03
EXPANSION CYLINDER BORE (M)                                 0.07010
EXPANSION MAXIMUM STROKE POSITION (M)                       0.02458
EXPANSION MINIMUM STROKE POSITION (M)                       0.05607
EXPANSION PISTON CURRENT POSITION (M)                       0.03028
EXPANSION PISTON TOTAL STROKE (M)                           0.03149
EXPANSION ZONE MAXIMUM DISPLACEMENT (CU M)                  1.215E-04

ACTIVE VOLUMES: (CU M)      1....................           9.396E-05
                            2....................           3.933E-05
                            3....................          -2.840E-05
                            4....................           7.389E-07
                            5....................           6.990E-05

                 EDIT COMMON GASPRP

GAS SPECIFIC HEAT AT CONSTANT PRESURE (J/KG-K)              1.456E+04
GAS SPECIFIC HEAT AT CONSTANT VOLUME  (J/KG-K)              1.043E+04
GAS SPECIFIC HEAT RATIO                                     1.39529
GAS CONSTANT (J/KG-K)                                        4124.6
```

```
    GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 1

         EDIT HEAT EXCHANGER VALUES PER NIHX    - COMMONS INCELL AND FLCELL
    CELL       *   1   *    *   2   *    *   3   *    *   4   *    *   5   *    *
   MHTRF   =       0            0            0            0            0
  MQFLOW   =       1            3            1            1            3
  MTLWAL   =      12           12           12           12           12
  MTLWIR   =       0            0            0            0            0
   MTYPE   =      25           15           10           10           15
    NIHX   =       1           40           40           40           40
   NOPT1   =       0            0            0            0            0
   AFLOW   =    3.860E-03    7.160E-06    7.160E-06    7.160E-06    7.160E-06
  ASPECT   =     0.0          0.0          0.0          0.0          0.0
  AXWALL   =    1.070E-03    1.116E-05    1.116E-05    1.116E-05    1.116E-05
    AWET   =    1.430E-02    3.670E-04    7.370E-04    7.370E-04    4.858E-04
      DH   =    7.010E-02    3.020E-03    3.020E-03    3.020E-03    3.020E-03
  FRICML   =      1.00         1.00         1.00         1.00         1.00
   HCEXT   =      0.0          0.0          0.0          0.0          0.0
  ORIFCE   =    0.0          0.0          0.0          0.0          0.0
    PGAS   =    4.212E+06    4.212E+06    4.212E+06    4.212E+06    4.212E+06
   POROS   =    0.0          0.0          0.0          0.0          0.0
   QFLOW   =      0.0          0.0          0.0          0.0          0.0
   SIGMA   =    0.0          0.0          0.0          0.0          0.0
    TGAS   =    927.00       927.00       927.00       927.00       927.00
   TWALL   =    927.00       927.00       927.00       927.00       927.00
     VOL   =    9.396E-05    2.420E-07    6.891E-07    6.891E-07    5.845E-07
   VOLWL   =    3.545E-05    4.431E-07    8.671E-07    8.671E-07    5.714E-07
  WALLTH   =  0.00456      0.00091      0.00091      0.00091      0.00091
  XLNGTH   =  0.03313      0.03970      0.07770      0.07770      0.05120

         EDIT REGENERATOR VALUES PER NIHX    - COMMONS INCEL AND FLCELL
    CELL       *   6   *    *   7   *    *   8   *    *   9   *    *  10   *    *
   MHTRF   =       0            0            0            0            0
  MQFLOW   =       1            1            1            1            1
  MTLWAL   =      12           12           12           12           12
  MTLWIR   =      11           11           11           11           11
   MTYPE   =      30           30           30           30           30
    NIHX   =       8            8            8            8            8
   NOPT1   =       0            0            0            0            0
   AFLOW   =    1.877E-04    1.877E-04    1.877E-04    1.877E-04    1.877E-04
  ASPECT   =     0.0          0.0          0.0          0.0          0.0
  AXWALL   =    1.322E-04    1.322E-04    1.322E-04    1.322E-04    1.322E-04
    AWET   =    5.412E-02    5.412E-02    5.412E-02    5.412E-02    5.412E-02
      DH   =    9.350E-05    9.350E-05    9.350E-05    9.350E-05    9.350E-05
  FRICML   =      2.00         2.00         2.00         2.00         2.00
   HCEXT   =      0.0          0.0          0.0          0.0          0.0
  ORIFCE   =     0.0          0.0          0.0          0.0          0.0
    PGAS   =    4.212E+06    4.212E+06    4.212E+06    4.212E+06    4.212E+06
   POROS   =    0.6970       0.6970       0.6970       0.6970       0.6970
   QFLOW   =      0.0          0.0          0.0          0.0          0.0
   SIGMA   =    0.4680       0.4680       0.4680       0.4680       0.4680
    TGAS   =    634.00       634.00       634.00       634.00       634.00
   TWALL   =    634.00       634.00       634.00       634.00       634.00
     VOL   =     1.265E-06    1.265E-06    1.265E-06    1.265E-06    1.265E-06
   VOLWL   =     5.975E-07    5.975E-07    5.975E-07    5.975E-07    5.975E-07
  WALLTH   =  0.00173      0.00173      0.00173      0.00173      0.00173
  XLNGTH   =  0.00452      0.00452      0.00452      0.00452      0.00452
```

```
      GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 1

            EDIT HEAT EXCHANGER VALUES PER NIHX    - COMMONS INCELL AND FLCELL
   CELL         *  11  *     *  12  *     *  13  *     *  14  *     *  15  *     *
  MHTRF  =         0            0            0            0            0
 MQFLOW  =         3            1            3            3            1
 MTLWAL  =        12           12           12           12           12
 MTLWIR  =         0            0            0            0            0
  MTYPE  =        15           10           15           15           20
   NIHX  =       312          312          312            8            1
  NOPT1  =         0            0            0            0            0
  AFLOW  =    9.160E-07    9.160E-07    9.160E-07    2.800E-05    3.770E-03
 ASPECT  =     0.0          0.0          0.0          0.0          0.0
 AXWALL  =    1.069E-06    1.069E-06    1.069E-06    3.869E-06    3.800E-05
   AWET  =    1.800E-05    1.200E-04    1.800E-05    8.583E-04    1.520E-02
     DH  =    1.080E-03    1.080E-03    1.080E-03    5.970E-03    6.040E-02
 FRICML  =     1.00         1.00         1.00         1.00         1.00
  HCEXT  =     0.0       3000.00         0.0          0.0          0.0
 ORIFCE  =     0.0          0.0          0.0          0.0          0.0
   PGAS  =    4.212E+06    4.212E+06    4.212E+06    4.212E+06    4.212E+06
  POROS  =    0.0          0.0          0.0          0.0          0.0
  QFLOW  =     0.0          0.0          0.0          0.0          0.0
  SIGMA  =    0.0          0.0          0.0          0.0          0.0
   TGAS  =    339.00       339.00       339.00       339.00       339.00
  TWALL  =    339.00       339.00       339.00       339.00       339.00
    VOL  =    2.013E-08    3.241E-08    4.838E-09    1.281E-06    1.093E-05
  VOLWL  =    5.668E-09    3.797E-08    5.668E-09    1.230E-07    1.208E-06
 WALLTH  =   0.00026      0.00026      0.00026      0.00020      0.00020
 XLNGTH  =   0.00530      0.03550      0.00530      0.03180      0.03180

                    EDIT ADDED VOLUME VALUES- COMMONS IADD AND FADD
 VOLUME      *   1  *    *  2  *    *

 IACOND =        2           1
 IALEAK =        1           1
  IAMTL =       12          12
 IATYPE =       10          20
 NRINGS =        0           2
 NSHELD =        2

 ADPGAS =    2.740E+06   2.740E+06
 ADRADA =    0.0
 ADTGAS =    890.00      315.00
  ADVOL =    1.511E-04   5.210E-04
 ADWALX =   0.04359
 ADWLDI =   0.06642     0.07010
 ADWLTH =   0.00159     0.00381
   EMIS =   0.30000
 RINGCL =    0.0         0.0
 RINGFR =   0.0         0.15000
 RINGHT =   0.0         0.00200
 RINGLK =    0.0         0.0
 SLNGTH =   0.04359
```

```
      GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 1

                RIOS/SEAM1 ANALYSIS  CONSOLIDATED INPUT INFORMATION FOR CASE   0

                            EXPANSION       HEATER     REGENERATOR       COOLER     COMPRESSION
      ZONE                      1              2             3              4             5

   VOLUME (CU M)          1.2153E-04    8.8190E-05    5.0600E-05    2.8150E-05    1.1453E-04
 FLOW AREA (SQ M)         3.8600E-03    2.8640E-04    1.5016E-03    2.8579E-04    3.7700E-03
WETTED AREA (SQ M)                      5.8960E-02    2.1647E+00    3.7440E-02
    HYD DIA (M)            0.070100      0.003020      0.000093      0.001080      0.060400
   TOT LNGTH (M)                         0.246300      0.022600      0.046100
 HT EXCH LNGTH (M)                       0.155400                    0.035500
  TEMPERATURE (K)            927.00        927.00        584.52        339.00        339.00

    REGENERATOR WALL DIAMETER (M)     0.02260     AVG HT TRANSF REYN EXPONENT      0.5503
      MINIMUM TOTAL VOLUME (CU M) 2.2992E-04     MAXIMUM TOTAL VOLUME (CU M) 3.4974E-04
   ANGLE AT MIN TOTAL VOLUME (DEG)   288.9998  ANGLE AT MAX TOTAL VOLUME (DEG)   127.9999
              AVERAGE PRESSURE (MPA)   2.7400              TOTAL GAS MASS (KGS) 4.0680E-04
```

```
    GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 1


              AT DESIGN CASE   1    FIND GRADIENT   1

                   OBJECTIVE                     CONSTRAINTS:

                            D-POWER  D-EFFIC PIST VEL  RPM MIN HTR FLUX COL FLUX RGN HIGT HTR RING COL SIZE HTR SIZE
                  1.0000  -0.0000   0.0      0.0109  49.0000   0.6867   1.2852   1.5312   1.0000   0.0000   0.4023

                           DUCT SIZ  R DRIVE RGN AXIL HTR CIRC DSPL DOM DSPL CYL BUKL COL PIST ROD EXPN HOP REGN HOP
                             0.5676   0.2295  -0.0142   0.8205   0.9644   0.9970   0.9748   0.9581   0.6624   0.8148

                           RGN TEMP HTR MACH COL MACH RGN REYN 20% WORK  5% WORK  PR DROP  STROKE  RGN WALL
                             0.8031   0.3855   0.4610   0.9266  -0.5541   5.2166   0.6456   0.1777   0.6173

                 OBJECTIVE FUNCTION =  1.0000  SUM OF THE WEIGHTED CONSTRAINTS =  0.0

DESIGN VARIABLE          RGN LNG   RGN DIA   RGN WAL   RGN SIG   RGN WIR
                       2.26D-02  2.26D-02  1.73D-03  4.68D-01  4.06D-05


              AT DESIGN CASE   2    FIND GRADIENT   2

                   OBJECTIVE                     CONSTRAINTS:

                            D-POWER  D-EFFIC PIST VEL  RPM MIN HTR FLUX COL FLUX RGN HIGT HTR RING COL SIZE HTR SIZE
                  0.9217   0.1114   0.1356   0.0109  49.0000   0.6923   1.2525   0.9622   0.8732   0.7823   0.4023

                           DUCT SIZ  R DRIVE RGN AXIL HTR CIRC DSPL DOM DSPL CYL BUKL COL PIST ROD EXPN HOP REGN HOP
                             0.5676   0.2295  -0.0054   0.8243   0.9674   0.9973   0.9698   0.9615   0.6657   0.7973

                           RGN TEMP HTR MACH COL MACH RGN REYN 20% WORK  5% WORK  PR DROP  STROKE  RGN WALL
                             0.6626   0.4067   0.4532   0.9145   0.2272   2.0911   0.8455   0.1777   0.6629

                 OBJECTIVE FUNCTION =  0.9217  SUM OF THE WEIGHTED CONSTRAINTS =  0.0013

DESIGN VARIABLE          RGN LNG   RGN DIA   RGN WAL   RGN SIG   RGN WIR
                       2.58D-02  2.48D-02  1.67D-03  5.86D-01  4.53D-05
```

```
     GPU-3 REGENERATOR OPTIMIZATION      STIRLING ENGINE DATA: CASE 1

                             ******   FINAL VALUES   ******          .

  LABEL      VALUE      RATIO      OBJF          CONSTRAINTS :

                                          D-POWER  D-EFFIC PIST VEL  RPM MIN HTR FLUX COL FLUX RGN HIGT HTR RING COL SIZE HTR SIZE
                                  0.8937   0.0072   0.1841   0.0109  49.0000   0.7270   1.2122  -0.0001   0.8066   1.5986   0.4023

                                         DUCT SIZ  R DRIVE RGN AXIL HTR CIRC DSPL DOM DSPL CYL BUKL COL PIST ROD EXPN HOP REGN HOP
                                           0.5676   0.2295   0.0000   0.8361   0.9717   0.9976   0.9648   0.9667   0.6668   0.7142

                                         RGN TEMP HTR MACH COL MACH RGN REYN 20% WORK  5% WORK  PR DROP   STROKE  RGN WALL
                                           0.6171   0.4414   0.4421   0.9146   0.1960   2.2161   0.8714   0.1777   0.7754

                                  .....    DERIVATIVES OF OBJCTV AND CONSTRAINTS WITH RESPECT TO DESIGN VARIBLES FOLLOW        .....

 RGN LNG   3.54D-02    1.5658   -0.0026  -0.2262   0.0044   0.0      0.0       0.0581  -0.0557  -1.6668   0.0       0.0       0.0
                                           0.0      0.0      0.2430   0.0184   0.0057   0.0005   0.0016   0.0067  -0.0007   0.0146
                                           0.0038   0.0472  -0.0157  -0.0033  -0.5692   2.2769  -0.0488   0.0       0.0
 RGN DIA   2.71D-02    1.1998   -0.0668  -0.3000   0.1157   0.0      0.0       0.1211  -0.1571  -1.6839  -1.1013   8.0000   0.0
                                           0.0      0.0     -0.6955   0.0419   0.0164   0.0014  -0.0576   0.0194   0.0048  -0.1854
                                           0.0123   0.1253  -0.0409   0.1329   0.6679  -2.6717   0.2197   0.0       0.1891
 RGN WAL   1.22D-03    0.7042    0.0391  -0.0027  -0.0677   0.0      0.0       0.0004  -0.0002   0.0     -0.1916   0.0       0.0
                                           0.0      0.0      0.5713   0.0000   0.0001   0.0000   0.0000   0.0001  -0.0000   0.3928
                                          -0.0005   0.0005   0.0005   0.0001   0.0001  -0.0004   0.0002   0.0      -0.3190
 RGN SIG   6.14D-01    1.3126    0.0082   0.0463  -0.0141   0.0      0.0      -0.0165   0.0165   0.0      0.0       0.0       0.0
                                           0.0      0.0      0.0034  -0.0008   0.0040   0.0003   0.0011   0.0047   0.0075   0.0087
                                          -0.8550   0.0324  -0.0029  -0.0921   1.1685  -4.6739   0.2060   0.0       0.0
 RGN WIR   4.93D-05    1.2124   -0.0031   0.0930   0.0054   0.0      0.0      -0.0249   0.0234   0.0      0.0       0.0       0.0
                                           0.0      0.0      0.0052  -0.0057   0.0002   0.0000   0.0000   0.0003   0.0038  -0.0002
                                          -0.0003   0.0027   0.0032  -0.0698   0.7306  -2.9222   0.1141   0.0       0.0

  INFO =  1         8  FUNCTION EVALUATIONS
  NVAR =  5         9  GRADIENT EVALUATIONS
 NCASE =  9       229  TOTAL CYCLES USED
  IBND =  0
```

```
            GPU-3 REGENERATOR OPTIMIZATION        STIRLING ENGINE DATA: CASE 1

                KEY      LABEL     VALUE       RATIO      FGRD       ERRROR      N-CNST      ERR CNSTR
VARIABLE- 1     18      RGN LNG  3.54D-02    1.5658 -2.55D-03 -5.74D-04      7          1.84D-02
VARIABLE- 2     19      RGN DIA  2.71D-02    1.1998 -6.68D-02 -1.42D-03     13          4.69D-02
VARIABLE- 3     20      RGN WAL  1.22D-03    0.7042  3.91D-02  5.97D-04     13          3.85D-02
VARIABLE- 4     21      RGN SIG  6.14D-01    1.3126  8.17D-03  7.94D-03     13          2.26D-04
VARIABLE- 5     22      RGN WIR  4.93D-05    1.2124 -3.09D-03 -3.44D-03     13          3.47D-04

 FINAL OBJECTIVE FUNCTION VALUE =  8.93651567D-01

 CONSTRAINTS EVALUATED AT FINAL DESIGN CONDITIONS:
 7.23001803D-03  1.84109060D-01  1.08526349D-02  4.90000000D+01  7.26966321D-01
 1.21221447D+00 -8.93051802D-05  8.06609083D-01  1.59858329D+00  4.02343750D-01
 5.67572594D-01  2.29479218D-01  4.20808792D-05  8.36121678D-01  9.71714437D-01
 9.97625351D-01  9.64791536D-01  9.66651142D-01  6.66847765D-01  7.14206696D-01
 6.17060959D-01  4.41367462D-01  4.42145079D-01  9.14635301D-01  1.95978859D-01
 2.21608456D+00  8.71421368D-01  1.77688742D-01  7.75381143D-01

  LAGRANGE MULTIPLIER ESTIMATES:
 0.0             0.0             0.0             0.0             0.0
 0.0             1.10091210D-02  0.0             0.0             0.0
 0.0             0.0             6.73666550D-02  0.0             0.0
 0.0             0.0             0.0             0.0             0.0
 0.0             0.0             0.0             0.0             0.0
 0.0             0.0             0.0             0.0

  LAGRANGIAN GRADIENT ERROR =  1.39778399D-02
  LAGRANGE MULTIPLIER ERROR =  0.0
        COMPLEMENTARY ERROR =  3.81801961D-06
           CONSTRAINT ERROR =  8.93051802D-05
  WEIGHTED CONSTRAINT ERROR =  1.07088057D-06
```

```
         GPU-3 REGENERATOR OPTIMIZATION      STIRLING ENGINE DATA: CASE 1

                   CASE TO CASE COMPARISON INFORMATION

 CASE NUMBER                 1          2          3          4          5          6
AVERAGE PRESSURE MPA      2.7400     2.7400     2.7400     2.7400     2.7400     2.7400
SPEED              RPM   3000.00    3000.00    3000.00    3000.00    3000.00    3000.00
GAS MASS            KG 0.0003297  0.0003654  0.0003927  0.0004415  0.0004410  0.0004377
INDIC HEAT IN        W   8876.03    8687.36    8275.81    7563.98    7523.75    7528.13
INDIC POWER OUT      W   3249.88    3612.21    3509.78    3267.23    3253.68    3260.32
EFFICIENCY                0.3661     0.4158     0.4241     0.4319     0.4325     0.4331

EXPANSION WALL T     K    826.81     832.02     836.94     844.77     844.80     844.37
EXPANSION GAS T      K    826.81     832.02     836.94     844.77     844.80     844.37
WALL TO GAS NET Q    W   -600.76    -613.04    -642.02    -687.45    -686.36    -683.24
P-V WORK             W   7778.18    7170.67    6754.84    6110.94    6118.50    6158.47
HYSTERESIS           W   0.0        0.0        0.0        0.0        0.0        0.0

COMBUSTOR TEMP       K    927.00     927.00     927.00     927.00     927.00     927.00
HEATER WALL T        K    923.16     923.23     923.40     923.69     923.70     923.68
HEATER GAS T         K    879.62     880.29     882.25     885.60     885.68     885.54
WALL TO GAS NET Q    W   8335.39    8186.12    7815.93    7183.52    7169.73    7197.36
WALL CONDUCTION      W    540.65     501.25     459.88     380.46     354.02     330.77
FRICTION P-DROP    MPA 0.0053084  0.0052315  0.0051245  0.0049279  0.0049108  0.0049118

REGENERATOR WALL T K      619.71     618.07     617.20     615.96     615.96     616.03
REGENERATOR GAS T  K      568.74     563.97     561.86     559.02     559.06     559.23
WALL TO GAS NET Q  W     -506.36    -649.57    -626.70    -563.42    -551.99    -549.29
WALL CONDUCTION    W      540.65     501.25     459.88     380.46     354.02     330.77
FRICTION P-DROP  MPA   0.0593651  0.0219085  0.0173106  0.0142134  0.0153500  0.0161521

WATER TEMPERATURE  K      287.00     287.00     287.00     287.00     287.00     287.00
COOLER WALL T      K      316.26     312.91     311.01     308.23     308.23     308.39
COOLER GAS T       K      341.49     334.37     330.39     324.70     324.71     325.03
WALL TO GAS NET Q  W    -4552.53   -4030.54   -3735.25   -3302.15   -3303.00   -3327.00
WALL CONDUCTION    W    -1073.63   -1044.61   -1030.79    -994.60    -967.07    -940.81
FRICTION P-DROP  MPA   0.0028594  0.0031505  0.0033333  0.0036235  0.0036172  0.0035972

COMPRESSION WALL T K      355.70     346.67     341.54     334.22     334.25     334.65
COMPRESSION GAS T  K      355.70     346.67     341.54     334.22     334.25     334.65
WALL TO GAS NET Q  W       67.77      69.68      71.11      73.31      73.31      73.19
P-V WORK           W    -3454.54   -3082.10   -2841.76   -2489.26   -2492.57   -2513.34
HYSTERESIS         W     0.0        0.0        0.0        0.0        0.0        0.0

MAX PRESSURE       MPA    3.6371     3.5499     3.4928     3.4078     3.4087     3.4139
MIN PRESSURE       MPA    1.9057     1.9607     1.9992     2.0604     2.0596     2.0558
INITIAL PRESSURE   MPA    3.3226     3.2844     3.2569     3.2124     3.2130     3.2159
P-VE PHASE ANGLE   DEG -250.2684  -250.9823  -251.3783  -251.8571  -251.8544  -251.8307
STATIC PWR LOSS      W   -836.11    -799.91    -766.45    -698.99    -669.74    -644.40
DYNAMIC PWR LOSS     W   -351.26    -352.41    -372.78    -405.12    -404.94    -403.03
FRICTION PWR LOSS    W  -1073.77    -476.36    -403.31    -354.45    -372.26    -384.81
REGENATR PWR LOSS    W   -460.39    -611.54    -589.37    -527.22    -517.69    -515.87
LEAKAGE PWR LOSS     W     0.0        0.0        0.0        0.0        0.0        0.0
NET Q TO COLD H2O    W  -5626.15   -5075.15   -4766.04   -4296.75   -4270.07   -4267.81
UNACCT PWR LOSS      W      0.00      -0.00      -0.00      -0.00       0.00      -0.00
REGENATR EFFECTVNS        0.9956     0.9932     0.9929     0.9929     0.9930     0.9931
```

```
          GPU-3 REGENERATOR OPTIMIZATION   FINAL DESIGN    OFF-DESIGN PERFORMANCE

                          CASE TO CASE COMPARISON INFORMATION

 CASE NUMBER                  1          2          3          4          5          6
AVERAGE PRESSURE MPA     2.7400     2.7400     2.7400     2.7400     2.7400     2.7400
SPEED            RPM    1000.02    1500.06    2000.10    2500.14    3000.18    3500.22
GAS MASS          KG  0.0004558  0.0004492  0.0004417  0.0004360  0.0004307  0.0004251
INDIC HEAT IN      W    2973.64    4194.45    5369.45    6473.50    7550.82    8608.14
INDIC POWER OUT    W    1121.53    1758.56    2340.06    2834.38    3273.64    3662.39
EFFICIENCY               0.3772     0.4193     0.4358     0.4378     0.4335     0.4255

EXPANSION WALL T   K     774.84     806.55     827.23     835.94     843.44     851.95
EXPANSION GAS T    K     774.84     806.55     827.23     835.94     843.44     851.95
WALL TO GAS NET Q  W    -548.93    -609.95    -648.09    -663.47    -676.21    -690.27
P-V WORK           W    2026.76    3115.88    4187.30    5221.39    6244.71    7263.84
HYSTERESIS         W     0.0        0.0        0.0        0.0        0.0        0.0

COMBUSTOR TEMP     K     890.00     899.00     909.00     918.00     927.00     937.00
HEATER WALL T      K     888.71     897.15     906.61     915.12     923.65     933.20
HEATER GAS T       K     812.71     846.55     868.40     877.44     885.19     894.00
WALL TO GAS NET Q  W    2704.59    3921.36    5091.72    6191.73    7265.03    8317.70
WALL CONDUCTION    W     269.05     273.09     277.74     281.77     285.79     290.44
FRICTION P-DROP  MPA  0.0006880  0.0013810  0.0023963  0.0035664  0.0049129  0.0064328

REGENERATOR WALL T K     591.38     597.28     603.80     609.95     616.22     623.08
REGENERATOR GAS T  K     526.14     536.30     546.31     553.16     559.67     566.69
WALL TO GAS NET Q  W    -143.10    -232.63    -332.06    -438.48    -551.65    -669.57
WALL CONDUCTION    W     269.05     273.09     277.74     281.77     285.79     290.44
FRICTION P-DROP  MPA  0.0042153  0.0070228  0.0102680  0.0138803  0.0178626  0.0221977

WATER TEMPERATURE  K     287.00     287.00     287.00     287.00     287.00     287.00
COOLER WALL T      K     294.04     297.42     300.98     304.78     308.78     312.96
COOLER GAS T       K     316.45     313.07     315.82     320.72     325.81     331.08
WALL TO GAS NET Q  W   -1095.86   -1620.47   -2174.61   -2765.97   -3388.10   -4038.96
WALL CONDUCTION    W    -756.25    -815.41    -854.79    -873.15    -889.09    -906.79
FRICTION P-DROP  MPA  0.0005772  0.0010430  0.0017151  0.0025616  0.0035540  0.0046809

COMPRESSION WALL T K     326.23     322.57     325.33     330.39     335.65     341.09
COMPRESSION GAS T  K     326.23     322.57     325.33     330.39     335.65     341.09
WALL TO GAS NET Q  W      61.73      67.63      71.04      72.09      72.92      73.92
P-V WORK           W    -876.52   -1283.21   -1697.14   -2126.18   -2559.47   -2994.22
HYSTERESIS         W     0.0        0.0        0.0        0.0        0.0        0.0

MAX PRESSURE     MPA     3.4127     3.4241     3.4278     3.4266     3.4250     3.4236
MIN PRESSURE     MPA     2.0683     2.0492     2.0430     2.0449     2.0476     2.0499
INITIAL PRESSURE MPA     3.1988     3.2200     3.2269     3.2248     3.2219     3.2193
P-VE PHASE ANGLE DEG  -251.3683  -251.7455  -251.8552  -251.8213  -251.7752  -251.7318
STATIC PWR LOSS    W    -533.41    -560.26    -578.29    -586.70    -594.22    -603.08
DYNAMIC PWR LOSS   W    -314.03    -352.61    -377.79    -389.15    -398.76    -409.02
FRICTION PWR LOSS  W     -28.71     -74.11    -150.10    -260.83    -411.61    -607.23
REGENATR PWR LOSS  W    -113.64    -202.81    -301.81    -407.86    -520.68    -638.18
LEAKAGE PWR LOSS   W       0.0        0.0        0.0        0.0        0.0        0.0
NET Q TO COLD H2O  W   -1852.11   -2435.88   -3029.40   -3639.12   -4277.18   -4945.75
UNACCT PWR LOSS    W       0.00       0.00      -0.01       0.00      -0.00      -0.00
REGENATR EFFECTVNS       0.9958     0.9949     0.9943     0.9937     0.9931     0.9926
```

F.2 SELECTED OUTPUT FROM EXAMPLE 1 - CASE 2

```
GPU-3 REGENERATOR OPTIMIZATION      STIRLING ENGINE DATA: CASE 2

    OPTIMIZATION INPUT:

    INTEGER INPUT -----------------------

NVAR  MXEVAL IPRINT NOBJTV NEQULC MCNSTR  MEQULC  MKEY1  NPRT2
 5     15      0      5      0      29      0      0     10

    FLOATING INPUT ----------------------

TOLERANCE     DELTA     DESIGN EFFIC   DESIGN POWER   DESIGN SOURCE   OBJCTV NORM
1.0000E-04    0.0100       0.0              0.0            0.0           1.0000

NUMBER    KEY    LABEL     VALUE     MINIMUM    MAXIMUM   NORMALIZATION
   1       18    RGN LNG   1.000     0.1000     10.00      2.260D-02
   2       19    RGN DIA   1.000     0.1000     10.00      2.260D-02
   3       20    RGN WAL   1.000     0.1000     10.00      1.730D-03
   4       21    RGN SIG   1.000     0.1068      2.03      4.680D-01
   5       22    RGN WIR   1.000     0.4920     10.00      4.065D-05
```

```
        GPU-3 REGENERATOR OPTIMIZATION      STIRLING ENGINE DATA: CASE 2

                  CASE TO CASE COMPARISON INFORMATION

 CASE NUMBER                1          2          3          4          5          6
AVERAGE PRESSURE MPA     2.7400     2.7400     2.7400     2.7400     2.7400     2.7400
SPEED            RPM    3000.00    3000.00    3000.00    3000.00    3000.00    3000.00
GAS MASS          KG  0.0003297  0.0003477  0.0003491  0.0003485  0.0003480  0.0003470
INDIC HEAT IN      W    8876.03    8985.44    9421.15    9374.80    9430.97    9604.98
INDIC POWER OUT    W    3249.88    3537.13    3541.10    3577.98    3600.03    3629.25
EFFICIENCY               0.3661     0.3937     0.3759     0.3817     0.3817     0.3779

EXPANSION WALL T   K     826.81     828.62     826.95     827.05     826.73     825.81
EXPANSION GAS T    K     826.81     828.62     826.95     827.05     826.73     825.81
WALL TO GAS NET Q  W    -600.76    -597.50    -589.91    -591.27    -590.65    -588.21
P-V WORK           W    7778.18    7433.71    7296.37    7331.30    7332.43    7317.20
HYSTERESIS         W     0.0        0.0        0.0        0.0        0.0        0.0

COMBUSTOR TEMP     K     927.00     927.00     927.00     927.00     927.00     927.00
HEATER WALL T      K     923.16     923.10     922.91     922.93     922.91     922.83
HEATER GAS T       K     879.62     878.84     876.22     876.56     876.24     875.23
WALL TO GAS NET Q  W    8335.39    8449.51    8871.54    8821.64    8874.91    9042.46
WALL CONDUCTION    W     540.65     535.94     549.62     553.16     556.06     562.51
FRICTION P-DROP  MPA  0.0053084  0.0053285  0.0053011  0.0053111  0.0053097  0.0053001

REGENERATOR WALL T K     619.71     619.14     620.41     620.14     620.23     620.64
REGENERATOR GAS T  K     568.74     566.68     569.29     568.68     568.83     569.66
WALL TO GAS NET Q  W    -506.36    -747.66   -1248.40   -1158.23   -1197.77   -1354.44
WALL CONDUCTION    W     540.65     535.94     549.62     553.16     556.06     562.51
FRICTION P-DROP  MPA  0.0593651  0.0318782  0.0236775  0.0231562  0.0215075  0.0179578

WATER TEMPERATURE  K     287.00     287.00     287.00     287.00     287.00     287.00
COOLER WALL T      K     316.26     315.18     317.91     317.34     317.55     318.45
COOLER GAS T       K     341.49     338.98     344.07     343.02     343.42     345.14
WALL TO GAS NET Q  W   -4552.53   -4383.36   -4808.14   -4720.14   -4751.87   -4892.28
WALL CONDUCTION    W   -1073.63   -1064.96   -1071.91   -1076.68   -1079.07   -1083.44
FRICTION P-DROP  MPA  0.0028594  0.0030488  0.0030859  0.0030765  0.0030738  0.0030709

COMPRESSION WALL T K     355.70     352.21     357.41     356.36     356.79     358.63
COMPRESSION GAS T  K     355.70     352.21     357.41     356.36     356.79     358.63
WALL TO GAS NET Q  W      67.77      68.47      67.62      67.75      67.64      67.28
P-V WORK           W   -3454.54   -3259.92   -3249.74   -3256.12   -3261.67   -3274.25
HYSTERESIS         W     0.0        0.0        0.0        0.0        0.0        0.0

MAX PRESSURE     MPA     3.6371     3.5884     3.5736     3.5776     3.5781     3.5773
MIN PRESSURE     MPA     1.9057     1.9373     1.9518     1.9482     1.9482     1.9502
INITIAL PRESSURE MPA     3.3226     3.2997     3.2853     3.2886     3.2883     3.2856
P-VE PHASE ANGLE DEG  -250.2684  -250.6203  -250.4998  -250.5160  -250.4964  -250.4312
STATIC PWR LOSS    W    -836.11    -811.56    -814.12    -819.90    -822.75    -827.89
DYNAMIC PWR LOSS   W    -351.26    -345.42    -340.99    -341.87    -341.64    -340.53
FRICTION PWR LOSS  W   -1073.77    -636.67    -505.53    -497.20    -470.74    -413.70
REGENATR PWR LOSS  W    -460.39    -724.11   -1232.81   -1140.89   -1180.10   -1336.74
LEAKAGE PWR LOSS   W      0.0        0.0        0.0        0.0        0.0        0.0
NET Q TO COLD H2O  W   -5626.15   -5448.31   -5880.05   -5796.82   -5830.94   -5975.72
UNACCT PWR LOSS    W      0.00       0.00       0.00       0.00      -0.00       0.00
REGENATR EFFECTVNS       0.9956     0.9935     0.9882     0.9891     0.9884     0.9862
```

```
GPU-3 REGENERATOR OPTIMIZATION      STIRLING ENGINE DATA: CASE 2


          AT DESIGN CASE  10    FIND GRADIENT  10

             OBJECTIVE                    CONSTRAINTS:

                        D-POWER  D-EFFIC PIST VEL  RPM MIN HTR FLUX COL FLUX RGN HIGT HTR RING COL SIZE HTR SIZE
               0.3299   0.1211   0.0001   0.0109  49.0000   0.6470   1.3278   1.5391   0.9930   0.0       0.4023

                       DUCT SIZ  R DRIVE RGN AXIL HTR CIRC DSPL DOM DSPL CYL BUKL COL PIST ROD EXPN HOP REGN HOP
                        0.5676   0.2295  -0.0000   0.8129   0.9667   0.9972   0.9752   0.9608   0.6707   0.8223

                       RGN TEMP HTR MACH COL MACH RGN REYN 20% WORK  5% WORK  PR DROP  STROKE  RGN WALL
                        0.0000   0.4072   0.4638   0.8453   0.4940   1.0242   0.8906   0.1777   0.6056

            OBJECTIVE FUNCTION =  0.3299  SUM OF THE WEIGHTED CONSTRAINTS =  0.0000

DESIGN VARIABLE          RGN LNG   RGN DIA   RGN WAL   RGN SIG   RGN WIR
                        2.25D-02  2.26D-02  1.78D-03  7.94D-01  3.75D-05
```

```
           GPU-3 REGENERATOR OPTIMIZATION           STIRLING ENGINE DATA: CASE 2

                              ******     FINAL VALUES      ******

 LABEL      VALUE      RATIO      OBJF            CONSTRAINTS :

                                          D-POWER   D-EFFIC  PIST VEL   RPM MIN  HTR FLUX  COL FLUX  RGN HIGT  HTR RING  COL SIZE  HTR SIZE
                                 0.3299    0.1211    0.0001    0.0109   49.0000    0.6470    1.3278    1.5391    0.9930    0.0          0.4023

                                         DUCT SIZ   R DRIVE  RGN AXIL  HTR CIRC  DSPL DOM  DSPL CYL  BUKL COL  PIST ROD  EXPN HOP  REGN HOP
                                           0.5676    0.2295   -0.0000    0.8129    0.9667    0.9972    0.9752    0.9608    0.6707    0.8223

                                         RGN TEMP  HTR MACH  COL MACH  RGN REYN  20% WORK   5% WORK   PR DROP    STROKE  RGN WALL
                                           0.0000    0.4072    0.4638    0.8453    0.4940    1.0242    0.8906    0.1777    0.6056

                                .....      DERIVATIVES OF OBJCTV AND CONSTRAINTS WITH RESPECT TO DESIGN VARIBLES FOLLOW               .....

RGN LNG   2.25D-02     0.9961    0.3312   -0.1093    0.2481    0.0       0.0       0.1088   -0.1604   -1.9999    0.0       0.0       0.0
                                           0.0       0.0       0.6103    0.0308    0.0055    0.0005    0.0012    0.0065   -0.0064    0.0110
                                          -0.0109    0.0322   -0.0268   -0.0115   -0.4623    1.8492   -0.0631    0.0       0.0
RGN DIA   2.26D-02     1.0000    0.6631   -0.0517    0.2997    0.0       0.0       0.1490   -0.2415   -3.5042   -1.4797    8.0000    0.0
                                           0.0       0.0      -0.7544    0.0475    0.0134    0.0011   -0.0518    0.0158   -0.0004   -0.1387
                                          -0.0062    0.0830   -0.0430    0.2836    0.5824   -2.3294    0.1973    0.0       0.3905
RGN WAL   1.78D-03     1.0306    0.0      -0.0035   -0.0595    0.0       0.0       0.0007   -0.0005    0.0      -0.2299    0.0       0.0
                                           0.0       0.0       0.0135    0.0001    0.0001    0.0000    0.0000    0.0001    0.0004    0.1599
                                           0.0014    0.0007    0.0005    0.0001   -0.0003    0.0012    0.0002    0.0      -0.3827
RGN SIG   7.94D-01     1.6960   -0.8012   -0.2965   -0.7117    0.0       0.0      -0.1702    0.3488    0.0       0.0       0.0       0.0
                                           0.0       0.0       0.1081   -0.0366    0.0052    0.0004    0.0008    0.0061    0.0291    0.0006
                                          -3.0880    0.0721    0.0393   -0.3221    0.6048   -2.4194    0.1313    0.0       0.0
RGN WIR   3.75D-05     0.9218    0.0      -0.0252   -0.2155    0.0       0.0      -0.0727    0.1281    0.0       0.0       0.0       0.0
                                           0.0       0.0       0.0419   -0.0162    0.0012    0.0001    0.0001    0.0014    0.0110   -0.0009
                                           0.0316    0.0205    0.0151   -0.1608    0.5457   -2.1828    0.1129    0.0       0.0

 INFO =   1        10  FUNCTION EVALUATIONS
 NVAR =   5        11  GRADIENT EVALUATIONS
NCASE =  11       293  TOTAL CYCLES USED
 IBND =   0
```

```
           GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 2

                KEY      LABEL     VALUE      RATIO     FGRD      ERRROR      N-CNST     ERR CNSTR
VARIABLE- 1  18     RGN LNG  2.25D-02    0.9961  3.31D-01  2.33D-02     13        2.81D-01
VARIABLE- 2  19     RGN DIA  2.26D-02    1.0000  6.63D-01 -3.36D-03      9        9.80D-01
VARIABLE- 3  20     RGN WAL  1.78D-03    1.0306  0.0        4.90D-04     2        7.07D-03
VARIABLE- 4  21     RGN SIG  7.94D-01    1.6960 -8.01D-01 -1.34D-03     21        7.65D-01
VARIABLE- 5  22     RGN WIR  3.75D-05    0.9218  0.0       -1.53D-03     2        2.56D-02

 FINAL OBJECTIVE FUNCTION VALUE =  3.299126630-01

 CONSTRAINTS EVALUATED AT FINAL DESIGN CONDITIONS:
 1.21086727D-01  1.12163451D-04  1.08526349D-02  4.90000000D+01  6.46996915D-01
 1.32778072D+00  1.53906303D+00  9.92964771D-01  0.0             4.02343750D-01
 5.67572594D-01  2.29479218D-01 -1.23977661D-05  8.12871814D-01  9.66713130D-01
 9.97205496D-01  9.75206256D-01  9.60754514D-01  6.70704484D-01  8.22269857D-01
 4.76837158D-07  4.07163039D-01  4.63775977D-01  8.45262945D-01  4.93959094D-01
 1.02416362D+00  8.90623742D-01  1.77688742D-01  6.05564244D-01

  LAGRANGE MULTIPLIER ESTIMATES:
 0.0             1.18755299D-01  0.0             0.0             0.0
 0.0             0.0             0.0             1.22498183D-01  0.0
 0.0             0.0             4.60747952D-01  0.0             0.0
 0.0             0.0             0.0             0.0             0.0
 2.47781395D-01  0.0             0.0             0.0             0.0
 0.0             0.0             0.0             0.0

  LAGRANGIAN GRADIENT ERROR =  2.99912393D-02
  LAGRANGE MULTIPLIER ERROR =  0.0
        COMPLEMENTARY ERROR =  1.91504009D-05
           CONSTRAINT ERROR =  1.23977661D-05
  WEIGHTED CONSTRAINT ERROR =  6.01099311D-06
```

F.3 SELECTED OUTPUT FROM EXAMPLE 1 - CASE 3

```
GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 3

    OPTIMIZATION INPUT:

    INTEGER INPUT -----------------------

NVAR  MXEVAL  IPRINT  NOBJTV  NEQULC  MCNSTR   MEQULC   MKEY1   NPRT2
  5     15      0       3       0       29       0        0       10

    FLOATING INPUT ----------------------

 TOLERANCE       DELTA      DESIGN EFFIC     DESIGN POWER     DESIGN SOURCE     OBJCTV NORM
1.0000E-04      0.0100         0.0                 0.0              0.0            1.0000

 NUMBER     KEY     LABEL      VALUE      MINIMUM     MAXIMUM   NORMALIZATION
    1        18    RGN LNG    1.000       0.1000      10.00       2.260D-02
    2        19    RGN DIA    1.000       0.1000      10.00       2.260D-02
    3        20    RGN WAL    1.000       0.1000      10.00       1.730D-03
    4        21    RGN SIG    1.000       0.1068       2.03       4.680D-01
    5        22    RGN WIR    1.000       0.4920      10.00       4.065D-05
```

```
      GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 3


                AT DESIGN CASE   9    FIND GRADIENT   9

                       OBJECTIVE                    CONSTRAINTS:

                                  D-POWER  D-EFFIC PIST VEL  RPM MIN HTR FLUX COL FLUX RGN HIGT HTR RING COL SIZE HTR SIZE
                          0.8490   0.1778   0.0510   0.0109  49.0000   0.6477   1.3148   1.5315   0.9865   0.0       0.4023

                                 DUCT SIZ  R DRIVE RGN AXIL HTR CIRC DSPL DOM DSPL CYL BUKL COL PIST ROD EXPN HOP REGN HOP
                                   0.5676   0.2295  -0.0000   0.8117   0.9651   0.9971   0.9749   0.9588   0.6683   0.8246

                                 RGN TEMP HTR MACH COL MACH RGN REYN 20% WORK  5% WORK  PR DROP   STROKE  RGN WALL
                                   0.7654   0.3931   0.4640   0.8306   0.5063   0.9749   0.8864   0.1777   0.5947

                     OBJECTIVE FUNCTION =  0.8490  SUM OF THE WEIGHTED CONSTRAINTS =  0.0000


DESIGN VARIABLE          RGN LNG   RGN DIA   RGN WAL   RGN SIG   RGN WIR
                        2.26D-02  2.26D-02  1.83D-03  5.08D-01  8.87D-05
```

```
          GPU-3 REGENERATOR OPTIMIZATION      STIRLING ENGINE DATA: CASE 3

                      CASE TO CASE COMPARISON INFORMATION

 CASE NUMBER               1           2           3           4           5           6
AVERAGE PRESSURE MPA    2.7400      2.7400      2.7400      2.7400      2.7400      2.7400
SPEED            RPM   3000.00     3000.00     3000.00     3000.00     3000.00     3000.00
GAS MASS          KG 0.0003297   0.0003396   0.0003406   0.0003406   0.0003401   0.0003384
INDIC HEAT IN      W   8876.03     9240.62     9438.70     9536.62     9601.43     9765.71
INDIC POWER OUT    W   3249.88     3718.42     3771.75     3782.44     3788.68     3799.77
EFFICIENCY              0.3661      0.4024      0.3996      0.3966      0.3946      0.3891

EXPANSION WALL T   K    826.81      826.14      825.39      824.96      824.61      823.66
EXPANSION GAS T    K    826.81      826.14      825.39      824.96      824.61      823.66
WALL TO GAS NET Q  W   -600.76     -596.56     -593.48     -591.94     -591.00     -588.71
P-V WORK           W   7778.18     7583.93     7524.98     7504.24     7499.54     7498.60
HYSTERESIS         W   0.0         0.0         0.0         0.0         0.0         0.0

COMBUSTOR TEMP     K    927.00      927.00      927.00      927.00      927.00      927.00
HEATER WALL T      K    923.16      922.98      922.89      922.85      922.82      922.75
HEATER GAS T       K    879.62      877.41      876.24      875.66      875.29      874.35
WALL TO GAS NET Q  W   8335.39     8710.12     8901.46     8995.43     9057.68     9213.35
WALL CONDUCTION    W    540.65      530.50      537.24      541.20      543.75      552.36
FRICTION P-DROP  MPA 0.0053084   0.0053205   0.0053165   0.0053115   0.0053074   0.0052961

REGENERATOR WALL T K    619.71      619.34      619.74      619.99      620.16      620.60
REGENERATOR GAS T  K    568.74      566.96      567.71      568.21      568.56      569.49
WALL TO GAS NET Q  W   -506.36     -798.47     -999.70    -1099.25    -1158.67    -1301.04
WALL CONDUCTION    W    540.65      530.50      537.24      541.20      543.75      552.36
FRICTION P-DROP  MPA 0.0593651   0.0242869   0.0177940   0.0157218   0.0146716   0.0126210

WATER TEMPERATURE  K    287.00      287.00      287.00      287.00      287.00      287.00
COOLER WALL T      K    316.26      315.69      316.59      317.14      317.50      318.44
COOLER GAS T       K    341.49      340.14      341.81      342.84      343.54      345.37
WALL TO GAS NET Q  W  -4552.53    -4463.00    -4603.77    -4688.39    -4745.20    -4891.70
WALL CONDUCTION    W  -1073.63    -1059.21    -1063.18    -1065.79    -1067.55    -1074.24
FRICTION P-DROP  MPA 0.0028594   0.0029661   0.0029876   0.0029921   0.0029902   0.0029795

COMPRESSION WALL T K    355.70      353.83      355.51      356.58      357.33      359.31
COMPRESSION GAS T  K    355.70      353.83      355.51      356.58      357.33      359.31
WALL TO GAS NET Q  W     67.77       67.85       67.54       67.35       67.20       66.82
P-V WORK           W  -3454.54    -3351.67    -3343.11    -3344.88    -3350.85    -3371.99
HYSTERESIS         W   0.0         0.0         0.0         0.0         0.0         0.0

MAX PRESSURE     MPA    3.6371      3.6100      3.6033      3.6013      3.6012      3.6025
MIN PRESSURE     MPA    1.9057      1.9236      1.9297      1.9320      1.9326      1.9333
INITIAL PRESSURE MPA    3.3226      3.3092      3.3033      3.3008      3.2998      3.2980
P-VE PHASE ANGLE DEG -250.2684   -250.4421   -250.4081   -250.3777   -250.3502   -250.2699
STATIC PWR LOSS    W   -836.11     -815.24     -817.75     -820.19     -822.35     -830.73
DYNAMIC PWR LOSS   W   -351.26     -346.77     -344.85     -344.02     -343.60     -342.72
FRICTION PWR LOSS  W  -1073.77     -513.84     -410.12     -376.92     -360.01     -326.85
REGENATR PWR LOSS  W   -460.39     -763.52     -967.82    -1068.19    -1127.46    -1268.64
LEAKAGE PWR LOSS   W       0.0         0.0         0.0         0.0         0.0         0.0
NET Q TO COLD H2O  W  -5626.15    -5522.20    -5666.95    -5754.18    -5812.74    -5965.94
UNACCT PWR LOSS    W      0.00        0.00        0.00        0.00        0.00       -0.00
REGENATR EFFECTVNS      0.9956      0.9920      0.9895      0.9831      0.9873      0.9853
```

```
          GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 3

                           ******    FINAL VALUES    ******

 LABEL     VALUE     RATIO     OBJF          CONSTRAINTS :

                                     D-POWER  D-EFFIC PIST VEL  RPM MIN HTR FLUX COL FLUX RGN HIGT HTR RING COL SIZE HTR SIZE
                             0.8490   0.1778   0.0510   0.0109  49.0000   0.6477   1.3148   1.5315   0.9865   0.0         0.4023

                                    DUCT SIZ  R DRIVE RGN AXIL HTR CIRC DSPL DOM DSPL CYL BUKL COL PIST ROD EXPN HOP REGN HOP
                                      0.5676   0.2295  -0.0000   0.8117   0.9651   0.9971   0.9749   0.9588   0.6683   0.8246

                                    RGN TEMP HTR MACH COL MACH RGN REYN 20% WORK  5% WORK  PR DROP   STROKE RGN WALL
                                      0.7654   0.3931   0.4640   0.8306   0.5063   0.9749   0.8864   0.1777   0.5947

                             .....    DERIVATIVES OF OBJCTV AND CONSTRAINTS WITH RESPECT TO DESIGN VARIBLES FOLLOW     .....

RGN LNG  2.26D-02   0.9998   0.0849  -0.1178   0.1890   0.0      0.0      0.0834  -0.1161  -2.0004   0.0      0.0      0.0
                                      0.0      0.0      0.6315   0.0241   0.0049   0.0004   0.0011   0.0058  -0.0039   0.0092
                                     -0.0017   0.0317  -0.0204  -0.0090  -0.4269   1.7074  -0.0665   0.0      0.0
RGN DIA  2.26D-02   1.0000   0.0241  -0.0334   0.2297   0.0      0.0      0.1125  -0.1839  -3.4962  -1.4732   8.0000   0.0
                                      0.0      0.0     -0.7305   0.0375   0.0122   0.0010  -0.0528   0.0144   0.0027  -0.1394
                                     -0.0032   0.0796  -0.0337   0.3152   0.6619  -2.6476   0.2148   0.0      0.4013
RGN WAL  1.83D-03   1.0589   0.0019  -0.0027  -0.0621   0.0      0.0      0.0006  -0.0005   0.0     -0.2299   0.0      0.0
                                      0.0      0.0     -0.0048   0.0000   0.0001   0.0000   0.0000   0.0001   0.0000   0.1501
                                     -0.0003   0.0007   0.0005   0.0001  -0.0004   0.0016   0.0002   0.0     -0.3827
RGN SIG  5.08D-01   1.0353  -0.0010   0.0014  -0.1237   0.0      0.0     -0.0446   0.0756   0.0      0.0      0.0      0.0
                                      0.0      0.0      0.0247  -0.0073   0.0039   0.0003   0.0007   0.0046   0.0112   0.0040
                                     -0.4929   0.0384   0.0052  -0.1273   0.8111  -3.2442   0.1961   0.0      0.0
RGN WIR  8.87D-05   2.1816   0.0021  -0.0029  -0.0754   0.0      0.0     -0.0261   0.0447   0.0      0.0      0.0      0.0
                                      0.0      0.0      0.0149  -0.0060   0.0004   0.0000   0.0001   0.0005   0.0039  -0.0003
                                      0.0023   0.0078   0.0052  -0.0750   0.2228  -0.8911   0.0493   0.0      0.0

 INFO =  1        9  FUNCTION EVALUATIONS
 NVAR =  5       10  GRADIENT EVALUATIONS
NCASE = 10      258  TOTAL CYCLES USED
 IBND =  0
```

```
           GPU-3 REGENERATOR OPTIMIZATION       STIRLING ENGINE DATA: CASE 3

                  KEY     LABEL    VALUE      RATIO    FGRD       ERRROR       N-CNST     ERR CNSTR
VARIABLE- 1  18     RGN LNG  2.26D-02    0.9998  8.49D-02 -2.04D-03     13        8.69D-02
VARIABLE- 2  19     RGN DIA  2.26D-02    1.0000  2.41D-02 -2.70D-04      9        1.25D-01
VARIABLE- 3  20     RGN WAL   1.83D-03    1.0589  1.95D-03  2.60D-03     13        6.57D-04
VARIABLE- 4  21     RGN SIG  5.08D-01    1.0853 -1.03D-03 -4.42D-03     13        3.40D-03
VARIABLE- 5  22     RGN WIR  8.87D-05    2.1816  2.06D-03  1.27D-05     13        2.05D-03

 FINAL OBJECTIVE FUNCTION VALUE =  8.49037349D-01

 CONSTRAINTS EVALUATED AT FINAL DESIGN CONDITIONS:
 1.77804487D-01  5.10270444D-02  1.08526349D-02  4.90000000D+01  6.47679806D-01
 1.31476688D+00  1.53151937D+00  9.86453680D-01  0.0             4.02343750D-01
 5.67572594D-01  2.29479218D-01 -5.72204590D-06  8.11691999D-01  9.65074122D-01
 9.97067869D-01  9.74883556D-01  9.58822191D-01  6.68317556D-01  8.24576735D-01
 7.65441358D-01  3.93088371D-01  4.63968500D-01  8.30592453D-01  5.06275496D-01
 9.74898016D-01  8.86436115D-01  1.77688742D-01  5.94723277D-01

  LAGRANGE MULTIPLIER ESTIMATES:
 0.0             0.0             0.0             0.0             0.0
 0.0             0.0             0.0             1.56054058D-02  0.0
 0.0             0.0             1.37604979D-01  0.0             0.0
 0.0             0.0             0.0             0.0             0.0
 0.0             0.0             0.0             0.0             0.0
 0.0             0.0             0.0             0.0

  LAGRANGIAN GRADIENT ERROR =  9.34801623D-03
  LAGRANGE MULTIPLIER ERROR =  0.0
        COMPLEMENTARY ERROR =  7.87382005D-07
           CONSTRAINT ERROR =  5.72204590D-06
  WEIGHTED CONSTRAINT ERROR =  8.43950040D-07
```

APPENDIX G. EXAMPLE 2 - OUTPUT

```
GPU-3  METHANE  877K/287K        STIRLING ENGINE DATA:      ( SAMPLE PROBLEM 2)

    OPTIMIZATION INPUT:

    INTEGER INPUT -----------------------

NVAR  MXEVAL IPRINT NOBJTV NEQULC MCNSTR  MEQULC  MKEY1  NPRT2
21     44      0      6      2     29       1       1     10

    FLOATING INPUT ----------------------

 TOLERANCE      DELTA     DESIGN EFFIC   DESIGN POWER   DESIGN SOURCE    OBJCTV NORM
1.0000E-04     0.0200        0.0           10000.0         25000.0         1.0000

 NUMBER     KEY     LABEL     VALUE      MINIMUM    MAXIMUM   NORMALIZATION
    1        1      PMEAN     1.000      0.1000     10.00      2.740D+06
    2        2       FREQ     1.000      0.1000     10.00      1.667D+01
    3        3    CRNK RAD    1.000      0.1000     10.00      1.380D-02
    4        4    CRNK L/R    1.000      0.6750     10.00      3.333D+00
    5        5    CRNK E-R    1.000      0.1000     19.71      7.000D-03
    6       10    EXPN DIA    1.000      0.1000     10.00      7.010D-02
    7       11    EXPN WAL    1.000      0.1000     10.00      4.562D-03
    8       12    DSPL LNG    1.000      0.1000     10.00      4.359D-02
    9       13    DSPL WAL    1.000      0.1000     10.00      1.590D-03
   10       14     HTR LNG    1.000      0.1000     10.00      1.554D-01
   11       15     HTR DIA    1.000      0.1000     10.00      3.020D-03
   12       16     HTR WAL    1.000      0.1000     10.00      9.050D-04
   13       18     RGN LNG    1.000      0.1000     10.00      2.260D-02
   14       19     RGN DIA    1.000      0.1000     10.00      2.260D-02
   15       20     RGN WAL    1.000      0.1000     10.00      1.730D-03
   16       21     RGN SIG    1.000      0.1068      2.03      4.680D-01
   17       22     RGN WIR    1.000      0.4920     10.00      4.065D-05
   18       24     COL LNG    1.000      0.1000     10.00      3.550D-02
   19       25     COL DIA    1.000      0.1000     10.00      1.080D-03
   20       26     COL WAL    1.000      0.1000     10.00      2.550D-04
   21       28    C-DCT DI    1.000      0.1000     10.00      5.970D-03
```

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               GPU-3  METHANE  877K/287K       STIRLING ENGINE DATA:     ( SAMPLE PROBLEM 2)

                       CASE TO CASE COMPARISON INFORMATION

 CASE NUMBER                 1          2          3          4          5          6          7          8          9         10
AVERAGE PRESSURE MPA     2.7400     5.4800     8.2288     9.3599     9.8335     9.7458    10.2108    10.8130    11.4377    12.2661
SPEED            RPM    1000.20    1319.07    1776.76    1173.82     658.78     934.64     910.93     840.35     874.43     864.56
GAS MASS          KG  0.0027202  0.0070529  0.0060542  0.0212093  0.0277793  0.0268277  0.0274766  0.0285125  0.0286604  0.0293566
INDIC HEAT IN      W    3898.92   23692.87   12453.59   24582.37   21174.79   23552.08   24881.33   24831.45   24846.29   24847.70
INDIC POWER OUT    W    1251.56    2181.29    4029.13    6783.91    7740.86    8759.44    9618.03    9981.14    9997.19    9971.03
EFFICIENCY               0.3210     0.0921     0.3235     0.2760     0.3656     0.3719     0.3866     0.4020     0.4024     0.4013

EXPANSION WALL T   K     830.23     746.06     824.71     811.30     795.98     818.39     820.29     822.35     824.75     826.21
EXPANSION GAS T    K     830.23     746.06     824.71     811.30     795.98     818.39     820.29     822.35     824.75     826.21
WALL TO GAS NET Q  W    -362.51    -486.23    -552.74    -763.06    -798.89    -708.67    -669.44    -631.43    -562.32    -498.16
P-V WORK           W    2505.50   19667.05    8431.87   20844.79   16396.30   19342.38   20417.34   20605.86   20381.08   20131.87
HYSTERESIS         W    0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0

COMBUSTOR TEMP     K     877.00     877.00     877.00     877.00     877.00     877.00     877.00     877.00     877.00     877.00
HEATER WALL T      K     875.36     867.71     874.19     869.91     864.66     866.63     866.67     867.06     867.14     866.82
HEATER GAS T       K     849.25     776.73     840.78     827.49     813.50     833.91     836.22     838.84     840.74     842.47
WALL TO GAS NET Q  W    3403.26   23468.34   11860.05   24118.92   20799.57   23048.64   24332.72   24211.00   24168.88   24008.59
WALL CONDUCTION    W     495.66     224.53     593.54     463.45     375.22     503.45     548.61     620.44     677.41     839.12
FRICTION P-DROP  MPA  0.0040877  0.0187213  0.0143799  0.0140211  0.0113830  0.0198562  0.0253049  0.0283436  0.0322656  0.0405891

REGENERATOR WALL T K     587.16     636.32     598.85     597.34     589.07     594.65     592.44     590.92     590.49     589.85
REGENERATOR GAS T  K     547.17     603.73     557.77     563.16     543.96     551.37     551.09     549.17     548.70     547.56
WALL TO GAS NET Q  W    -647.06   -5310.21   -3169.94   -4624.34   -4160.20   -3795.26   -4116.26   -3818.72   -4063.44   -4241.19
WALL CONDUCTION    W     495.66     224.53     593.54     463.45     375.22     503.45     548.61     620.44     677.41     839.12
FRICTION P-DROP  MPA  0.0346561  0.4695345  0.1182946  1.0669012  0.5677188  0.4889356  0.4656526  0.4176638  0.3790459  0.3017350

WATER TEMPERATURE  K     287.00     287.00     287.00     287.00     287.00     287.00     287.00     287.00     287.00     287.00
COOLER WALL T      K     298.95     404.93     323.52     324.77     313.47     322.66     318.20     314.78     313.85     312.89
COOLER GAS T       K     326.94     458.55     346.75     362.19     341.77     341.24     339.57     335.37     333.67     331.11
WALL TO GAS NET Q  W   -1859.67  -20847.16   -7310.87  -16619.65  -12300.83  -13614.20  -14077.20  -13627.36  -13635.10  -13561.33
WALL CONDUCTION    W    -787.70    -664.40   -1113.58   -1178.81   -1133.10   -1178.43   -1186.09   -1222.95   -1214.01   -1315.35
FRICTION P-DROP  MPA  0.0022798  0.0766407  0.0340847  0.0057166  0.0073551  0.0826961  0.0300556  0.0234545  0.0215115  0.0214765

COMPRESSION WALL T K     335.96     484.20     358.75     379.41     360.90     358.21     355.90     351.07     348.07     343.34
COMPRESSION GAS T  K     335.96     484.20     358.75     379.41     360.90     358.21     355.90     351.07     348.07     343.34
WALL TO GAS NET Q  W      70.48      46.35      32.70      47.70      41.02      33.69      31.96      28.92      25.72      21.92
P-V WORK           W   -1036.82  -12883.59   -3689.61   -9829.57   -7526.95   -8546.23   -8925.85   -8854.65   -8652.39   -8419.48
HYSTERESIS         W    0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0

MAX PRESSURE     MPA     3.6033     9.0747    10.5968    12.9651    14.2415    13.4663    13.9496    14.6501    15.1790    15.8834
MIN PRESSURE     MPA     1.9261     3.4313     6.3960     6.9471     6.9893     7.2437     7.6673     8.1659     8.7806     9.5224
INITIAL PRESSURE MPA     3.3152     5.4162     8.9969     9.2346     9.2357     9.5065    10.2817    11.1643    12.0308    13.3822
P-VE PHASE ANGLE DEG  -249.8528  -207.8483  -229.1270  -204.9996  -200.1539  -202.7547  -206.7558  -210.8838  -214.8294  -225.3381
STATIC PWR LOSS    W    -797.84    -372.37    -780.42    -837.36    -775.31    -869.82    -900.84    -951.19    -988.06   -1134.37
DYNAMIC PWR LOSS   W    -103.05    -367.99    -407.09    -431.57    -426.01    -375.38    -350.42    -335.53    -286.01    -238.90
FRICTION PWR LOSS  W    -217.12   -4602.16    -713.13   -4231.31   -1128.48   -2036.71   -1873.45   -1770.07   -1731.49   -1741.36
REGENATR PWR LOSS  W    -604.35   -5280.60   -3128.72   -4581.92   -4132.99   -3762.19   -4083.05   -3783.87   -4029.10   -4205.19
LEAKAGE PWR LOSS   W        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
NET Q TO COLD H2O  W   -2647.37  -21511.57   -8424.45  -17798.45  -13433.92  -14792.64  -15263.29  -14850.30  -14849.11  -14876.68
UNACCT PWR LOSS    W        0.0       0.01       0.01       0.01        0.0       0.01       0.01      -0.00      -0.01        0.0
REGENATR EFFECTVNS       0.9927     0.9915     0.9888     0.9976     0.9981     0.9979     0.9973     0.9971     0.9965     0.9952
```

```
GPU-3  METHANE  877K/287K        STIRLING ENGINE DATA:      ( SAMPLE PROBLEM 2)


            AT DESIGN CASE  10    FIND GRADIENT  10

                OBJECTIVE                     CONSTRAINTS:

                           D-EFFIC  D-POWER PIST VEL  RPM MIN HTR FLUX COL FLUX RGN HIGT HTR RING COL SIZE HTR SIZE
                    0.4061  0.0061  -0.0029   0.3641  13.4094   0.4414   1.2403   0.0132   0.3992   0.0003   0.3880

                          DUCT SIZ  R DRIVE RGN AXIL HTR CIRC DSPL DOM DSPL CYL BUKL COL PIST ROD EXPN HOP REGN HOP
                           -0.0007   1.7425  -0.0064   0.4326   0.6107   0.8829   0.3336   0.7840   0.0198   0.6646

                          RGN TEMP HTR MACH COL MACH RGN REYN 20% WORK  5% WORK  PR DROP   STROKE  RGN WALL
                            0.1453  -0.0070   0.3645  -0.0091  -0.0341   3.1365   0.6180   0.3093   0.0003

              OBJECTIVE FUNCTION =  0.4061  SUM OF THE WEIGHTED CONSTRAINTS =  0.0192

DESIGN VARIABLE          PMEAN     FREQ   CRNK RAD CRNK L/R CRNK E-R EXPN DIA EXPN WAL DSPL LNG DSPL WAL  HTR LNG
                      1.23D+07 1.44D+01 3.44D-02 6.72D+00 1.25D-02 8.44D-02 8.57D-03 1.19D-01 1.13D-03 2.11D-01

DESIGN VARIABLE        HTR DIA  HTR WAL  RGN LNG  RGN DIA  RGN WAL  RGN SIG  RGN WIR  COL LNG  COL DIA  COL WAL
                      3.31D-03 1.22D-03 7.70D-02 3.24D-02 6.48D-03 2.28D-01 9.96D-05 1.00D-01 1.15D-03 4.24D-04

DESIGN VARIABLE       C-DCT DI
                      5.07D-03
```

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          GPU-3  METHANE  877K/287K        STIRLING ENGINE DATA:     ( SAMPLE PROBLEM 2)

                CASE TO CASE COMPARISON INFORMATION

 CASE NUMBER              11         12         13         14         15         16         17         18         19         20
AVERAGE PRESSURE MPA   12.1603    12.1299    12.0693    12.0186    11.9909    11.9156    11.7368    11.7556    11.7381    11.7261
SPEED            RPM    862.76     860.04     838.09     812.86     811.70     809.14     827.25     822.31     826.52     824.12
GAS MASS          KG 0.0293059  0.0291803  0.0287926  0.0282403  0.0280421  0.0274994  0.0260643  0.0261174  0.0260468  0.0260553
INDIC HEAT IN      W  25020.83   25014.47   25003.44   24978.67   24997.09   24987.64   24955.79   25000.73   25000.99   25001.69
INDIC POWER OUT    W   9995.39    9996.99    9998.23    9993.11   10004.06   10001.06    9963.90    9994.49    9998.47    9999.32
EFFICIENCY              0.3995     0.3996     0.3999     0.4001     0.4002     0.4002     0.3993     0.3998     0.3999     0.3999

EXPANSION WALL T   K    825.41     824.48     821.66     816.83     815.42     810.54     795.96     797.50     797.82     797.71
EXPANSION GAS T    K    825.41     824.48     821.66     816.83     815.42     810.54     795.96     797.50     797.82     797.71
WALL TO GAS NET Q  W   -506.45    -506.79    -524.20    -544.07    -547.99    -552.50    -552.54    -556.46    -553.24    -552.48
P-V WORK           W  20135.47   20145.74   20186.02   20211.30   20300.74   20395.74   20666.22   20725.12   20723.75   20728.85
HYSTERESIS         W    0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0

COMBUSTOR TEMP     K    877.00     877.00     877.00     877.00     877.00     877.00     877.00     877.00     877.00     877.00
HEATER WALL T      K    866.67     866.50     866.08     865.37     865.07     863.96     860.47     861.05     861.18     861.19
HEATER GAS T       K    841.69     840.90     838.68     834.57     833.32     828.80     814.73     816.37     816.65     816.59
WALL TO GAS NET Q  W  24184.24   24170.45   24146.76   24105.16   24115.89   24083.70   24014.39   24058.14   24057.09   24055.31
WALL CONDUCTION    W    836.59     844.03     856.68     873.51     881.19     893.95     941.40     942.59     943.90     946.38
FRICTION P-DROP  MPA 0.0393562  0.0390227  0.0380376  0.0362357  0.0356861  0.0340094  0.0302089  0.0303584  0.0303065  0.0302634

REGENERATOR WALL T K    590.05     589.97     589.83     589.54     589.35     588.71     586.74     587.04     587.10     587.11
REGENERATOR GAS T  K    547.78     547.28     546.08     543.96     543.33     541.24     535.23     535.94     536.09     536.06
WALL TO GAS NET Q  W  -4373.87   -4346.02   -4252.43   -4127.44   -4054.09   -3918.30   -3568.08   -3563.39   -3568.04   -3562.14
WALL CONDUCTION    W    836.59     844.03     856.68     873.51     881.19     898.95     941.40     942.59     943.90     946.38
FRICTION P-DROP  MPA 0.2907384  0.2818089  0.2622610  0.2406909  0.2427071  0.2395597  0.2467837  0.2492831  0.2503744  0.2489037

WATER TEMPERATURE  K    287.00     287.00     287.00     287.00     287.00     287.00     287.00     287.00     287.00     287.00
COOLER WALL T      K    313.43     313.44     313.58     313.70     313.63     313.46     313.00     313.02     313.02     313.02
COOLER GAS T       K    331.84     331.55     331.03     330.20     329.96     329.40     328.39     328.52     328.58     328.58
WALL TO GAS NET Q  W -13704.70  -13688.61  -13645.66  -13588.39  -13584.09  -13554.89  -13516.65  -13526.06  -13524.34  -13522.47
WALL CONDUCTION    W  -1320.74   -1328.87   -1359.55   -1397.17   -1408.94   -1431.69   -1475.24   -1480.18   -1478.18   -1479.89
FRICTION P-DROP  MPA 0.0224918  0.0240870  0.0285894  0.0371716  0.0385475  0.0425712  0.0487080  0.0470963  0.0463413  0.0463563

COMPRESSION WALL T K    344.17     343.76     343.06     342.14     341.94     341.46     340.89     340.98     341.05     341.02
COMPRESSION GAS T  K    344.17     343.76     343.06     342.14     341.94     341.46     340.89     340.98     341.05     341.02
WALL TO GAS NET Q  W     22.30      21.95      21.32      20.42      20.25      19.75      18.71      18.87      18.97      18.97
P-V WORK           W  -8449.30   -8454.68   -8489.04   -8534.43   -8583.16   -8666.22   -8928.77   -8940.39   -8937.58   -8940.99
HYSTERESIS         W    0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0

MAX PRESSURE     MPA   15.7644    15.7208    15.6806    15.6898    15.6787    15.6393    15.5406    15.5702    15.5437    15.5296
MIN PRESSURE     MPA    9.4355     9.4007     9.2935     9.1738     9.1323     9.0312     8.8265     8.8306     8.8224     8.8082
INITIAL PRESSURE MPA   13.2547    13.2667    13.3305    13.4069    13.3991    13.3614    13.1767    13.2191    13.1908    13.1898
P-VE PHASE ANGLE DEG -224.9108  -225.8950  -228.4249  -230.6956  -231.0106  -231.5511  -230.8911  -231.3260  -231.1439  -231.4012
STATIC PWR LOSS    W  -1135.63   -1140.20   -1147.66   -1155.60   -1160.87   -1171.39   -1198.53   -1201.77   -1202.74   -1204.66
DYNAMIC PWR LOSS   W   -242.90    -246.42    -269.61    -299.24    -306.09    -318.90    -336.98    -338.91    -335.10    -336.01
FRICTION PWR LOSS  W  -1690.78   -1694.07   -1698.74   -1683.76   -1713.52   -1728.45   -1773.55   -1790.24   -1787.69   -1788.53
REGENATR PWR LOSS  W  -4338.38   -4310.23   -4216.05   -4090.19   -4016.31   -3879.46   -3526.52   -3521.76   -3526.35   -3520.33
LEAKAGE PWR LOSS   W      0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
NET Q TO COLD H2O  W -15025.44  -15017.48  -15005.21  -14985.56  -14993.03  -14986.57  -14991.88  -15006.24  -15002.52  -15002.36
UNACCT PWR LOSS    W      0.0        0.00       0.0        0.0        0.0        0.01       0.01      -0.00       0.01       0.01
REGENATR EFFECTVNS      0.9951     0.9949     0.9946     0.9943     0.9943     0.9944     0.9948     0.9948     0.9949     0.9948
```

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GPU-3  METHANE  877K/287K        STIRLING ENGINE DATA:     ( SAMPLE PROBLEM 2)


             AT DESIGN CASE  20    FIND GRADIENT  20

                   OBJECTIVE                     CONSTRAINTS:

                             D-EFFIC  D-POWER PIST VEL  RPM MIN HTR FLUX COL FLUX RGN WIGT HTR RING COL SIZE HTR SIZE
                     0.3767  -0.0001  -0.0001   0.4276  12.7354  -0.0001   1.2372   0.0001   0.0000   0.0001   0.5597

                            DUCT SIZ  R DRIVE RGN AXIL HTR CIRC DSPL DOM DSPL CYL BUKL COL PIST ROD EXPN HOP REGN HOP
                             -0.0000   1.3687   0.0000   0.3155   0.5830   0.8676   0.1328   0.7914   0.0001   0.6736

                            RGN TEMP HTR MACH COL MACH RGN REYN 20% WORK  5% WORK  PR DROP   STROKE  RGN WALL
                              0.3355  -0.0000   0.0693  -0.0001  -0.0002   3.0007   0.6304   0.3695  -0.0000

                 OBJECTIVE FUNCTION =  0.3767  SUM OF THE WEIGHTED CONSTRAINTS =  0.0001

DESIGN VARIABLE           PMEAN     FREQ    CRNK RAD  CRNK L/R  CRNK E-R  EXPN DIA  EXPN WAL  DSPL LNG  DSPL WAL   HTR LNG
                      1.17D+07  1.37D+01  3.21D-02  6.27D+00  1.74D-02  8.38D-02  7.47D-03  1.25D-01  1.12D-03  1.25D-01

DESIGN VARIABLE        HTR DIA   HTR WAL   RGN LNG   RGN DIA   RGN WAL   RGN SIG   RGN WIR   COL LNG   COL DIA   COL WAL
                      3.26D-03  1.08D-03  7.24D-02  3.36D-02  6.72D-03  2.06D-01  1.11D-04  1.05D-01  9.84D-04  4.70D-04

DESIGN VARIABLE       C-DCT DI
                      4.34D-03
                      4.56D-03
```

```
           GPU-3  METHANE  877K/287K        STIRLING ENGINE DATA:     ( SAMPLE PROBLEM 2)

                            ******    FINAL VALUES    ******

  LABEL     VALUE     RATIO     OBJF          CONSTRAINTS :

                                      D-EFFIC  D-POWER PIST VEL  RPM MIN HTR FLUX COL FLUX RGN HIGT HTR RING COL SIZE HTR SIZE
                              0.3652   0.0001   0.0001   0.3885  13.0711   0.0002   1.2748   0.0001   0.0000  -0.0000   0.5808

                                     DUCT SIZ  R DRIVE RGN AXIL HTR CIRC DSPL DOM DSPL CYL BUKL COL PIST ROD EXPN HOP REGN HOP
                                      -0.0001   1.3266  -0.0000   0.3321   0.6796   0.9209   0.4591   0.7695  -0.0000   0.6825

                                     RGN TEMP HTR MACH COL MACH RGN REYN 20% WORK  5% WORK  PR DROP   STROKE RGN WALL
                                       0.4482  -0.0001   0.0834  -0.0001  -0.0004   3.0017   0.6286   0.2301   0.0000

                              .....     DERIVATIVES OF OBJCTV AND CONSTRAINTS WITH RESPECT TO DESIGN VARIBLES FOLLOW       .....

  PMEAN   1.14D+07   4.1427  -0.0740  -0.2296   0.2060   0.0      0.0     -0.2402   0.0747   0.0      0.0      0.0      0.0
                                       0.0      0.0     -0.1182  -0.1593  -0.0756  -0.0187  -0.1303  -0.0544  -0.2261  -0.0777
                                      -0.1453   0.0067   0.0078  -0.2248   0.1220  -0.4881   0.0396   0.0      0.0
  FREQ    1.41D+01   0.8441  -0.2020  -0.9853   0.5595  -0.7245  16.6701  -1.0309   0.3860   0.0      0.0      0.0      0.0
                                       0.0      0.0      0.0568  -0.2674   0.0083   0.0020   0.0013   0.0060   0.0519  -0.0064
                                      -0.0522  -1.1461  -1.0425  -1.0845  -1.7708   7.0831  -0.6864   0.0      0.0
CRNK RAD  3.33D-02   2.4100   0.0283  -0.4045   0.3201  -0.2439   0.0     -0.4231   0.1408   1.6008   0.0      0.0      0.0
                                       0.0      0.3146   0.0311  -0.1342  -0.0622  -0.0154  -0.0324  -0.1355  -0.0148  -0.0214
                                      -0.0426  -0.3650  -0.2031  -0.2243  -0.0511   0.2046  -0.1760  -0.2671   0.0
CRNK L/R  7.10D+00   2.1307   0.0469   0.3188  -0.1655   0.0231   0.0      0.3335  -0.1261  -0.1518   0.0      0.0      0.0
                                       0.0      1.1389  -0.0264   0.1000   0.0484   0.0120   0.0178  -0.0627  -0.0165   0.0128
                                      -0.0973   0.2024   0.4285   0.2930   0.6917  -2.7668   0.1952   0.0253   0.0
CRNK E-R  3.08D-02   4.4039  -0.0105  -0.0795   0.0377  -0.0055   0.0     -0.0832   0.0323   0.0359   0.0      0.0      0.0
                                       0.0     -0.1734   0.0067  -0.0240  -0.0090  -0.0022  -0.0031  -0.0080   0.0066  -0.0024
                                       0.0213  -0.0470  -0.1015  -0.0694  -0.1426   0.5703  -0.0440  -0.0060   0.0
EXPN DIA  8.08D-02   1.1525   0.0053  -2.2661   1.7040   0.0      0.0     -2.3702   0.7831   2.0294  -3.0545   0.0      1.8189
                                       0.0      0.0      0.1739  -0.7340  -0.8846  -0.2520  -0.1573  -0.2354  -0.8026  -0.1057
                                      -0.0166  -1.8730  -1.5966  -1.4819  -1.2682   5.0727  -1.0383   0.0      0.0
EXPN WAL  6.95D-03   1.5237   0.0007  -0.0036  -0.0019   0.0      0.0     -0.0038  -0.0003   0.0     -0.3975   0.0      0.2367
                                       0.0      0.0      0.0002  -0.0008   0.0006   0.0001   0.0002   0.0004   0.6100   0.0002
                                      -0.0005   0.0001   0.0005  -0.0004  -0.0007   0.0030   0.0002   0.0      0.0
DSPL LNG  1.03D-01   2.3686   0.0116   0.0194  -0.0318   0.0      0.0      0.0203  -0.0057   2.5298   0.0      0.0      0.0
                                       0.0      0.0     -0.0015   0.0087   0.0096  -0.0311   0.0045   0.0069   0.0051   0.0027
                                       0.0014   0.0118  -0.0001  -0.0072  -0.0666   0.2666  -0.0024   0.2886   0.0
DSPL WAL  1.21D-03   0.7601   0.0002  -0.0011  -0.0005   0.0      0.0     -0.0012   0.0007   0.0      0.0      0.0      0.0
                                       0.0      0.0      0.0001  -0.0003   0.9052   0.2814  -0.0000  -0.0000   0.0002  -0.0000
                                      -0.0003  -0.0000  -0.0001   0.0002   0.0003  -0.0010   0.0000   0.0      0.0
 HTR LNG  1.28D-01   0.8207  -0.0189   0.0140   0.0519   0.0      0.0      1.2054  -0.0180   0.0      0.0      0.0      0.0
                                       0.0      0.0     -0.0771   0.3218   0.0207   0.0051   0.0109   0.0149  -0.1644   0.0022
                                       0.0856   0.0423   0.0086   0.0614  -0.2218   0.8872  -0.0107   0.0      0.0
 HTR DIA  3.29D-03   1.0885   0.0692   0.1496  -0.1887   0.0      0.0      0.7177  -0.0350   0.0      4.0728   0.0     -0.4989
                                       0.0      0.0      0.0016   0.0217   0.0655   0.0162   0.0304   0.0471   0.0762   0.0155
                                      -0.0230   1.8534  -0.0019  -0.0655  -0.1319   0.5275   0.1416   0.0      0.0
 HTR WAL  1.01D-03   1.1148   0.0110   0.0160  -0.0303   0.0      0.0      0.3595  -0.0012   0.0      2.4411   0.0     -1.7520
                                       0.0      0.0      0.0133   0.0466   0.0034   0.0008   0.0014   0.0025   0.0273   0.0036
                                      -0.0041   0.0013  -0.0014  -0.0137  -0.0153   0.0611  -0.0019   0.0      0.0
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          GPU-3  METHANE  877K/287K      STIRLING ENGINE DATA:     ( SAMPLE PROBLEM 2)

LABEL      VALUE     RATIO     OBJF             CONSTRAINTS :
                                        1         2         3         4         5         6         7         8         9         10

                              0.3652    0.0001    0.0001    0.3885   13.0711    0.0002    1.2748    0.0001    0.0000   -0.0000    0.5808
                                       -0.0001    1.3266   -0.0000    0.3321    0.6796    0.9209    0.4591    0.7695   -0.0000    0.6825
                                        0.4482   -0.0001    0.0834   -0.0001   -0.0004    3.0017    0.6286    0.2301    0.0000

                               .....  DERIVATIVES OF OBJCTV AND CONSTRAINTS WITH RESPECT TO DESIGN VARIBLES FOLLOW      .....

RGN LNG   7.08D-02   3.1349   0.0416    0.1291   -0.1131    0.0       0.0       0.1217   -0.0346   -1.3116    0.0       0.0       0.0
                                        0.0       0.0       0.1297    0.0433    0.0328    0.0081    0.0156    0.0236    0.0019    0.0097
                                       -0.0070    0.0123   -0.0017   -0.0127   -0.3832    1.5330   -0.0898    0.0       0.0
RGN DIA   3.45D-02   1.5250   0.0098    0.3407   -0.0269    0.0       0.0       0.3798   -0.1718   -1.3383   -0.3356    5.2459    0.0
                                        0.0       0.0      -0.5566    0.1535    0.1546    0.0382   -0.6573    0.1112    0.0430   -0.1181
                                       -0.0282    0.0662   -0.0081    1.2645    0.9675   -3.8699    0.6201    0.0       0.6691
RGN WAL   6.89D-03   3.9845   0.0002   -0.0114   -0.0005    0.0       0.0       0.0005   -0.0001    0.0      -0.1507    0.0       0.0
                                        0.0       0.0       0.0816    0.0002    0.0002    0.0001    0.0001    0.0002   -0.0000    0.0642
                                       -0.0000    0.0001    0.0002   -0.0000   -0.0004    0.0017    0.0001    0.0      -0.2510
RGN SIG   1.89D-01   0.4033   0.0234   -0.1305   -0.0643    0.0       0.0      -0.1368    0.0895    0.0       0.0       0.0       0.0
                                        0.0       0.0       0.0152    0.0400    0.2207    0.0545    0.1015    0.1588    0.1643    0.0575
                                       -2.0574    0.1322    0.0315   -0.4679    3.0778  -12.3112    1.4851    0.0       0.0
RGN WIR   1.19D-04   2.9294  -0.0215   -0.0443    0.0592    0.0       0.0      -0.0463    0.0105    0.0       0.0       0.0       0.0
                                        0.0       0.0       0.0043   -0.0120    0.0004    0.0001    0.0001    0.0003    0.0073   -0.0001
                                       -0.0005    0.0010    0.0012   -0.3402    0.3956   -1.5324    0.1460    0.0       0.0
COL LNG   8.76D-02   2.4672  -0.0259   -0.0186    0.0706    0.0       0.0      -0.0187   -0.1198   -2.3678    0.0       0.0       0.0
                                        0.0       0.0      -0.0145   -0.0030    0.0036    0.0009   -0.4316    0.0026    0.0043    0.0051
                                        0.0118   -0.0191   -0.0049   -0.0444   -0.0529    0.2116   -0.0102    0.0       0.0
COL DIA   1.03D-03   0.9560  -0.0017    0.1563    0.0048    0.0       0.0       0.1645   -0.2288    0.0       0.0      -2.0157    0.0
                                       -2.1593    0.0      -0.0174    0.0691    0.0741    0.0183    1.1977    0.0533    0.0234    0.0269
                                       -0.0399    0.0314    2.0417   -0.0230    0.5558   -2.2230    0.2424    0.0       0.0
COL WAL   4.79D-04   1.8772  -0.0082   -0.0078    0.0225    0.0       0.0      -0.0078   -0.0704    0.0       0.0      -3.1916    0.0
                                        0.0       0.0      -0.0026   -0.0021   -0.0008   -0.0002    0.5820   -0.0006    0.0007    0.0014
                                        0.0034   -0.0046   -0.0049   -0.0132   -0.0122    0.0488   -0.0020    0.0       0.0
C-DCT DI  4.56D-03   0.7635   0.0149    0.0417   -0.0408    0.0       0.0       0.0436   -0.0121    0.0       0.0       0.0       0.0
                                        2.5849    0.0      -0.0045    0.0178    0.0184    0.0046    0.0086    0.0133    0.0052    0.0052
                                       -0.0088    0.0093    0.0164   -0.0008   -0.0373    0.1492    0.0069    0.0       0.0

 INFO =  1        39  FUNCTION EVALUATIONS
 NVAR = 21        38  GRADIENT EVALUATIONS
NCASE = 38      3412  TOTAL CYCLES USED
 IBND =  0
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            GPU-3  METHANE  877K/287K       STIRLING ENGINE DATA:     ( SAMPLE PROBLEM 2)

               KEY     LABEL     VALUE       RATIO     FGRD      ERRROR     N-CNST     ERR CNSTR
VARIABLE- 1     1      PMEAN    1.14D+07    4.1427 -7.40D-02  2.61D-03    13        8.08D-02
VARIABLE- 2     2      FREQ     1.41D+01    0.8441 -2.02D-01 -2.36D-03    24        1.64D-01
VARIABLE- 3     3    CRNK RAD   3.33D-02    2.4100  2.83D-02  1.58D-03     2        8.25D-02
VARIABLE- 4     4    CRNK L/R   7.10D+00    2.1307  4.69D-02 -1.09D-03    24        4.43D-02
VARIABLE- 5     5    CRNK E-R   3.08D-02    4.4039 -1.05D-02  4.33D-04    24        1.05D-02
VARIABLE- 6    10    EXPN DIA   8.08D-02    1.1525  5.32D-03  1.19D-02     2        4.39D-01
VARIABLE- 7    11    EXPN WAL   6.95D-03    1.5237  6.85D-04  6.20D-04    19        1.67D-03
VARIABLE- 8    12    DSPL LNG   1.03D-01    2.3686  1.16D-02 -5.13D-04     7        2.27D-02
VARIABLE- 9    13    DSPL WAL   1.21D-03    0.7601  1.91D-04  2.89D-04     2        1.36D-04
VARIABLE-10    14     HTR LNG   1.28D-01    0.8207 -1.89D-02 -4.92D-04    13        5.26D-02
VARIABLE-11    15     HTR DIA   3.29D-03    1.0885  6.92D-02 -4.62D-03    22        1.07D-01
VARIABLE-12    16     HTR WAL   1.01D-03    1.1148  1.10D-02  7.47D-04    13        9.08D-03
VARIABLE-13    18     RGN LNG   7.08D-02    3.1349  4.16D-02 -6.03D-04    13        8.86D-02
VARIABLE-14    19     RGN DIA   3.45D-02    1.5250  9.83D-03 -1.00D-02    13        3.80D-01
VARIABLE-15    20     RGN WAL   6.89D-03    3.9845  1.66D-04 -5.19D-04    13        5.57D-02
VARIABLE-16    21     RGN SIG   1.89D-01    0.4033  2.34D-02 -1.73D-02    25        1.21D-01
VARIABLE-17    22     RGN WIR   1.19D-04    2.9294 -2.15D-02 -1.39D-04    24        5.14D-02
VARIABLE-18    24     COL LNG   8.76D-02    2.4672 -2.59D-02 -1.49D-03     7        2.12D-02
VARIABLE-19    25     COL DIA   1.03D-03    0.9560 -1.75D-03  2.10D-03    25        2.18D-02
VARIABLE-20    26     COL WAL   4.79D-04    1.8772 -8.21D-03 -1.83D-04     9        8.62D-03
VARIABLE-21    28    C-DCT DI   4.56D-03    0.7635  1.49D-02  5.24D-04    11        2.54D-02

 FINAL OBJECTIVE FUNCTION VALUE =  3.65188698D-01

 CONSTRAINTS EVALUATED AT FINAL DESIGN CONDITIONS:
 1.38282776D-04  6.60156250D-05  3.88470471D-01  1.30711498D+01  1.78158283D-04
 1.27482605D+00  6.60636935D-05  1.39062333D-06 -3.03858747D-05  5.80829620D-01
-1.47104263D-04  1.32656708D+00 -2.67028809D-05  3.32087815D-01  6.79587841D-01
 9.20864642D-01  4.59058166D-01  7.69500017D-01 -1.52587891D-05  6.82536542D-01
 4.48194683D-01 -7.26580620D-05  8.34479779D-02 -8.96453857D-05 -4.14002975D-04
 3.00165601D+00  6.28592223D-01  2.30073953D-01  5.06592914D-06

  LAGRANGE MULTIPLIER ESTIMATES:
 6.99371830D-02  2.57624123D-01  0.0             0.0             1.43140012D-02
 0.0             8.95712631D-03  2.16440127D-03  2.70198515D-03  0.0
 9.83594664D-03  0.0             6.82955515D-01  0.0             0.0
 0.0             0.0             0.0             2.73710826D-03  0.0
 0.0             5.75115588D-02  0.0             1.51050786D-01  3.91941501D-02
 0.0             0.0             0.0             2.14297217D-01

  LAGRANGIAN GRADIENT ERROR =  6.01382497D-02
  LAGRANGE MULTIPLIER ERROR =  0.0
        COMPLEMENTARY ERROR =  8.46626841D-05
           CONSTRAINT ERROR =  9.34041007D-04
  WEIGHTED CONSTRAINT ERROR =  6.59152867D-05
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      GPU-3  METHANE  877K/287K       STIRLING ENGINE DATA:     ( SAMPLE PROBLEM 2)

                CASE TO CASE COMPARISON INFORMATION

 CASE NUMBER               31         32         33         34         35         36         37         38
AVERAGE PRESSURE MPA    11.2314    11.2641    11.2802    11.3172    11.3722    11.3440    11.3510    11.3510
SPEED            RPM     828.40     830.75     832.23     836.02     842.18     841.79     844.27     844.27
GAS MASS          KG  0.0249462  0.0249444  0.0249352  0.0249598  0.0249435  0.0248678  0.0248152  0.0248151
INDIC HEAT IN      W   24998.46   24996.05   24995.67   24986.68   24979.00   25003.61   24996.54   24996.50
INDIC POWER OUT    W   10000.32   10001.32   10001.83   10004.92   10007.44    9998.66   10000.66   10000.61
EFFICIENCY               0.4000     0.4001     0.4001     0.4004     0.4006     0.3999     0.4001     0.4001

EXPANSION WALL T   K     796.59     796.83     796.97     797.36     797.92     797.72     797.90     797.91
EXPANSION GAS T    K     796.59     796.83     796.97     797.36     797.92     797.72     797.90     797.91
WALL TO GAS NET Q  W    -492.18    -485.51    -482.50    -473.64    -460.41    -461.23    -456.83    -456.83
P-V WORK           W   21079.07   21105.92   21116.21   21140.57   21180.56   21182.25   21193.61   21193.61
HYSTERESIS         W    0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0

COMBUSTOR TEMP     K     877.00     877.00     877.00     877.00     877.00     877.00     877.00     877.00
HEATER WALL T      K     862.11     862.15     862.18     862.25     862.38     862.37     862.43     862.43
HEATER GAS T       K     816.53     816.75     816.88     817.20     817.69     817.54     817.72     817.72
WALL TO GAS NET Q  W   23969.34   23970.41   23972.15   23966.54   23964.93   23986.47   23980.58   23980.54
WALL CONDUCTION    W    1029.12    1025.65    1023.52    1020.14    1014.07    1017.15    1015.96    1015.96
FRICTION P-DROP  MPA  0.0283978  0.0284951  0.0285452  0.0286609  0.0288202  0.0287610  0.0287748  0.0287747

REGENERATOR WALL T K     588.84     589.03     589.12     589.32     589.67     589.71     589.81     589.81
REGENERATOR GAS T  K     539.32     539.75     539.94     540.41     541.15     541.16     541.36     541.36
WALL TO GAS NET Q  W   -3249.05   -3232.42   -3227.83   -3209.53   -3184.80   -3203.97   -3191.64   -3191.59
WALL CONDUCTION    W    1029.12    1025.65    1023.52    1020.14    1014.07    1017.15    1015.96    1015.96
FRICTION P-DROP  MPA  0.2414592  0.2434049  0.2444950  0.2469979  0.2508307  0.2490339  0.2502936  0.2502931

WATER TEMPERATURE  K     287.00     287.00     287.00     287.00     287.00     287.00     287.00     287.00
COOLER WALL T      K     315.56     315.92     316.06     316.39     316.96     317.06     317.19     317.19
COOLER GAS T       K     333.32     333.83     334.03     334.53     335.34     335.44     335.62     335.62
WALL TO GAS NET Q  W  -13500.38  -13507.21  -13511.37  -13511.62  -13521.03  -13550.34  -13546.79  -13546.80
WALL CONDUCTION    W   -1497.76   -1487.53   -1482.41   -1470.13   -1450.53   -1454.61   -1449.09   -1449.09
FRICTION P-DROP  MPA  0.0358137  0.0357260  0.0357178  0.0354013  0.0353838  0.0357827  0.0358815  0.0358815

COMPRESSION WALL T K     344.95     345.45     345.66     346.16     346.98     347.06     347.26     347.26
COMPRESSION GAS T  K     344.95     345.45     345.66     346.16     346.98     347.06     347.26     347.26
WALL TO GAS NET Q  W      23.54      23.63      23.61      23.65      23.66      23.76      23.70      23.70
P-V WORK           W   -9232.12   -9254.31   -9262.44   -9281.00   -9312.68   -9319.07   -9326.81   -9326.86
HYSTERESIS         W    0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0

MAX PRESSURE     MPA    14.9012    14.9400    14.9592    14.9979    15.0599    15.0279    15.0370    15.0370
MIN PRESSURE     MPA     8.3625     8.3924     8.4074     8.4452     8.4995     8.4720     8.4799     8.4799
INITIAL PRESSURE MPA    12.9058    12.9378    12.9518    12.9836    13.0291    13.0092    13.0115    13.0113
P-VE PHASE ANGLE DEG -236.3046  -236.1748  -236.0692  -235.8608  -235.4724  -235.7244  -235.5545  -235.5547
STATIC PWR LOSS    W   -1280.16   -1275.97   -1273.47   -1269.10   -1261.47   -1264.52   -1262.76   -1262.76
DYNAMIC PWR LOSS   W    -287.43    -281.34    -278.62    -270.63    -258.42    -259.67    -255.82    -255.82
FRICTION PWR LOSS  W   -1846.62   -1850.29   -1851.89   -1854.64   -1860.44   -1864.52   -1866.13   -1866.13
REGENATR PWR LOSS  W   -3202.76   -3186.26   -3181.77   -3163.58   -3139.09   -3158.15   -3145.85   -3145.80
LEAKAGE PWR LOSS   W        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
NET Q TO COLD H2O  W  -14998.14  -14994.73  -14993.78  -14981.75  -14971.55  -15004.96  -14995.88  -14995.89
UNACCT PWR LOSS    W        0.0       0.01       0.01       0.00       0.01        0.0        0.0       0.01
REGENATR EFFECTVNS       0.9949     0.9949     0.9950     0.9950     0.9951     0.9951     0.9951     0.9951
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           GPU-3  METHANE  877K/287K    FINAL DESIGN    OFF-DESIGN PERFORMANCE

                      CASE TO CASE COMPARISON INFORMATION

 CASE NUMBER                  1          2          3          4          5          6
AVERAGE PRESSURE MPA    11.3510    11.3510    11.3510    11.3510    11.3510    11.3510
SPEED            RPM     100.00     300.00     500.00     700.00     900.00    1099.99
GAS MASS          KG  0.0259134  0.0256187  0.0253326  0.0250362  0.0247273  0.0244031
INDIC HEAT IN      W    4559.16   10389.69   15998.93   21348.39   26360.16   31008.90
INDIC POWER OUT    W    1643.23    4602.42    7077.54    8997.16   10295.38   10904.82
EFFICIENCY               0.3604     0.4430     0.4424     0.4214     0.3906     0.3517

EXPANSION WALL T   K     819.46     811.93     805.88     800.91     796.84     793.48
EXPANSION GAS T    K     819.46     811.93     805.88     800.91     796.84     793.48
WALL TO GAS NET Q  W    -465.78    -453.20    -436.30    -444.70    -462.82    -489.57
P-V WORK           W    2691.93    7914.46   12947.82   17800.96   22478.83   26979.09
HYSTERESIS         W     0.0        0.0        0.0        0.0        0.0        0.0

COMBUSTOR TEMP     K     877.00     877.00     877.00     877.00     877.00     877.00
HEATER WALL T      K     874.91     871.38     867.97     864.68     861.59     858.70
HEATER GAS T       K     840.22     832.39     826.09     820.88     816.59     813.02
WALL TO GAS NET Q  W    3478.10    9325.40   14951.97   20319.29   25349.37   30016.97
WALL CONDUCTION    W    1081.06    1064.29    1046.96    1029.10    1010.80     991.93
FRICTION P-DROP  MPA  0.0005711  0.0042768  0.0109670  0.0203885  0.0323543  0.0466938

REGENERATOR WALL T K     582.51     583.98     585.80     587.99     590.58     593.62
REGENERATOR GAS T  K     524.89     528.80     532.96     537.65     542.88     548.72
WALL TO GAS NET Q  W    -326.16   -1025.24   -1800.42   -2610.91   -3420.45   -4234.20
WALL CONDUCTION    W    1081.06    1064.29    1046.96    1029.10    1010.80     991.93
FRICTION P-DROP  MPA  0.0156754  0.0578069  0.1165687  0.1895968  0.2754204  0.3728901

WATER TEMPERATURE  K     287.00     287.00     287.00     287.00     287.00     287.00
COOLER WALL T      K     290.11     296.57     303.63     311.30     319.57     328.55
COOLER GAS T       K     300.47     309.88     319.05     328.53     338.46     349.03
WALL TO GAS NET Q  W   -1395.62   -4295.43   -7463.03  -10901.62  -14614.68  -18645.44
WALL CONDUCTION    W   -1520.32   -1491.84   -1458.37   -1449.62   -1450.10   -1458.64
FRICTION P-DROP  MPA  0.0007372  0.0054105  0.0137636  0.0254757  0.0403208  0.0581014

COMPRESSION WALL T K     311.10     320.82     330.26     339.99     350.17     360.98
COMPRESSION GAS T  K     311.10     320.82     330.26     339.99     350.17     360.98
WALL TO GAS NET Q  W      26.53      25.66      24.89      24.19      23.52      22.86
P-V WORK           W   -1036.80   -3169.37   -5372.72   -7645.12   -9986.33  -12399.24
HYSTERESIS         W     0.0        0.0        0.0        0.0        0.0        0.0

MAX PRESSURE     MPA    15.0788    15.0641    15.0525    15.0430    15.0351    15.0288
MIN PRESSURE     MPA     8.3606     8.3979     8.4302     8.4599     8.4873     8.5131
INITIAL PRESSURE MPA    13.2797    13.2005    13.1287    13.0599    12.9927    12.9260
P-VE PHASE ANGLE DEG  -239.6679  -238.4825  -237.3877  -236.3172  -235.2536  -234.1787
STATIC PWR LOSS    W   -1354.88   -1329.95   -1305.33   -1280.64   -1255.78   -1230.47
DYNAMIC PWR LOSS   W    -240.83    -235.62    -225.19    -239.58    -263.38    -295.67
FRICTION PWR LOSS  W     -11.90    -142.67    -497.56   -1158.68   -2197.12   -3675.02
REGENATR PWR LOSS  W    -277.30    -977.16   -1753.16   -2564.50   -3374.91   -4189.57
LEAKAGE PWR LOSS   W        0.0        0.0        0.0        0.0        0.0        0.0
NET Q TO COLD H2O  W   -2915.94   -5787.27   -8921.40  -12351.24  -16064.79  -20104.08
UNACCT PWR LOSS    W      -0.00       0.00        0.0      -0.00        0.0      -0.00
REGENATR EFFECTVNS       0.9974     0.9964     0.9958     0.9953     0.9950     0.9948
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