## Thermal Power Systems

Advanced Solar Thermal Technology Project

## HEAP: <br> Heat Energy Analysis Program

 A Computer Model Simulating Solar ReceiversF. L. Lansing



January 15, 1979

Prepared for
U.S. Department of Energy

Through an agreement with
National Aeronautics and Space Administration
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Jet Propulsion Laboratory
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## ABSTRACT

Thermal design of solar receivers is commonly accomplished via approximate models, where the receiver is treated as an isothermal box with lumped quantities of heat losses to the surroundings by radiation, conduction and convection.

These approximate models, though adequate for preliminary design purposes, are not detailed enough to distinguish between different receiver designs, or to predict transient performance under variable solar flux, ambient temperatures, etc. A computer code has been written for this purpose and is given the name "HEAP", an acronym for Heat Energy Analysis Program. HEAP has a basic structure that fits a general heat transfer problem, but with specific features that are custom-made for solar receivers. The code is written in MBASIC computer language.

This document explains the detailed methodology followed in solving the heat transfer problem, and includes a program flow chart, an explanation of input and output tables, and an example of the simulation of a cavity-type solar receiver.

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## SECTION 1

## INTRODUCTION

## BACKGROUND

The need for an accurate prediction of the thermal behavior of solar receivers, to be used as a basic tool for their performance comparison, has become increasingly important with the rapid development of solar energy technology. A sufficiently general analytical model is needed which is both an adequate idealization of the physical system and is capable of expressing itself by simple mathematical description.

Thermal design of solar receivers is commonly accomplished with approximate models where the receiver is treated as an isothermal box with lumped quantities of heat losses by radiation, conduction and convection to the surroundings. These approximate models, though adequate for preliminary design purposes, are not detailed enough to distinguish between different receiver designs, to predict transient performance, or to accurately express the performance sensitivity to variations occurring in the system parameters.

Since the solar receiver is part of the energy collection subsystem, and located in a very strategic spot in between the concentrator and the energy conversion subsystem, its thermal performance and concept selection affect the system installation and operation cost. A detailed model is, therefore, needed not only to predict the performance under varying solar flux, ambient temperatures, and local heat transfer rates, but also to detect locations of hot spots and metallurical difficulties, and to predict the performance sensitivity to neighboring component parameters.

The computer code written for this purpose is presented in this report and is given the name HEAP, an acronym for Heat Energy Analysis Program. HEAP has a basic structure that fits any general heat transfer problem, but with specific features that are "custom-made" for solar receivers. The processes that led to the writing of this new code and the criteria to be satisfied are explained in the following sections.
1.2 SURVEY OF OTHER AVAILABLE PROGRAMS

Before constructing the HEAP code, a computer library search was conducted to review current programs and select one that could be applied. Five computer programs were identified in the search:
(1) LOHARP: Lockheed Orbital Heat Rate Package, contains three subprograms for calculation of radiation view factors, radiation constant, and radiation orbital heat rate. The program is applicable to outer space related problems. The program is documented in Reference 1.
(2) TAS: Thermal Analysis System, a JPL-developed program for steady state problems only, written in FORTRAN IV. No convection heat transfer or fluid flow across nodes are considered (see Reference 2).
(3) THERM: Thermal Analyzer Computer Code is another code in FORTRAN IV, which is capable of handling 1000 nodes of a problem with steady state or transient heat transfer. Available documentation (Reference 3) is not complete or clear in its assumptions and limitations.
(4) SINDA: Systems Improved Numerical Differencing Analyzer is a code expanded by TRW from a version written by Chrysler Corporation. It is a general,
versatile but expensive code which handles both steady and transient problems. Users may add their own subroutines after becoming familiar with its massive documentation (Reference 4).
(5) HEAT: HEAT Transfer Computing Program, developed by W. A. Beckman at the University of Wisconsin. HEAT is basically the same as TAS except that transient solutions are added. HEAT is written in FORTRAN V language (see Reference 5).

## 1.3 <br> SELECTION CRITERIA

Before selecting the computer program most suitable for advanced receiver studies, some criteria were set to bound the selection process of any one of the above computer codes:
(1) The computer program should include analyses of all modes of heat transfer by conduction, convection, longwave (infrared) radiation, solar radiation and the convective energy carried in and out the receiver by flowing fluids. The program should also allow for possible heat generation or loss due to chemical reactions, electrical resistances, etc.
(2) The computer program must handle both steady state (or quasi-steady state) and transient problems. This requirement is essential in testing receivers of different response times under variable daily solar flux, ambient temperatures, or flow rates.
(3) For ease of use, the computer program should be a self-sufficient and complete package with minimum or no dependence on other computer program outputs. This means that the user should not be required to put together the input data for one program using subroutines of other programs as a prerequisite or to
reach the final answers of the problem by working in more than one program simultaneously.
(4) The program documentation should be clear, simple, and self explanatory to shorten the user's start-up time and should explicitly show its flexibility and limitations.
(5) The computer program should be closely associated with this particular application to solar receivers, but generally expressed to handle many possible receiver configurations. This criterion saves time consumed in running complicated, general programs that have too many variables irrelevant to the problem under study.

The five computer codes previously identified were evaluated against these selection criteria. The first program, LOHARP, was dropped because of the fifth criterion since it handles mostly outer space objects with no convection heat transfer. The second program, TAS, was also dropped because of the first and second criteria since it handles steady state problems only, with no heat or flow convection. The third program, THERM, was omitted for lack of good documentation, required by the fourth criterion. The fourth and fifth programs, SINDA and HEAT, though comprehensive at times, both lack the completion specified by the third criterion. They both require radiation view factor data which the user may either calculate or obtain from the output of the LOHARP program, for example. HEAT assumes that the user will provide all thermal and fluid conductances, which can be quite a tedious process. For instance, for a 30 -node problem, at least 450 conductances are needed. The SINDA program was further downgraded for being too complex and still needs extra subroutines to be added. The fifth, and the only hopeful program, HEAT, was then carefully investigated to see how additional receiver subroutines could be tied to it. Unfortunately, the HEAT structure and assumptions were found inadequate in handling many computational procedures. For the Euler's minimum time step of the transient solution, HEAT approximates the radiation effect, which
is dominant in solar receivers, as equal to one-half of all other effects, an approximation which may not be adequate. Also, since the user provides all conductances, HEAT makes no distinction between different surface areas and neighboring nodes. For each node, only one surface area for all modes of heat transfer is used in HEAT, which is not adequate for a detailed heat transfer analysis.

Finally, a decision was made to save time and effort consumed in correcting, adding, or deleting in the candidate programs to suit solar receivers, by writing a new computer code that satisfies the above criteria and fits the special needs. The following sections of this report explain the detailed methodology followed in solving the heat transfer problem and show the flow chart of the program. A solved example is also provided in Section 4 to illustrate the program use for a cavity-type solar receiver.

## PROGRAM METHODOLOGY

GENERAL LAYOUT

The methodology used in the heat transfer calculations is a well-known numerical technique which has enjoyed wide application since the advent of high-speed digital computers. The system (receiver) under consideration is discretized in space by nodes as shown in Figure 2-1. Heat exchange between any two arbitrary nodes (i) and (j) is illustrated in Figure 2-2. The heat transfer rate equations are written with all modes of heat transfer included. The net energy stored in each node (sometimes called the energy "residual") is calculated using the first law of thermodynamics, including the energy exchange to and from each node as a result of neighboring nodes.

Spatial nodes are divided into two categories: a) equilibrium nodes (or finite heat reservoirs) whose temperature is dependent on the net energy stored; and b) source or sink nodes (or infinite heatreservoirs) which gain or lose any amount of heat without changing temperature. The program structure differentiates between these node categories for both steady state and transient solutions. Another advantage was taken from the axisymmetric geometry of receiver nodes by adding a special radiation view factor subroutine. To avoid excessive precalculations of thermal conductances, the program is built to compute these conductances with minimum information requested from the user.

For the steady state solution, the energy residual to each equilibrium node must be zero. A Newton-Raphson iteration solution is used to solve for: a) the temperature distribution of equilibrium nodes; and b) the residuals of source/sink nodes.


Figure 2-1. Receiver Discretization into Nodes


Figure 2-2. Heat Exchange Between Nodes (i) and (j)

For the transient solution, the "forward" finite difference numerical technique is used. By this technique, the new temperature vector at time $(\tau+\Delta \tau)$ is calculated using the old temperature vector at time ( $\tau$ ) and all other parameters evaluated at time ( $\tau$ ) . To ensure stable computations (convergence) a special formula is used for the critical time step $(\Delta \tau)$ based on the methodology stated in Reference 7. To minimize computational time consumed due to nodes having a very small time step $(\Delta \tau)$, the concept of zero-capacity nodes was included in the transient solution.

### 2.2 BASIC ASSUMPTIONS

The following assumptions and idealizations were made in the heat transfer calculations:
(1) The physical system under study (solar receiver) is treated as a collection of a number of isothermal nodes with uniform optical and physical properties throughout each node. Nodes are preferred to be of axisymmetric geometry (plane discs, plane rings, cylindrical discs, cylindrical rings etc.) to make use of the special radiation view factor subroutine. The axis of symmetry must coincide with the receiver axis. The condition of axial symmetry is preferred, but is not mandatory.
(2) The nodal specific heat, thermal conductivity, and density are assumed independent of operating temperature. The properties may be taken at an estimated average working temperature as an approximation. Also, nodal infrared and solar emittances are assumed independent of both directivity and operating temperature. A second phase of the program would consider the above variations if they showed serious effects on the computational accuracy.
(3) The solar flux incident on each node is assumed uniform. Therefore, the user should be careful when dividing the receiver into nodes to avoid distorting the flux profile inside or outside the receiver body.
(4) The radiation emitted by a node surface is assumed to be diffuse. In general, the radiation reflected from a node surface can be diffuse, specular or a combination of diffuse and specular parts. In the present model, only diffuse reflection was assumed, with a zero specular component.
(5) The optical properties of a node surface are assumed to be divided into two bands in the electromagnetic spectrum. This "semi gray" assumption allows separate calculations of radiation energy exchange in the short wave (solar) band and the longwave (infrared) band. Consequently, the radiation energy in the program is allowed to be absorbed, reflected or emitted in the longwave region, and is only absorbed or reflected in the short wave region.
(6) The nodal internal energy generation or dissipation by electrical, chemical or nuclear effects is assumed uniform over the node surface and independent of the temperature.

Based on the above assumptions and idealizations, the heat transfer rate equations are written and analyzed as described in the following sections.

### 2.3 CONDUCTION HEAT TRANSFER

For two neighboring nodes (i) and (j), exchanging heat by conduction across the interface area $A(i, j)$, the heat transfer $Q(i)$ cond is given by

$$
\begin{align*}
& Q(i)_{\text {cond }}=\sum_{j} C(i, j)_{\text {cond }}[T(j)-T(i)]  \tag{1}\\
& C(i, j)_{\text {cond }}=\left[\frac{A(i, j)}{\frac{L(i)}{K(i)}+\frac{L(j)}{K(j)}}\right] \tag{2}
\end{align*}
$$

where
$K$ is the thermal conductivity, and $L$ is the conduction distance between the center of each node and the separating surface.

The value of $C(i, j)$ could be a function of time and/or temperature or any other variable. However, the conductance $C(i, j)$ is assumed constant and uniform throughout the node. Because of the symmetric nature of the conductance matrix, only half the input data are needed for $A(i, j)$, where the half-cut is made along the diagonal.

### 2.4 CONVECTION HEAT TRANSFER

The convection heat transfer across the boundary layer of solid-liquid, liquid-gas or solid-gas interfaces, can be put in the following form between two neighboring nodes (i) and ( $j$ ) of different phase:

$$
\begin{equation*}
Q(i){ }_{c o n v}=\sum_{j} C(i, j)_{c o n v} \cdot[T(j)-T(i)] \tag{3}
\end{equation*}
$$

where the conductance $C(i, j)$ conv is given by:

$$
\begin{equation*}
C(i, j)_{\text {conv }}=h(i, j) \quad . A(i, j) \tag{4}
\end{equation*}
$$

and where $h(i, j)$ is the convective heat transfer coefficient.
$C(i, j)$ conv is treated in a manner that allows its value to be changed with time. This is made simply because the convection coefficient $h(i, j)$ is a dominant function of the fluid mass flow rate which may not be constant during the receiver operation. Furthermore, because of the symmetric nature of the $h(i, j)$ matrix, only half of it (cut along the diagonal) is needed as input data.

Consider a node (i) surrounded by two adjacent nodes (j) and (k) with a fluid flowing at a rate $\dot{m}_{f}$ from (s) to (i) to (k) as shown in Figure 2-3. An energy balance on node (i) taking into account the energy entering at $T(s)$ and leaving at $T(i)$ would yield

$$
\begin{equation*}
Q(i)_{f}=\dot{\mathrm{m}}_{\mathrm{f}} \mathrm{C}_{P_{f}}(i)[T(s)-T(i)] \tag{5}
\end{equation*}
$$

which will be written as

$$
\begin{equation*}
Q(i)_{f}=C(i, s)_{f}[T(s)-T(i)] \tag{6}
\end{equation*}
$$

or

$$
\begin{equation*}
Q(i)_{f}=\sum_{j} C(i, j)_{f}[T(j)-T(i)] \tag{7}
\end{equation*}
$$

where $T(j)$ is the temperature of all neighboring nodes in direct contact with node (i). For a single fluid stream crossing node (i), there will be only two surrounding nodes, one upstream and the other downstream. The flow conductance $C(i, j)$ will then be taken as $\left[\dot{m}_{f}(i) C_{f}(i)\right]$ if the flow goes from (j) to (i), and zero if the flow goes from (i) to $(j)$. The matrix $C(i, j)_{f}$ will then be asymmetric such that $C(i, j) \neq$ $C(j, i)$ and all elements are needed as input data. For boundary nodes that present the inlet fluid zone or outlet fluid zone as shown in Figure 2-4 the next neighboring node in the closed loop should always be considered. In the program, one input number ( +1 ) will be entered in the [MASFLO] matrix representing the direction of the flow to each node. Care must be taken when these boundary nodes are treated as source/sink nodes and when dealing with more than one fluid loop inside the receiver.

(b) Temperature Profile Across Nodes

Figure 2-3. Fluid Convection Between Nodes


Figure 2-4. Closed Sequence of Fluid Nodes in Receiver Loop

## 2.6 <br> INFRARED (IR) AND SOLAR RADIATION

The analysis of heat transfer by radiation is divided into two wavelength regions: a) an infrared or long wavelength region in which infrared properties apply; and b) solar (or short wavelength) region in which solar radiation properties apply (a wavelength region from $0.3-5 \mu)$. The solution methodology is:

For an enclosure composed of $N$ surfaces as shown in Figure $2-5$, the radiosity ( $J$ ) streaming out of the $i^{\text {th }}$ node is given by

$$
\begin{equation*}
J(i)=X(i)+\rho(i) G(i) \tag{8}
\end{equation*}
$$

where $\rho(i)$ is the gray reflectance of the $i^{\text {th }}$ node, $G(i)$ is the irradiation or flux streaming onto the node and $X(i)$ is the thermal excitation at the $i^{\text {th }}$ node. For infrared or longwave radiation, the excitation (X) can be written as

$$
\begin{equation*}
X(i)=\sigma \varepsilon(i) T^{4}(i) \tag{9}
\end{equation*}
$$

where $\sigma$ is the Stefan-Boltzman radiation constant.

On the other hand, the relationship between the radiosity [J] and irradiation [G] for each node can be determined using energy conservation principle as

$$
\begin{align*}
A(i) G(i)= & A(1) F(1, i) J(1)+A(2) F(2, i) J(2)+ \\
& A(K) F(K, i) J(K)+\ldots+A(N) F(N, i) J(N) \tag{10}
\end{align*}
$$

where A's represent surface areas and $F(k, i)$ is the view factor (sometimes called geometric, configuration or form factor) from node (K) to node (i). Using the view factors reciprocity relation presented by

$$
\begin{equation*}
A(K) F(K, i)=A(i) F(i, K) \tag{11}
\end{equation*}
$$



Figure 2-5. Radiosity and Irradiation in Enclosure

Equation (10) is then reduced to the form

$$
\begin{align*}
G(i)= & F(i, 1) J(1)+F(i, 2) J(2)+\ldots \\
& +F(i, K) J(K)+\ldots+F(i, N) J(N) \tag{12}
\end{align*}
$$

or in the matrix form to

$$
[\mathrm{G}]=[\mathrm{F}][\mathrm{J}]
$$

Equation (12) can be also put in the indicial form

$$
\begin{equation*}
G(i)=\sum_{k} F(i, k) \cdot J(k) \tag{13}
\end{equation*}
$$

Combining Equations (8) and (13), we can write

$$
\begin{equation*}
J(i)=X(i)+\rho(i)[F(i, 1) J(1)+F(i, 2) J(2)+\ldots] \tag{14}
\end{equation*}
$$

Equation (14) could be arranged in the matrix form

$$
\begin{aligned}
& {\left[\begin{array}{ccc}
{[1-\rho(1) F(1,1)]} & -\rho(1) F(1,2) & \ldots \\
-\rho(2) F(2,1) & {[1-\rho(2) F(2,2)]} & \ldots \\
\ldots & \ldots & -\rho(2) F(2, N) \\
-\rho(N) F(N, 1) & -\rho(2) F(N, 2) & {[1-\rho(N) F(N, N)]}
\end{array}\right]\left[\begin{array}{l}
J(1) \\
J(2) \\
J(N)
\end{array}\right]\left[\begin{array}{l}
X(1) \\
X(2) \\
X(N)
\end{array}\right]} \\
& \longrightarrow \text { Transfer Matrix } \longrightarrow \\
& \text { Excitation } \\
& \text { Vector }
\end{aligned}
$$

The transfer matrix [A] would then be formed such that

$$
\begin{equation*}
A(i, k)=\delta(i, K)-\rho(i) F(i, k) \tag{15}
\end{equation*}
$$

where $\delta(i, k)$ is the Kronecker delta matrix defined as

$$
\begin{align*}
\delta(i, k) & =0 & & i \neq K \\
& =I & & i=K \tag{16}
\end{align*}
$$

Equation 14 in the matrix form will be reduced to

$$
[\mathrm{A}][\mathrm{J}]=[\mathrm{X}]
$$

By using the matrix algebra, the radiosity vector [ J ] is expressed from Equation (14) as:

$$
\begin{equation*}
[\mathrm{J}]=[\mathrm{A}]^{-1}[\mathrm{X}] \tag{17}
\end{equation*}
$$

Combining Equations (13) and (17), then

$$
\begin{equation*}
[G]=[F] \cdot[J]=[F] \cdot[A]]^{-1}[X]=[\hat{F}] \cdot[X] \tag{18}
\end{equation*}
$$

where $[\hat{F}]$, an $F$ circumflex matrix, is defined as

$$
\begin{equation*}
[\hat{F}]=[F] \cdot\left\lceil\left. A\right|^{-1}\right. \tag{19}
\end{equation*}
$$

The element of the $[\hat{F}]$ matrix, $\hat{F}(i, j)$, is sometimes called "Total" view factor since it includes the effect of the neighboring nodes as well.

The net radiant flux $Q$ "(i) retained in a node (i) is expressed as

$$
\begin{equation*}
Q^{\prime \prime}(i)=G(i)-J(i) \tag{20}
\end{equation*}
$$

Using Equation (9), the net radiant flux into $i^{\text {th }}$ node is expressed as

$$
\begin{equation*}
Q^{\prime \prime}(i)=[1-p(i)] G(i)-X(i) \tag{21}
\end{equation*}
$$

Combining with Equation (18) written in the form

$$
G(i)=\sum_{k} \hat{F}(i, K) X(K)
$$

then

$$
\begin{equation*}
Q^{\prime \prime}(i)=\left\{\sum_{k}[1-\rho(i)] \hat{F}(i, k) \cdot X(k)\right\}-X(i) \tag{22}
\end{equation*}
$$

Equation (22) can be applied to both the solar band and IR band of radiation. These cases are discussed below.

### 2.6.1 Energy Exchange in the IR Band

The excitation $\mathrm{X}(\mathrm{i})$ is given by Equation (9). Accordingly, the net radiant flux into the $i^{\text {th }}$ node becomes

$$
\begin{equation*}
\frac{Q^{\prime \prime}(i)}{\sigma}=\left\{\sum_{k} \varepsilon(i): \hat{F}(i, k) \cdot \varepsilon(k) \cdot T^{4}(k)\right\}-\varepsilon(i) T^{4}(i) \tag{23}
\end{equation*}
$$

In a uniform temperature enclosure, the net radiant flux $Q$ "(i) is zero. Therefore,

$$
T(1)=T(2)=T(3)=\ldots=T(N)=T
$$

For example, for the first node in this case

$$
\frac{Q(1)}{\sigma T^{4}}=\varepsilon(1)[\hat{F}(1,1) \varepsilon(1)+\hat{F}(1,2) \varepsilon(2)+\ldots]-\varepsilon(1)=0
$$

or, in general terms

$$
\varepsilon(i) \sum_{k} \hat{F}(i, k) \varepsilon(k)=\varepsilon(i)
$$

The left hand side is sometimes called Hottel's F-script matrix, $\mathcal{F}(i, k)$, where

$$
\left.\begin{array}{l}
\sum_{k} \mathcal{F}(i, k)=\varepsilon(i)  \tag{24}\\
\sum_{k} \hat{F}(i, k) \varepsilon(k)=1
\end{array}\right\}
$$

Equation (24) can still hold for a non-uniform temperature enclosure and by combining Equations (23) and (24), then the IR radiation energy exchange to node (i) of area $A(i)$ is

$$
\begin{equation*}
Q_{I R}=\sigma A(i) \cdot \sum_{k} \varepsilon(i) \cdot \hat{F}(i, k) \cdot \varepsilon(k) \cdot\left[T^{4}(k)-T^{4}(i)\right] \tag{25}
\end{equation*}
$$

2.6.2 Energy Exchange in the Solar Radiation Band

For the solar band, the IR properties $\rho, \varepsilon$, are replaced by $\rho^{*}, \varepsilon^{*}$ for differentiation. The solar excitation $X^{*}$ on node (i) in this case is as shown in Figure 2-6 and is given by

$$
\begin{equation*}
\stackrel{*}{\mathrm{X}}(i)=\stackrel{*}{\rho}(i) \cdot \text { Flux }(i) \tag{26}
\end{equation*}
$$

Equations (15), (17), (18), and (19) still apply, but use the optical properties with an asterisk for the solar spectrum.

$$
\begin{align*}
& A^{*}(i, k)=\delta(i, k)-\rho *(i) F(i, k)  \tag{27}\\
& {\left[J^{*}\right]=\left[A^{*}\right]^{-1} \cdot\left[X^{*}\right]}  \tag{28}\\
& {\left[G^{*}\right]=\left[\hat{F}^{*}\right] \cdot\left[X^{*}\right]}  \tag{29}\\
& {\left[\hat{F}^{*}\right]=[F] \cdot\left[A^{*}\right]^{-1}} \tag{30}
\end{align*}
$$



Figure 2-6. Radiosity and Irradiation in the Solar Band

The net solar radiation flux retaining in the $i^{\text {th }}$ node is expressed as

$$
\begin{align*}
& Q^{\prime \prime *}(i)=G^{*}(i)-\rho *(i) G *(i)+F 1 u x(i)-\rho^{*}(i) F 1 u x(i)  \tag{31}\\
& Q^{\prime \prime *}(i)=\left[1-\rho^{*}(i)\right] G^{*}(i)+\alpha *(i) F 1 u x(i)
\end{align*}
$$

Using Equation (29), then

$$
Q^{\prime \prime *}(i)=\sum_{k} \varepsilon *(i) \hat{F} *(i, k) \stackrel{*}{X}(k)+\varepsilon *(i) F l u x \quad(i)
$$

From Equation (26), the net solar energy exchange on node (i), $Q$ (i) ${ }_{s}$ is $Q(i)_{S}=\left\{\sum_{k} \varepsilon *(i) \cdot \hat{F} *(i, k) \cdot \stackrel{*}{\rho}(k) \cdot F \operatorname{lux}(k)+\stackrel{*}{\varepsilon}(i) F l u x(i)\right\} A(i)$
where

$$
\begin{equation*}
\hat{F}^{*}(i, k)=\sum_{S} F(i, s) A^{*-1}(s, k) \tag{33}
\end{equation*}
$$

From the above analysis, Equation (25) is the final form to be used for the longwave (infrared) radiation exchange, and Equation (32) is the final form to be used for shortwave (solar) radiation exchange.

### 2.7 NODAL ENERGY BALANCE

For a particular receiver node (i), the energy is exchanged with the neighboring nodes by many modes of heat transfer such ds: 1) longwave (infrared) radiation, $Q(i) I R$; 2) shortwave (solar) radiation, $Q(i)_{s}$; 3) conduction, $Q(i)_{\text {cond }}$; 4) thermal convection, $Q(i)$ conv; 5) fluid convection, $Q(i)_{f}$; and 6) internal generation (or dissipation) by electrical, chemical or nuclear effects, $Q(i)_{p}$. The final mathematical form of each heat transfer mode is written as follows:

The net longwave (infrared) radiation received by the $i^{\text {th }}$ node from all neighboring nodes is given by

$$
\begin{equation*}
Q(i)_{I R}=\sum_{j} C(i, j) I R\left[T^{4}(j)-T^{4}(i)\right] \tag{34}
\end{equation*}
$$

where $T$ is the absolute temperature and $C(i, j)$ iR is the $I R$ radiation conductance between nodes (i) and (j). $C(i, j){ }_{I R}$ can be derived from Equation (25).

The final forms of the shortwave (solar) radiation, $Q(i)$ and the internal energy generation $Q(i)$ operating temperature and are constants. The energy transferred by conduction to the $i^{\text {th }}$ node from neighboring nodes is given by Equation (1) as

$$
Q(i)_{c o n d}=\sum_{j} C(i, j)_{c o n d}[T(j)-T(i)]
$$

Also, for thermal convection, the net energy convected to the $i^{\text {th }}$ node is given by Equation (3) stated before as

$$
Q(i)_{\text {conv }}=\sum_{j}\left(C(i, j)_{c o n v}[T(j)-T(i)]\right.
$$

where $C(i, j)$ conv is the thermal conductance for convection.

For fluid convection, the heat extracted or dumped to the $i^{\text {th }}$ node is given by Equation (7) as

$$
Q(i)_{f}=\sum_{j} C(i, j)_{f}[T(j)-T(i)]
$$

where $C(i, j)_{f}$ is the fluid conductance for flowing fluid.
The first law of thermodynamics applied to the $i^{\text {th }}$ node gives:

$$
\begin{align*}
& \operatorname{DN}(i) V(i) \operatorname{CP}(i) \frac{\Delta T(i)}{\Delta \tau}=\operatorname{Res}(i)= \\
& Q(i)_{I R}+Q(i)_{\text {cond }}+Q(i)_{\text {conv }}+Q(i)_{f}+Q(i)_{s}+Q(i)_{p} \tag{35}
\end{align*}
$$

where Res(i), $D N(i), V(i)$ are the net energy stored, density and volume of the $i^{\text {th }}$ node, respectively, and $\Delta \tau$ is the elapsed time increment.
2.8 MATRIX FORMATS

To facilitate the manipulation processes, Equations (1), (3), (7), and (34) are put in matrix or vector forms. Generally,

$$
\begin{equation*}
Q(i)=\sum_{j} C(i, j)\left[T^{n}(j)-T^{n}(i)\right] \tag{36}
\end{equation*}
$$

where $n$ is an exponent that can be either 1 or 4 , and $C(i, j)$ is a general indicial form for conductance matrix. Equation (36) can be written in the expanded form
$Q(i)=C(i, 1)\left[T^{n}(1)-T^{n}(i)\right]+C(i, 2)\left[T^{n}(2)-T^{n}(i)\right]+\ldots$
or
$Q(i)=C(i, 1) T^{n}(1)+C(i, 2) T^{n}(2)+\ldots-[C(i, 1)+C(i, 2)+C(i, 3)+\ldots] T^{n}(i)$

In the matrix form, Equation (36) may also be expressed as

$$
\left[\begin{array}{r}
\cdot  \tag{37}\\
Q(i) \\
.
\end{array}\right] \cdot\left[\begin{array}{cccc}
\cdot & \cdot & \cdot \\
C(i, 1) & C(i, 2) & \ldots & -[C(i, 1)+C(i, 2)+.] \ldots \\
\cdot & \cdot & \cdot
\end{array}\right]\left[\begin{array}{c}
\mathrm{T}^{\mathrm{n}}(1) \\
\mathrm{T}^{\mathrm{n}}(2) \\
\mathrm{T}^{\mathrm{n}}(\mathrm{i})
\end{array}\right]
$$

or

$$
\begin{aligned}
{[Q] } & =[\overline{\mathrm{C}}]\left[\mathrm{T}^{\mathrm{n}}\right] \\
Q(i) & =\sum_{\mathrm{k}} \overline{\mathrm{C}}(\mathrm{i}, \mathrm{k}) \mathrm{T}^{\mathrm{n}}(\mathrm{k})
\end{aligned}
$$

where [ $\overline{\mathrm{C}}$ ] is a modified conductance matrix developed from the original [c] matrix with diagonal elements equal to the negative sum of the corresponding row.
$[\bar{C}]=\bar{C}(i, j)=\left[\begin{array}{ccc}-[C(1,2)+C(1,3)+. .] & C(1,2) & C(1,3) \\ c(2,1) & -[C(2,1)+c(2.3)+\ldots] & C(2,3) \\ C(3,1) & C(3,2)-[(3,1)+c(3,2)+\ldots]\end{array}\right]$

In indicial form, the new matrix $[\overline{\mathrm{C}}]$ can be written as

$$
\left.\begin{array}{rl}
\bar{c}(i, i)= & -\sum_{j} C(i, j)  \tag{38}\\
& i \neq j \\
\bar{C}(i, j)= & C(i, j) \\
o r \\
\bar{C}(i, j)=C(i, j)-\delta(i, j) \quad \sum_{j} C(i, j)
\end{array}\right\}
$$

where $\delta(i, j)$ is the Kroencker delta defined by Equation (16) as

$$
\begin{aligned}
\delta(i, j) & =1 & & i=j \\
& =0 & & i \neq j
\end{aligned}
$$

Based on the above discussion; Equations (1), (3), (7), and (34) can be written in terms of the modified conductance matrices as

$$
\left.\begin{array}{l}
Q(i)_{I R}=\sum_{j} \bar{c}(i, j)_{I R} T^{4}(j)  \tag{39}\\
Q(i)_{c o n d}=\sum_{j} \bar{c}(i, j)_{c o n d} T(j) \\
Q(i)_{c o n v}=\sum_{j} \bar{c}(i, j)_{c o n v} T(j) \\
Q(i)_{f}=\sum_{j} \bar{c}(i, j)_{f} T(j)
\end{array}\right\}
$$

and the nodal energy balance, Equation (35) is written as:

$$
\begin{align*}
\operatorname{Res}(i) & =\sum_{j} \bar{C}(i, j)_{I R} T^{4}(j) \\
& +\sum_{j}\left[\bar{C}(i, j)_{c o n d}+\bar{C}(i, j)_{c o n v}+\bar{C}(i, j)_{f}\right] \cdot T(j)  \tag{40}\\
& +Q(i)_{s}+Q(i)_{p}
\end{align*}
$$

2.9

STEADY STATE SOLUTION

The steady state solution differentiates between equilibrium nodes and source/sink nodes when handling Equation (35). The solution has two functions: 1) to find the steady state temperature for each equilibrium node; and 2) to find the energy residue for source/sink nodes. Numbering of the nodes is arbitrary but should be in sequence to facilitate tracking the results.

For the equilibrium nodes, the set of non-linear energy equations are solved by iteration starting from an initial estimate of the temperature vector. The residuals will be equal to zero at steady state. The most convenient method used is the iterative Newton-Raphson method. The method linearizes Equation (39) in the region of the initial temperature estimate using the Taylor series.

Therefore,

$$
\begin{equation*}
\delta \operatorname{Res}(i)=\sum_{j} \frac{\partial \operatorname{Res}(i)}{\partial T(j)} \delta T(j) \tag{41}
\end{equation*}
$$

or

$$
\begin{align*}
{[\delta \operatorname{Res}] } & =[J A C] \cdot[\delta T] \\
\delta \operatorname{Res}(i) & =\sum_{j} \operatorname{JAC}(i, j) \cdot \delta T(j) \tag{42}
\end{align*}
$$

where [JAC] is the Jacobian matrix, given by using Equations (40) and (41) as

$$
\begin{align*}
\operatorname{JAC}(i, j)=\frac{\partial \operatorname{Res}(i)}{\partial T(j)}= & 4 \overline{\mathrm{C}}(i, j)_{\operatorname{IR}} T(j)^{3}+\overline{\mathrm{C}}(i, j)_{\text {cond }}+\overline{\mathrm{C}}(i, j)_{\text {conv }} \\
& +\overline{\mathrm{C}}_{f}(i, j) \tag{43}
\end{align*}
$$

Equation (42) can be written as:
or

$$
\begin{align*}
& \begin{array}{cc}
(K+1) & (k) \\
{[\operatorname{Res}(i)-\operatorname{Res}(i)]}
\end{array}=\begin{array}{cc}
(K) \\
J A C(i, j)
\end{array}\left[\begin{array}{cc}
(K+1) & (K) \\
T(j) & -T(j) \\
{[\operatorname{Res}]-[\operatorname{Res}]^{(K)}} & =[J A C] *\left[T^{(K+1)}-[T](K)\right]
\end{array}\right\} \tag{44}
\end{align*}
$$

where $K$ is the number of iteration process. Equation 44 can also be written as

$$
\begin{equation*}
[\mathrm{T}]^{(\mathrm{K}+1)}=[\mathrm{T}]^{(\mathrm{K})}-[\mathrm{JAC}]^{-1} *[\operatorname{Res}]^{(\mathrm{K})} \tag{45}
\end{equation*}
$$

Reference 6 states that six iterations or less are usually sufficient when initial estimates are far from actual steady state values. An exceptionally good initial guess does not necessarily save computer time or cost since it may eliminate only one or two iterations. Therefore, the user should not put much effort into the initial temperature estimate accuracy.
2.10

TRANSIENT SOLUTION

After the elapse of a small time interval $\Delta \tau$, Equation (3b) gives the slope of equilibrium nodes temperature array $T$ as an approximation of the derivative $\partial T / \partial \tau$ using the finite difference. The "forward" finite difference method will be used as it is the most adequate one. The new nodal temperature $T$ (i) at ( $\tau+\Delta \tau$ ) is calculated using the old temperature $T$ (i) and all other properties at the old time ( $\tau$ ) , i.e.,

$$
\stackrel{(\tau+\Delta \tau)}{T(i)} \stackrel{(\tau)}{=} T(i)+\frac{\Delta \tau}{M(i) \operatorname{Cp}(i)} \begin{gather*}
(\tau)  \tag{46}\\
\operatorname{Res}(i)
\end{gather*}
$$

where $M(i), C p(i)$ and Res(i) are the mass, specific heat and energy residue at the $i^{\text {th }}$ node. The time step $(\Delta \tau)$ is subject to computation convergence and stability criteria. To use the simple method applied in Reference 7 the residual Res(i) is written in the following linearized form

$$
\begin{equation*}
\operatorname{Res}(i)=\sum_{j} D(i, j)[T(j)-T(i)]+E(i) \tag{47}
\end{equation*}
$$

where

$$
\begin{gather*}
D(i, j)=C(i, j)_{I R}\left[\frac{T^{4}(j)-T^{4}(i)}{T(j)-T(i)}\right]+C(i, j)_{c o n d}+C(i, j)_{c o n v}+C(i, j)_{f} \\
E(i) \quad=Q_{S}(i)+Q_{P}(i) \geq 0 \tag{48}
\end{gather*}
$$

combining Equations (47) and (48), then

$$
\begin{align*}
(\tau+\Delta \tau) & (\tau) \\
T(i) & T(i)\left[1-\frac{\Delta \tau}{M(i) C p(i)} \cdot \sum_{j} D(i, j)\right]+\left[\sum_{j} D(i, j) T(j)\right.  \tag{49}\\
& \times\left[\frac{(\tau)}{M(i) C p(i)}\right]
\end{align*}
$$

Since all the coefficients $D(i, j)$ and $E(i)$ are always positive or at least zero, the coefficient of $T(i)$ should be selected to be either positive or at least zero in order not to violate the thermodynamic principles. Accordingly

$$
\underset{\min }{\Delta \tau(i)} \leq \frac{M(i) C p(i)}{D(i, j)}
$$

the denominator $\sum_{i} D(i, j)$ can be expressed in terms of the modified matrices $[\overline{\mathrm{C}}]$ in Equation (38) as:

$$
\begin{aligned}
\sum_{j} D(i, j)= & \sum_{j} C(i, j)_{I R}\left\{\frac{T^{4}(j)-T^{4}(i)}{T(j)-T(i)}\right\}+\sum_{j} C .(i, j)_{\text {cond }} \\
& +C(i, j)_{\text {conv }}+C(i, j)_{f}
\end{aligned}
$$

or

$$
\begin{equation*}
\sum_{j} D(i, j)=-\left[\overline{L C}(i, i){ }_{I R}+\bar{C}(i, i)_{\text {cond }}+\bar{C}(i, i)_{\text {conv }}+\bar{C}(i, i)_{f}\right] \tag{51}
\end{equation*}
$$

where [LC] is the linearized IR radiation conductance given by

$$
\operatorname{LC}(i, j)=C(i, j)_{I R}\left(\frac{T(j)^{4}-T(i)^{4}}{T(j)-T(i)}\right)
$$

The matrices [LC] and [LD] are related by the same expression, Equation (38). For each node, Equation (50) is used to find the minimum time increment $\Delta \tau$ (i) for stable computations of the $i^{\text {th }}$ node. A scanning process is then made to find the minimum of these minima to be used for the iteration process for all nodes, excluding source/sink nodes.

### 2.10.1 Zero-Capacity Nodes

Some nodes have a very small time increment, $\Delta \tau$ (i), which reduces the overall time increment $\Delta \tau$ to such a small value that the computer execution time and cost increase dramatically. These types of nodes have either a very small thermal capacity [M(i).Cp(i)] or a small thermal resistance and they do not store a significant amount of energy during the exchange with the surrounding nodes. The transient analysis in the program considers a minimum time increment (MINDTU) below which such nodes can be treated as zero-capacity nodes.

Zero capacity nodes are momentarily in thermal equilibrium with neighboring nodes with no energy storage during intervals of $\Delta \tau$. The temperature vector at time $\tau+\Delta \tau$ is determined by the NewtonRaphson iteration process similar to the one described under steady state (Section 2.9) using the old neighboring node temperature at time t. For other equilibrium nodes that are not "zero-capacity" nodes, the new temperature vector at time $\tau+\Delta \tau$ is determined by Equation (46) based on the old temperature vector at time $\tau$. In both cases, source/ sink nodes are not affecting the minimum time increment selection. The detailed flow chart of calculation is presented in Appendix A.

This subroutine is used whenever the program output is printed at times that do not coincide with the built-in program clock. Linear interpolation between two values (X1, Y1) and (X2, Y2) is used as illustrated in Figure 2-7. For a value $\overline{\mathrm{X}}$ between X 1 and X 2 , the corresponding value $\overline{\mathrm{Y}}$ is given by the linear relation:

$$
\begin{equation*}
\bar{Y}=Y_{1}+\left(\frac{\bar{X}-X_{1}}{X_{2}-X_{1}}\right)\left(Y_{2}-Y_{1}\right) \tag{52}
\end{equation*}
$$

The variable $X$ represents the time and $Y$ the variable to be interpolated.

### 2.12 VIEW (CONFIGURATION) FACTORS

To take advantage of the axisymmetric nature of a solar receiver, a special view (or configuration, geometric) factor model is used. The possible node shapes would be a plane disc, plane ring, cylindrical ring, conical ring, etc., as shown in Figure 2-8.

Each axisymmetric node surface is entered by the user in the cylindrical coordinate system by four coordinates $r_{1}, r_{2}$ in the radial direction and $Z_{1}, Z_{2}$ in the axial direction, respectively. The radial coordinate ( $r$ ) will be measured from the receiver centerline and the axial coordinate $Z$ can be measured from the cavity aperture for cavity receivers or from the bottom of the external receiver connecting with the engine section as shown in Figure 2-9. The selection of the $Z$ datum is arbitrary.

Each line ring as shown in Figure $2-10$ will be defined by its radius ( $r$ ) and the corresponding axial distance ( $Z$ ); thus the sets $\left(r_{1}, Z_{1}\right),\left(r_{2}, Z_{2}\right)$ correspond to the two line ring coordinates that constitute each node. The set ( $\mathrm{r}_{1}, \mathrm{Z}_{1}$ ) will always refer to the bottom ring and ( $\mathrm{r}_{2}, \mathrm{Z}_{2}$ ) will always refer to the top ring. Two cases will be discussed as follows:


Figure 2-7. Linear Interpolation


Figure 2-8. Possible Axisymmetric Nodes


EXTERNAL RECEIVER


Figure 2-9. Possible Coordinate System Location


Figure 2-10. Cylindrical Coordinates of a Ring Wire

For two axisymmetric nodes $i$, $j$, as shown in Figure 2-11, each node will be presented by four coordinates ( $r_{1}, r_{2}, Z_{1}, Z_{2}$ ). The view factors from bottom surfaces to top surfaces are computed as follows:

$$
\begin{aligned}
F(i, j) & =F\left(i, j_{1}\right)-F\left(i, j_{2}\right) \\
& =\frac{A\left(j_{1}\right)}{A(i)} F\left(j_{1}, i\right)-\frac{A\left(j_{2}\right)}{A(i)} F\left(j_{2}, i\right) \\
& =\frac{A\left(j_{1}\right)}{A(i)}\left[F\left(j_{1}, i_{2}\right)-F\left(j_{1}, i_{1}\right)\right]-\frac{A\left(j_{2}\right)}{A(i)}\left[F\left(j_{2}, i_{2}\right)-F\left(j_{2}, i_{1}\right)\right]
\end{aligned}
$$

or

$$
\begin{gather*}
F(i, j)=\frac{1}{A(i)}\left[\left(A\left(i_{2}\right) F\left(i_{2}, j_{1}\right)-A\left(i_{1}\right) F\left(i_{1}, j_{1}\right)\right)-\left(A\left(i_{2}\right) F\left(i_{2}, j_{2}\right)-\right.\right. \\
\left.\left.A\left(i_{1}\right) F\left(i_{1}, j_{2}\right)\right)\right] \tag{53}
\end{gather*}
$$

where
$A\left(i_{2}\right)=\pi r^{2}\left(i_{2}\right)$,
$A(i)=\pi\left[r\left(i_{1}\right)+r\left(i_{2}\right)\right] \sqrt{\left[r\left(i_{2}\right)-r\left(i_{1}\right)\right]^{2}+\left[Z\left(i_{2}\right)-Z\left(i_{1}\right)\right]^{2}}$
and
$A\left(i_{1}\right)=\pi r^{2}\left(i_{1}\right)$

The above four view factors $F\left(i_{2}, j_{1}\right), F\left(i_{2}, j_{2}\right), F\left(i_{1}, j_{2}\right)$, and $F\left(i_{1}, j_{1}\right)$, between the plane discs forming node boundaries should be calculated first before the view factor $F(i, j)$ between nodes (i) and ( $j$ ) can be calculated. The view factor between two plane discs, as shown in Figure 2-12, can be expressed by the closed analytical form:


Figure 2-11. Calculation of View Factor, Interior-to-Interior Nodes


Figure 2-12. View Factor Between Two Coaxial Discs
where $r_{H}$ and $r_{L}$ are the radii of the higher and lower discs, respectively and X is given by

$$
\begin{equation*}
X=1+\left(r_{H} / r_{L}\right)^{2}+\left(s / r_{L}\right)^{2} \tag{55}
\end{equation*}
$$

To facilitate the view factor calculations, it is necessary to define the radii vector (R) with a dimension of ( $2 * \mathrm{NN}$ ), where (NN) is the number of axisymmetric nodes that exchange energy by radiation. Each node will be represented by two elements of the vector (R). For example, the $i^{\text {th }}$ node will have a lower radius located at the ( $2 i-1$ ) th element of (R), and a higher radius located at the (2i) ${ }^{\text {th }}$ element of (R).


Similarly, it is necessary to define the axial vector ( $Z$ ) with a dimension of $(2 * N N)$ to present the axial distances of the lower and higher rings that bound each node. The calculation of the four elementary disc-to-disc view factors, $\operatorname{DDF}(4)$, previously mentioned in Equation (53) is done using the 4-element vectors $\mathrm{X}(4), \mathrm{RH}(4)$, $\mathrm{RL}(4), \mathrm{S}(4)$. Use is made of the view factor reciprocity relations and the view factor summation rule to complete the calculation process of the matrix $F(i, j)$.

Equation (53) is applicable to an interior node surface looking into another interior node surface above it in the $Z$ direction or an exterior node surface looking into another exterior node surface above it in the $Z$ direction. The sign of $F(i, j)$ in Equation (53) is reversed if node ( $j$ ) is below node (i). For an interior node surface looking into another exterior node surface or vice versa, the Flag matrix V (i,j) is added to distinguish between these two cases and separate the view factor calculations.
2.12.2 Calculation of Interior-to-Exterior Node View Factor

For this case, a rough set of calculations is followed. A modification of the view factor presented in Reference 8 for two concentric cylinders of the same finite length, illustrated in Figure 2-13, is made to approximate the case of two dislocated conic rings of unequal lengths.

Although the view factor calculations are approximated in this case, they could be changed by the user and replaced by other expressions if they are more accurate. The procedure followed is outlined below.

For the two concentric cylinder nodes (i) and ( $j$ ) sketched in Figure 2-13 where the node (i) is the inner surface node and the node ( $j$ ) is the outer surface node, the view factor $F(i, j)$ is given by:

$$
\begin{align*}
F(i, j)=\frac{1}{\bar{R}}-\frac{1}{\pi \bar{R}} & \left.\left\{\begin{array}{c}
\cos ^{-1} \frac{B}{A}-\frac{1}{2 L}\left[\sqrt{(A+2)^{2}-4 \bar{R}^{2}} \cos ^{-1} \frac{B}{\bar{R} A}\right. \\
\\
\end{array}+B \sin \left(\frac{1}{\bar{R}}\right)-\frac{\pi A}{2}\right]\right\}
\end{align*}
$$



Figure 2-13. Two Concentric Cylinders of Same Finite Length
where

$$
\left.\begin{array}{rl}
\bar{R} & =r(i) / r(j)  \tag{57}\\
L & =\ell / r(j) \\
A & =L^{2}+\bar{R}^{2}-1 \\
B & =L^{2}-\bar{R}^{2}+1
\end{array}\right\}
$$

To allow for conic rings that are not of equal length and also dislocated axially with respect to each other, as shown in Figure 2-14, the view factor in Equation (56) was modified to use average radii for the nodes (i) and ( $j$ ), and an average length for both nodes. Using vectors $R(2 * N N)$ and $Z(2 * N N)$, the equivalent radii and length can be approximated as:


Figure 2-14. Approximation of Two Conical Nodes in View Factor Calculation
$r(j)=\operatorname{INR}=\frac{R(2 j)+R(2 j-1)}{2}=$ equivalent radius of node ( $j$ )
$r(i)=O T R=\frac{R(2 i)+R(2 i-1)}{2}=$ equivalent radius of node (i)

$$
\ell=E Q V L=\left[\frac{Z(2 i)-Z(2 i-1)}{2}\right]+\left[\frac{Z(2 j)-Z(2 j-1)}{2}\right]=\begin{gather*}
\text { equivalent }  \tag{58}\\
\text { length }
\end{gather*}
$$

For dislocated concentric nodes, the cosine of the angle of dislocation (KOST) is used to modify the view factor $F(i, j)$ given by Equation (56) and written as:

$$
\begin{equation*}
\operatorname{KOST}=\frac{(O T R-I N R)}{\sqrt{(X X)^{2}+(O T R-I N R)^{2}}} \tag{59}
\end{equation*}
$$

where $X X$ is the dislocation or shift given by

$$
\begin{equation*}
x X=\left(\frac{z(2 j)+z(2 j-1)}{2}\right)-\left(\frac{z(2 i)+z(2 i-1)}{2}\right) \tag{60}
\end{equation*}
$$

Furthermore, to account for the small slope of each conical ring, the view factor between two dislocated cylinders is multiplied by the cosine of the total angle between the two conical surfaces:
$\cos \left(\phi_{i}+\phi_{j}\right)$ where:
$\operatorname{Cos}\left(\phi_{i}+\phi_{j}\right)=\operatorname{Cos}\left\{\tan ^{-1} \frac{R(2 i)-R(2 i-1)}{Z(2 i)-Z(2 i-1)}+\tan ^{-1} \frac{R(2 j-1)-R(2 j)}{Z(2 j)-Z(2 j-1)}\right\}$

It should be noted that the above view factors calculated for interior-interior nodes or interior-exterior nodes may still need some minor alterations by the user to correct for partial shading by other nodes. The latter is sometimes inevitable in view factor calculations and the user should make several trials to make sure that the rules of summation and reciprocity of view factors are applied correctly and the errors in satisfying these rules are minimized.

## SECTION 3

EXPLANATION OF INPUT DATA AND OUTPUT FORMS


#### Abstract

3.1 INPUT DATA

To facilitate the use of the program, a set of tables is provided in Appendix $B$, with a brief description, to explain how each input variable shall be entered. Table 3-1 lists alphabetically all the input data, whether they are in free format or not. More data explanation can be found in Appendix C.


The program language is written in MBASIC. The structure of the program is divided into two files and is given in Appendix D. These two files are the [FIRSTPART] file, which includes all dimensioning and initialization statements and a large space for entering input data, and the [LASTPART] file which contains the set of computational statements. Merging of the two files in the program execution is done using the command [MERGE] in MBASIC. The first file, [FIRSTPART], should be loaded by the user using the command [LOAD] to enter all data. Tele-computer terminals are chosen for their fast communication with the user. Statement numbers 450-2000 are used to enter data, as will be explained in the example in the next section.

Vectors are entered either in an element-by-element form or in scattered forms. For example, if the vector [ESOL] has 5 elements such that

ESOL (1) $=0.2$
$\operatorname{ESOL}(2)=0.0$
ESOL (3) $=0.5$
ESOL (4) $=1.0$
ESOL (5) $=0.0$
either one of the following statements can be typed:

Table 3-1. Input Variables

| VARIABLE | DESCRIPTION |
| :---: | :---: |
| ADJ | Conduction/convection identifier |
| AMBNN | Number of node representing ambient air |
| ASOL | Nodal surface area for solar/IR radiation exchange |
| ASRF | Nodal surface area for conduction/convection heat transfer |
| CONDUC | Flag for conduction heat transfer in problem |
| CONVEC | Flag for convection heat transfer in problem |
| DEN | Node density |
| DTOUT | Time interval for printing out results |
| EIR | Nodal infrared emittance |
| ESOL | Nodal emittance in the solar band |
| F | View factor matrix |
| FLOW | Flag for fluid flow in problem |
| FNDIN | Number of node that represents the fluid inlet section of receiver |
| FNDOT | Number of node that represents the fluid outlet section of receiver |
| HCONV | Convective heat transfer coefficient |
| HRIN | Time sequence vector for time-varying variables |
| HROUT | Starting time for printing out program results |
| IFLUX | Solar flux incident on each node |
| KCOND | Nodal thermal conductivity |
| LNGTH | Conduction path length between nodes |
| MASFLO | Flow direction identifier |
| MINDTU | Minimum time increment for zero-capacity nodes |
| MFM | Mass flow rate or multiplier |
| MNIT | Maximum number of iterations in Newton-Raphson method |
| NDIN | Number of data points in time-varying input or dimension of HRIN vector |
| NN | Total number of nodes |
| POWR | Internal power generated in each node |
| R | Radial coordinates of nodes |
| SAIRR | Flag for solar/infrared radiation |
| SIGMA | Stefan-Boltzman radiation constant |
| SORSIN | Source/sink node identifier |
| SPHT | Nodal specific heat |
| SS | Flag for steady state/transient solutions |
| SSTEM | Temperature data of source/sink nodes |
| TEM | Nodal temperature |
| $\checkmark$ | View factor flag |
| VOL | Nodal volume |
| VPRNT | Flag for printing view factor matrix |
| Z | Nodal axial coordinates |

$>500$ ESOL $(1)=0.2, \operatorname{ESOL}(3)=0.5, \operatorname{ESOL}(4)=1.0$ (scattered form)
or
$>500$ REAL ESOL (5) / 0.2, 0.0, 0.5, 1.0/
or
$>500$ DIM ESOL (5) / 0.2, 0.0, 0.5, 1.0/
Remember that statements in MBASIC can take more than one instruction but separated with commas as shown above. Default values are always equal to zero.

Overwriting data is also possible in MBASIC, and could be used to correct for typing errors. For example:
$>600$ ESOL (1) $=0.9, \quad$ ESOL (3) $=0.5$
$>700$ ESOL (3) $=0.4$
is equivalent to

```
> ESOL (1) = 0.9, ESOL (3) = 0.4
```

For vector or matrix elements that have identical values, the following shorthand notation could be used in MBASIC:

$$
\begin{aligned}
& >800 \quad A(1), A(2), A(3)=1 \\
& >900 \quad B(2,3), B(3,4), B(3,10)=0.5 \\
& \text { is equivalent to } \\
& \begin{array}{ll}
A(1)=1 & B(2,3)=0.5 \\
A(2)=1 & B(3,4)=0.5 \\
A(3)=1 & B(3,10)=0.5
\end{array}
\end{aligned}
$$

Also, the [FOR] modifier can be used to assign values for vector or matrix elements having identical values. For example:

$$
\begin{aligned}
& >1000 \quad C(I, 3)=2 \quad \text { FOR } I=7 \text { TO } 12 \\
& \text { is equivalent to } \\
& C(7,3)=2, \quad C(10,3)=2 \\
& C(8,3)=2, \quad C(11,3)=2 \\
& C(9,3)=2 \quad, \quad C(12,3)=2
\end{aligned}
$$

To input a large NXN matrix, it is preferred to input only the elements that are non-zero in the scattered form. For some symmetrical matrices, only half the matrix is needed as input, and the other half is completed by the program structure. The solved example given in the next section illustrates how the data are collected, entered, and used to calculate the performance of a cavity receiver. More information about using the MBASIC programming language can be found in Reference 9.

### 3.2 OUTPUT FORMS

The program results in the following output:
(1) Applicability of the summation rule of view factors The output of $\sum F(i, j)$ for the $i^{\text {th }}$ node should be j either 1.0 or zero. Zero is only applicable for nodes that are assumed not exchanging heat by radiation. The user should make the necessary changes in the view factors after the program is first run to make sure that the summation rule is applied to all radiation nodes whether they are partially shaded or not.
(2) View-factor matrix

This output is presented if the user selects the value 1 for [VPRNT]. The view factors are printed in column form for each node. Non-zero view factors are only printed.

Nodal energy residue
This is applicable to steady state solutions only. Equilibrium nodes should show a very small energy residue (around $10^{-4}-10^{-3}$ Btu for example) at steady state. Source/sink nodes should give a nonzero energy residue with a proper sign. Positive energy residue means that there is a net energy accumulation to the node and vice versa. As a check on computational accuracy, using the first law of thermodynamics, the algebraic sum of energy residues to source/sink nodes must equal the sum of all external energy entering the receiver through solar flux or other internal heat generation.
(4) Accumulated energy extracted

This variable is summed over all the time steps starting from the initial time of computations up to the printout time ( $\tau$ ). The accumulated energy extracted is computed for both quasi-steady state and transient solutions. The daily accumulated energy extracted when the solar receiver is modeled over one day is useful in determining the integrated effects of transients.
(5) Accumulated solar energy into receiver

This variable is also summed over all the time steps, starting from the initial time of computations up to the printout time ( $\tau$ ). The accumulated solar energy received is computed for both quasi-steady state and transient solutions.
(6)
(7)

Receiver efficiency
This is the ratio of the accumulated energy extracted from the receiver up to time ( $\tau$ ) to the accumulated solar energy into the receiver up to time ( $\tau$ ). For steady state solutions, the time interval is taken as one hour.

Nodal temperature profile
The temperature distribution of the receiver nodes at time ( $\tau$ ) is given in tabular form. This output is desirable in observing the location of high temperature spots, outlet fluid temperature, and the determination of isothermal contours.

Nodal minimum transient time, $(\triangle \tau)$ min
For each node, the minimum time increment allowed for stable transient computations is printed. Also, the time increment (MINDTU) specified by the user to distinguish between zero capacity nodes and equilibrium nodes is reprinted.

Indirect Output results
In addition to the above direct program printouts, the program can be used, either through several runs or by minor code modifications, to give a complete performance analysis of a particular receiver in the following areas:
(a) Sensitivity Analysis

This will include the sensitivity of solar receiver performance to variations occuring in the major receiver parameters such as nodal emissivity in the IR band (EIR), solar band emissivity (ESOL), fluid mass flow rate (MFM),
nodal solar flux (IFLUX), ambient temperature [SSTEM (1)] or the fluid inlet temperature, TEM (FNDIN).

The effects on system efficiency and output can be studied in each case.
(b) Itemization of Heat Losses

The total heat loss from the receiver can be divided into 3 main parts: 1) a radiation heat loss through the aperture (for cavityshaped receivers) and from the external boundary surface; 2) a convection heat to the ambient air outside the receiver and/or inside the receiver; 3) a conduction heat loss through the insulation layers or because of lateral exchange between neighboring nodes.

Two runs could be made to quantize the magnitude of each part lost. The first run assumes zero thermal conductivity of the insulation layers or other layers that the user sees as causing a significant heat loss to the surroundings. The difference between the heat losses from this first run and the total heat losses would give the conduction loss. The second run assumes zero convective heat transfer coefficient between all the nodes exposed to ambient air and the ambient air node. The difference between the heat losses from the second run and the total heat losses would give the convection loss. The radiation loss will then be the third part left after subtracting the conduction and convection parts.
(c) Parameterization Study

This study may include the effects on receiver performance because of changes made in receiver geometry, interior piping arrangements, different working fluids or operating conditions. Studies of full-day performance under varying solar flux, inlet fluid conditions and ambient temperatures could be made with results specific to the receiver under study. Also, system optimization taking into consideration the receiver and engine performance is another indirect output of the receiver program.

In this section, the input and output procedure used in the computer program is applied to an experimental receiver. A brief description of the receiver, input data and the output results are presented.

## RECEIVER DESCRIPTION

The receiver understudy was built at JPL in 1977 as a part of an energy program aimed at studying solar-thermal electric power plants. The receiver is placed at the focal spot of a $2.9-\mathrm{m}$ ( $9.5-\mathrm{ft}$ ) diameter paraboloid dish; both are located at the JPL Table Mountain facility. The receiver is referred to as the Table Mountain receiver for identification.

The receiver body is a cavity-type composed of four primary parts as shown in Figure 4-1. These parts are: 1) a heat exchanger section consisting of 34 pairs of $U$-tubes made of Inconel 718. The tubes are brazed to a head plate which is also made of Incone1 718; 2) a ceramic receiver countour that forms the inside surface; 3) an aperture piece with changeable aperture sizes; and 4) an insulation layer in the form of a stainless steel canister filled with calcium silicate.

The receiver performance was tested using helium as the working fluid at various inlet temperatures and pressures. Only one set of operating conditions is presented here in the receiver simulation.

The overall receiver is cylindrical, $34.8-\mathrm{cm}$ (13.7-in) in diameter and $12.7-\mathrm{cm}$ ( $5-\mathrm{in}$ ) high. The working receiver aperture is $5.08-\mathrm{cm}$ (2-in) in diameter. The theoretical focal size (sun spot) is $1.59-\mathrm{cm}$ ( $0.625-\mathrm{in}$ ) and the actual focal size is $2.03-\mathrm{cm}$ ( $0.8-\mathrm{in}$ ).


Figure 4-1. Cavity-Type Receiver Understudy

Tables 4-1 through 4-13 include all the receiver data required to run the program. The tables are constructed following the instructions given for each parameter in Appendix C. The receiver was discretized into 32 nodes as shown in Figure 4-2. Nodes 1, 2, 3, and 4 represent the interior ceramic surface of the cavity. Nodes 5 and 6 represent the outer insulation layer. Node 7 represents the head plate-supporting structure. To make use of the view factor subroutine for axisymmetric nodes, the 34 pairs of U-tubes were treated as two coaxial annulus channels. Nodes $9,10,11,12,13$, and 14 represent the annulus channel through which the fluid enters, while nodes $16,17,18,19,20$, and 21 represent the annulus channel through which the fluid exits. Nodes 8 and 15 represent the tip of the U-shape connecting the two annulus channels. Nodes 22 through 28 represent the fluid in its passage through the tubes. Node 29 represents the state of the working fluid leaving the receiver and node 31 represents the state at entrance. Since the receiver inlet temperature was kept constant during the receiver operation, regardless of the exit temperature variations or ambient conditions, node 31 was considered as a source/sink node. Node 30 represents the head plate through which the fluid paths are grooved, and node 32 represents the ambient air temperature around the outer receiver body as well as the interior cavity. The following subsections describe the input data.
4.2.1 Data for Table 4-1

The problem at hand includes all four modes of heat transfer, i.e., radiation, thermal convection, conduction, and fluid convection. Therefore, the variables SAIRR, CONVEC, CONDUC, and FLOW were assigned the value 1. For a steady state solution, SST is assigned the value 1 , keeping in mind that the program is capable of also handing transient solutions. With a total of 32 nodes, two of which are source/sink nodes, the rest of the variables in Table $4-1$ are assigned. Since only the steady state case was considered at one particular hour of operation, one element of the (NDIN) vector was given. The maximum number of iterations (MNIT) was assumed as 6 , based on past experience in iteration


Figure 4-2. Nodal Representation of Receiver

## Table 4-1 Scalar Variables

| No. | Variable | Value | Type |
| :---: | :---: | :---: | :---: |
| 1 | SAIRR | 1 | Integer |
| 2 | CONVEC | 1 | Integer |
| 3 | CONDUC | 1 | Integer |
| 4 | FLOW | 1 | Integer |
| 5 | SST | 1 | Integer |
| 6 | VPRNT | 1 | Integer |
| 7 | NN | 32 | Integer |
| 8 | SS | 2 | Integer |
| 9 | MINDTU | 1/60 | Real |
| 10 | AMBNN | 32 | Integer |
| 11 | FNDIN | 31 | Integer |
| 12 | FNDOT | 29 | Integer |
| 13 | FTTME | 1.0 | Real |
| 14 | NDIN | 1 | Integer |
| 15 | MNIT | 6 | Integer |
| 16 | HROU'T | 0.0 | Real |
| 17 | DTOUT | 1.0 | Real |

Table 4-2. SORSIN, TEM, EIR, ESOL \& POWR

| Mode | SORSIN | $\begin{aligned} & \text { TEM } \\ & \left({ }^{\circ} \mathrm{F}\right) \end{aligned}$ | EIR | ESOL | POWR |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | 4 | 0.75 | 0.156 | 4 |
| 2 |  |  | 0.75 | 0.156 |  |
| 3 |  |  | 0.75 | 0.156 |  |
| 4 |  |  | 0.75 | 0.156 |  |
| 5 |  |  | 0.0 | 0.0 |  |
| 6 |  |  | 4 | 4 |  |
| 7 |  |  |  |  |  |
| 8 |  |  |  |  |  |
| 9 |  |  |  |  |  |
| 10 | 0 | 77 |  |  | 0.0 |
| 11 |  |  |  | 0.5 |  |
| 12 |  |  |  |  |  |
| 13 |  |  | 0.5 |  |  |
| 14 |  |  |  |  |  |
| 15 |  |  |  |  |  |
| 16 |  |  |  |  |  |
| 17 |  |  |  |  |  |
| 18 |  |  |  |  |  |
| 19 |  |  |  |  |  |
| 20 |  |  |  |  |  |
| 21 |  |  | $\dagger$ | $\dagger$ |  |
| 22 |  |  |  | $4$ |  |
| 23 |  |  |  |  |  |
| 24 |  |  |  | 0.0 |  |
| 25 |  |  |  |  |  |
| 26 |  |  | 0.0 |  |  |
| 27 |  |  |  |  |  |
| 28 |  |  |  |  |  |
| 29 |  |  | 1 | $\dagger$ |  |
| 30 | 1 | $\dagger$ | 0.5 | 0.5 |  |
| 31 | 1 | 652 | 0.0 | 0.0 |  |
| 32 | 1 | 77 | 1.0 | 1.00 | $\dagger$ |

convergence. The minimum time increment (MINDTU), below which nodes are considered zero-capacity nodes, was entered as $1 / 60$-hour and is only applicable when the user requests the transient solution.
4.2.2 Data for Table 4-2

Nodes 31 (fluid inlet section) and 32 (ambient air) were the only source/sink nodes as indicated in the SORSIN column. The initial temperature for all nodes, excluding node 31 , was taken as $25^{\circ} \mathrm{C}\left(77^{\circ} \mathrm{F}\right)$. The inlet fluid temperature was kept fixed at $344.44^{\circ} \mathrm{C}\left(652^{\circ} \mathrm{F}\right)$. The emissivity values in the infrared range (EIR) or the solar band (ESOL) were best estimates. No attempt was made to measure any of these emissivities. For the aperture, or ambient air node (node 32), perfect black body values must be entered, i.e., ESOL $=\operatorname{EIR}=1$. Since there is no internal heat generation in any node, the (POWR) vector is set to zero.

### 4.2.3 Data for Table 4-3

Nodal specific heat, density, and thermal conductivity were abstracted from the literature for the materials used in the receiver construction. Helium properties were taken at a pressure of $206.73 \mathrm{~N} / \mathrm{cm}^{2}$ ( 300 psia ) and an average temperature of $373^{\circ} \mathrm{C}\left(703.4^{\circ} \mathrm{F}\right.$ ). The volume of each node is calculated from the receiver dimensions in Figure 4-2. The volume of nodes 31 and 32 (infinite capacity nodes) were set arbitrarily at 1. A1though the volume of each node is not needed for a steady-state solution, it was given for reference or for use in a transient solution. The surface area exposed to either direct solar radiation or to infrared energy exchange (ASOL) was also computed from the receiver geometry.

Table 4-3. SPHT, DEN, KCOND, VOL \& ASOL

| Node | $\begin{gathered} \text { SPHT } \\ \left(\mathrm{Btu} / 1 \mathrm{~b}^{\mathrm{o}} \mathrm{~F}\right) \end{gathered}$ | $\begin{gathered} \text { DEN } \\ \left(1 \mathrm{~b} / \mathrm{ft}^{3}\right) \end{gathered}$ | $\begin{gathered} \text { KCOND } \\ \left(\mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{\mathrm{O}} \mathrm{~F}\right) \end{gathered}$ | $\begin{gathered} \text { VOL } \\ \left(\mathrm{ft}^{3}\right) \end{gathered}$ | $\begin{aligned} & \text { ASOL } \\ & \left(\mathrm{ft}^{2}\right) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.12 | 511 | 6.5 | 0.00364 | 0.017429 |
| 2 | 0.316 | 140 | 0.83 | 0.00592 | 0.080505 |
| 3 | 0.316 | 140 | 0.83 | 0.00509 | 0.12344 |
| 4 | 0.316 | 140 | 0.83 | 0.01233 | 0.21764 |
| 5 | 0.2 | 11 | 0.04 | 0.14199 | 0 |
| 6 | 0.2 | 11 | 0.04 | 0.22998 | 0 |
| 7 | 4 | 4 | 4 | 0.02119 | 0 |
| 8 |  |  |  | 0.00008 | 0.047305 |
| 9 |  |  |  | 0.00012 | 0.0705 |
| 10 | 0.12 | 511 | 6.5 | 0.00017 | 0.10104 |
| 11 |  |  |  | 0.00015 | 0.08722 |
| 12 |  |  |  | 0.00013 | 0.07503 |
| 13 |  |  |  | 0.00015 | 0.088802 |
| 14 |  |  |  | 0.00008 | 0.04909 |
| 15 |  |  |  | 0.00005 | 0.02727 |
| 16 |  |  |  | 0.00006 | 0.03466 |
| 17 |  |  |  | 0.00011 | 0.06877 |
| 18 |  |  |  | 0.00011 | 0.064117 |
| 19 |  |  |  | 0.00009 | 0.054522 |
| 20 | , |  |  | 0.00010 | 0.058179 |
| 21 | $\dagger$ | $\dagger$ | $\dagger$ | 0.00006 | 0.03633 |
| 22 | 4 | 4 | 4 | 0.00097 |  |
| 23 | ${ }_{1.24}$ |  |  | 0.00116 |  |
| 24 |  | 0.0961 | 0.1457 | 0.00069 |  |
| 25 |  |  |  | 0.00059 | 0.0 |
| 26 |  |  |  | 0.00041 |  |
| 27 |  |  |  | 0.00078 |  |
| 28 |  |  |  | 0.00070 |  |
| 29 | - | $\dagger$ |  | 0.00070 |  |
| 30 | 0.12 | 511 | 6.5 | 0.01615 | 0.021814 |
| 31 | 1.24 | 0.0961 | 0.1457 | 1.0 | 0 |
| 32 | 0.24 | 0.075 | 0.015 | 1.0 | 0.021814 |

Tables 4-4 and 4-5 were constructed simultaneously to complete the radiation view factors. Nodes $5,6,7,22,23,24,25,26,27,28$ and 29 were excluded entirely from the radiation exchange and were left blank. The elements of the view factor matrix that have a flag equal to [3] in Table 4-4, i.e., $V(i, j)=3$, were marked (X) in Table 4-5 and were given as:

$$
\begin{array}{ll}
F(2,3) & =0.1 \\
F(3,4) & =0.1 \\
F(3,9) & =0.448 \\
F(13,16) & =0.083 \\
F(13,17) & =0.563 \\
F(13,18) & =0.051
\end{array}
$$

The above view factors were estimated after a preliminary run of the program to allow for shading by other nodes and to satisfy the summation and reciprocity rules. This procedure may or may not be needed in most cases, but was introduced in this document to familiarize the user with this iteration process.
4.2.5 Conduction and Convection Between Nodes

Table 4-6 shows the distinction between nodes exchanging heat by conduction or by convection. The surface area in contact (ASRF) was computed for these two modes of heat transfer and is marked by ( $X$ ) in Table 4-7. The elements of the ASRF matrix are given below for the example receiver. Additional conditions imposed for filling out the tables are given in Appendix C. The English system of units is used throughout this example, but can be readily replaced by the $S I$ (metric) system if necessary.

Table 4-4. View Factor Flag [V]


```
Table 4-5. View Factor Matrix [F]
```



Table 4-6. Adjacent Condition [ADJ]


Table 4-7. Surface Area [ASRF]


| ASRF | $(1,2)=$ | 0.11454 | , | ASRF | $(5,6)=$ | 1.01450 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| " | $(1,5)=$ | 0.04363 | , | " | $(5,7)$ | 0.33540 |
| " | $(1,32)=$ | 0.11454 | , | " | $(5,32)=$ | 0.39540 |
| " | $(2,3)$ | 0.05760 | , | " | $(6,32)=$ | 2.59836 |
| " | $(2,5)$ | 0.09817 | , | " | $(7,30)$ | 0.10123 |
| " | $(2,32)=$ | 0.07592 | , | " | $(7,32)=$ | 0.48380 |
| " | $(3,4)$ | 0.04014 | , | " | $(8,25)=$ | 0.04729 |
| " | $(3,5)$ | 0.15272 | , | " | $(8,32)$ | 0.04729 |
| " | $(3,32)=$ | 0.12217 | , | " | $(9,24)=$ | 0.07150 |
| " | $(4,5)=$ | 0.20180 | , | " | $(9,32)=$ | 0.07150 |
| " | $(4,7)$ | 0.08727 | , | " | $(10,23)=$ | 0.10079 |
| " | $(4,30)=$ | 0.08050 | , | " | $(10,32)=$ | 0.10079 |
| " | $(4,32)=$ | 0.02220 | , | " | $(12,22)=$ | 0.07200 |
| " | $(11,22)=$ | 0.08727 | , | " | $(13,23)=$ | 0.08857 |
| " | $(11,32)=$ | 0.08727 | , | " | $(14,24)=$ | 0.05236 |
| " | $(15,25)=$ | 0.02730 | , | " | $(16,26)=$ | 0.03436 |
| " | $(17,27)=$ | 0.06872 | , | " | $(18,28)=$ | 0.06152 |
| " | $(19,28)=$ | 0.05236 | , | " | $(20,27)=$ | 0.05803 |
| " | $(19,32)=$ | 0.05236 | , | " | $(20,32)=$ | 0.05803 |
| " | $(21,26)=$ | 0.03436 |  | " | $(30,32)=$ | 0.12960 |
| " | $(21,32)=$ | 0.03436 | , |  | --- |  |

To complete the computations of the thermal conductance needed for the conduction heat transfer, the matrix (LNGTH) in Table 4-8 has to be completed in full. However, the LNGTH matrix is completed for those nodes showing $A D J=1$. The elements of the non-symmetric (LNGTH) were computed (in ft) and were given as tabulated below.

Table 4-8. [LNGTH] Matrix


| LGNTH | $(1,2)$ | = | 0.0167 | LNGTH | $(2,1)$ | = | 0.0375 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| " | $(1,5)$ | $=$ | 0.0583 | " | $(5,1)$ | = | 0.0917 |
| " | $(2,3)$ | = | 0.0375 | " | $(3,2)$ | $=$ | 0.0583 |
| " | $(2,5)$ | = | 0.0417 | " | $(5,2)$ | = | 0.0917 |
| " | $(3,4)$ | = | 0.0583 | " | $(4,3)$ | $=$ | 0.1125 |
| " | $(3,5)$ | = | 0.0208 | " | $(5,3)$ | = | 0.0917 |
| " | $(4,5)$ | = | 0.0260 | " | $(5,4)$ | = | 0.0917 |
| " | $(4,7)$ | $=$ | 0.0333 | " | $(7,4)$ | = | 0.0917 |
| " | $(5,6)$ | = | 0.0917 | " | $(6,5)$ | $=$ | 0.0917 |
| " | $(5,7)$ | $=$ | 0.2083 | " | $(7,5)$ | $=$ | 0.0333 |
| " | $(4,30)$ | $=$ | 0.1104 | " | $(30,4)$ | $=$ | 0.0375 |
| " | $(7,30)$ | $=$ | 0.0333 | " | $(30,7)$ | $=$ | 0.03755 |

For thermal convection, the convective heat transfer coefficient (HCONV) was assembled as shown in Table 4-9. The heat transfer coefficient was taken as approximately 1 Btu/ $\mathrm{hr}^{\circ} \mathrm{Fft}^{2}$ between node 32 , which is the ambient air, and nodes $1,2,3,4,8,9,10,11,19,20$, and 21 . For nodes 5, 6, 7, and 30 , the convective coefficient was considered as approximately $2 \mathrm{Btu} / \mathrm{hr}^{\circ} \mathrm{Fft}^{2}$ to allow for the unaccounted radiation loss from the receiver exterior surface. Note that the above values could be better refined if more analytical information was given about the convection mechanism inside and outside the receiver cavity. For the convective coefficient between the working fluid (he1ium) and the receiver tubes, exemplified in the set of nodes $(8,25),(9,24),(10,23),(11,22)$, $(12,22),(13,23),(14,24),(15,25),(16,26),(17,27),(18,28),(19,28)$, $(20,27)$ and $(21,26)$, the following formula for turbulent flow inside tubes was used:

$$
\mathrm{Nu}=0.023 \quad \mathrm{R}_{\mathrm{n}}^{0.8} \operatorname{Pr}^{0.4}
$$

Table 4-9. [HCONV] Matrix

Node

where $N u$ is the Nusselt number, $\operatorname{Pr}$ is the $\operatorname{Prandtl}$ number and Rn is the Reynolds number. The equivalent, or "Hydraulic" diameter ( $D_{h}$ ) to be used in the above formula is determined from the expression

$$
D_{h}=4 \times \frac{\text { cross sectional area }}{\text { wetted perimeter }}
$$

For an annulus area of an inside diameter $d_{i}$ and an outside diameter $d_{o}$, the hydraulic diameter is found to be $\left(d_{o}-d_{i}\right)$.

In the present example, helium gas flows at a rate of $42.11 \mathrm{~kg} / \mathrm{hr}$ ( $92.9 \mathrm{lb} / \mathrm{hr}$ ) at $206.73 \mathrm{~N} / \mathrm{cm}^{2}$ ( 300 psia ) pressure. Since the inlet helium temperature was $344.4^{\circ} \mathrm{C}\left(652^{\circ} \mathrm{F}\right)$, the property of helium was taken arbitrarily at an average temperature of approximately $372.8{ }^{\circ} \mathrm{C}\left(703^{\circ} \mathrm{F}\right)$ based on about $55.6^{\circ} \mathrm{C}\left(100^{\circ} \mathrm{F}\right)$ temperature rise through the receiver tubes. The Reynolds and the Prandtl numbers were 52,400 and 0.659 , respectively. The convective heat transfer coefficient was then calculated as $580 \mathrm{Btu} / \mathrm{hr}^{\circ} \mathrm{Fft}^{2}$. For different helium flow rates, the convective coefficient (H), in $B t u / h{ }^{\circ}{ }^{\circ} \mathrm{Fft}^{2}$, is written in the form

$$
\mathrm{H}=15.453 \dot{\mathrm{~m}}_{\mathrm{f}} 0.8
$$

where $\dot{m}_{f}$ is the flow rate in $1 b / h r$ inside the receiver tubes and $H$ is the convective coefficient between the helium gas and the tubes in the example receiver.

### 4.2.6 Fluid Convection

The flow direction matrix (MASFLO) was developed as in Table 4-10. The closed fluid loop (only one in this example) was considered to go from nodes 31-22-23-24-25-26-27-28-29-31. Either zero or 1 is entered to identify the flow direction. In Table 4-11, variations of the mass flow rate (MFM) with time (HRIN) were allowed. Only one value of the flow rate was imposed in the fluid loop (92.9 lb/hr in Table 4-11) and was assumed to remain constant thereafter. Table 4-11 was completed by entering the fixed temperature for source/sink nodes SSTEM(1) and SSTEM(2). Note the two sources/sink nodes in this example

Table 4-10. [MASFLO] Matrix

were the ambient temperature (node 32) and the inlet fluid (node 31), respectively.

Table 4-11. SSTEM(1), SSTEM(2) and MFM

| HRIN | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SSTEM <br> $(1)$ | 77 |  |  |  |  |  |  |  |  |  |  |
| SSTEM <br> $(2)$ | 652 |  |  |  |  |  |  |  |  |  |  |
| MFM | 92.9 |  |  |  |  |  |  |  |  |  |  |

### 4.2.7 F1ux Distribution

The solar flux distribution in Table 4-12 was predetermined using an optics computer program developed for the on-going solar-thermal power system analyses. The intensity concentration ratio (c) divided by the parabolic mirror reflectivity ( $\rho$ ) is given in Figure 4-3 together with a sketch of the cavity receiver made to fit the optics program. Table 4-13 was used to determine the solar flux on nodes $3,8,19,20$, 21, and 30 in terms of the values given in Figure 4-3.

The total solar energy retained inside the cavity receiver was found from a previous calorimeter test to range from $4.097 \mathrm{~kW}_{t}$ ( $13,983.7 \mathrm{Btu} / \mathrm{hr}$ ) to $4.47 \mathrm{~kW}_{\mathrm{t}}(15,256.1 \mathrm{Btu} / \mathrm{hr})$. Arbitrarily taking the lower end of the scale, the equivalent solar intensity falling on the mirror surface would be $13,983.7 / 83.07$ or $168.3 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{2}$. Note that the latter is always less than the actual solar intensity falling on the mirror to allow for the receiver intercept factor caused by the energy spill-over from the limited receiver aperture size. The last

Table 4-12. Solar Flux [IFLUX]

|  | NDIN |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 3 | X |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 4 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 6 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0 | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| D | 8 | X |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | . |  |  |  |
| E | 9 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 10 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 11 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 12 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 13 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 14 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 15 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 16 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 17 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 18 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 19 | $x$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 20 | X |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 21 | $x$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 22 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 23 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 24 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 25 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 26 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 27 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 28 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 29 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 30 | X |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 31 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 32 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |



Figure 4-3. Solar F1ux Distribution on Example Receiver

Table 4-13. Solar Flux Distribution

| Node | $\mathrm{C} / \rho$ | $\mathrm{C}^{(1)}$ | $\mathrm{ASOL}^{(2)}$ <br> $\mathrm{ft}^{2}$ | $\mathrm{C} * \mathrm{ASOL}$ | IFLUX <br> $\mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 300 | 270 | 0.12344 | 33.33 | 45,441 |
| 8 | 800 | 720 | 0.047305 | 34.06 | 121,176 |
| 19 | 0 | 0 | 0.054522 | 0.0 | 0.0 |
| 20 | 100 | 90 | 0.058179 | 5.24 | 15,147 |
| 21 | 200 | 180 | 0.03633 | 6.51 | 30,294 |
| 30 | 200 | 180 | 0.021814 | 3.93 | 30,294 |
| Tota1 |  |  |  | 83.07 |  |

(1) Assuming a mirror reflectivity ( $\rho$ ) of 0.9
(2) Taken from Table 4-3
(3) Product of (C) column multiplied by $168.3 \mathrm{Btu} / \mathrm{hr} \mathrm{ft}^{2}$
column in Table 4-13 is thus completed and the data entered back into Table 4-12. Again, in Table 4.12, only one column is used corresponding to one set of solar flux values which remain constant thereafter. The user has the option of completing Table 4-12 for a transient solution by repeating this procedure at preselected time intervals during the solar day.

### 4.2.8 Nodal Coordinates

In Table 4-14, the axial and radial coordinates of each node were given based on a reference point taken arbitrarily at the cavity aperture center with the Z-direction taken along the receiver centerline. Each node in the cavity interior is represented by two bounding circular rings. Therefore, four coordinates ( $r_{1}, r_{2}, Z_{1}$, and $Z_{2}$ ) are needed. Only axisymmetric nodes that exchange heat by radiation need to be considered to make use of the special view-factor subroutine embedded in the code. For example, nodes 5, 6, 7, 22-29, and 31 were assumed not exchanging heat by radiation to neighboring nodes and their coordinates were entered as zero.

After completing the input data tables, (LOAD) the [FIRSTPART] to enter the data, then (MERGE) the [LASTPART] of the program and the program is ready for execution using the (RUN) command.

### 4.3 ANALYSIS OF PROGRAM RESULTS

A copy of the program results is shown in Figures $4-4$ and 4-5. The output for the steady state solution starts by a list of the radiation view factors (if requested by the user), a check on the summation rule of the radiation view factors, the accumulated thermal energy extracted by the receiver, the accumulated solar energy incident on the receiver interior, the receiver efficiency, and the nodal residual energy. The user may repeat this exercise using different values for the receiver parameters for the performance evaluation, the transient solution and the sensitivity analysis.

Table 4-14. Nodal Coordinates

Radial
Coordinates

Axial
Coordinates

| Node | $\mathrm{r}_{1}$ | $\mathrm{r}_{2}$ | $\mathrm{Z}_{1}$ | $\mathrm{Z}_{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.0833 | 0.0833 | 0 | 0.0333 |
| 2 | 0.0833 | 0.1583 | 0.0333 | 0.1083 |
| 3 | 0.1583 | 0.175 | 0.1083 | 0.225 |
| 4 | 0.175 | 0.1333 | 0.225 | 0.4458 |
| 5 | $1$ | $4$ | $4$ | $1$ |
| 6 | 0.0 | 0.0 | 0.0 | 0.0 |
| 7 | $t$ | $t$ | $\downarrow$ | $1$ |
| 8 | 0.14167 | 0.0708 | 0.1458 | 0.1458 |
| 9 | 0.14167 | 0.14167 | 0.1458 | 0.225 |
| 10 | 0.14167 | 0.1333 | 0.225 | 0.341667 |
| 11 | 0.1333 | 0.1333 | 0.341667 | 0.4458 |
| 12 | 0.11667 | 0.1125 | 0.341667 | 0.4458 |
| 13 | 0.125 | 0.11667 | 0.225 | 0.341667 |
| 14 | 0.125 | 0.125 | 0.1625 | 0.225 |
| 15 | 0.0833 | 0.125 | 0.1625 | 0.225 |
| 16 | 0.0833 | 0.091667 | 0.1625 | 0.225 |
| 17 | 0.091667 | 0.095833 | 0.225 | 0.341667 |
| 18 | 0.095833 | 0.10 | 0.341667 | 0.4458 |
| 19 | 0.08333 | 0.08333 | 0.341667 | 0.4458 |
| 20 | 0.075 | 0.08333 | 0.225 | 0.341667 |
| 21 | 0.0708 | 0.075 | 0.1458 | 0.225 |
| 22 | $4$ | $4$ | $4$ |  |
| 23 |  |  |  |  |
| 24 |  |  |  |  |
| 25 | 0.0 | 0.0 | 0.0 | 0.0 |
| 26 |  |  |  |  |
| 27 |  |  |  |  |
| 28 |  |  |  |  |
| 29 | $\dagger$ | $\dagger$ |  | $\dagger$ |
| 30 | 0.0833 | 0.0 | 0.4458 | 0.4458 |
| 31 | 0.0 | 0.0 | 0.0 | 0.0 |
| 32 | 0.0 | 0.0833 | 0.0 | 0.0 |

```
ENTER ZERD FOR TRANSIENT ANAL'YSIS QR ENTER 1 FGR GUARSI STEAD'Y STATE ONE:1
ENTER NUMBER DF NODES:3E
ENTER NUMBER OF SQURCE-SINK MUDES:E
ENTER MD OF TIME DEFENDENT DATA POINTS:1
EMTER ZERD IF THERE IS NG RADIATIGN EXICHANGE ELSE ENTER 1:1
ENTER ZERO IF THERE IS NU EONDUICTIDN ELSE ENTER 1:1
EMTER ZERD IF THERE IS ND COMVECTIUN ELSE EMTER 1:1
EHTER ZERD IF THERE IS ND FLUID FLDN ELSE ENTER 1:1
ENTER 1 TQ PRINT VIE| FACTORS ELSE ENTER O:1
MON-ZERD ELEMENTS DF THE VIEN FACTDR MATRIX ARE :
F(1, 1)= .13035E6
F(1, E)= . 23497491
F< 1, 3)= .G5010917E-01
F< 1, E)= .69947446E-01
F( 1, 19)= .69775335E-02
F< 1, 20)= .1924146E-01
F< 1, E1)= .43644969E-01
F(1, 32)= .40995019
FC E, 1)= .50349009E-01
F( こ. こ)= .36236416
F(E, 3)= .1
F&E,G)=.30967289
F(E, 19)= . 26434366E-01
F( E, 20)= .40119394E-01
F(E, E1)= .E0979597E-01
F(E, 32)= .93315902E-02
F( 3, 1)= .1E00egr3E-01
F( 3, 2)= .65217652E-01
F(3, )}=.1285893
F( 3, 4)=.1
F( 3, 9) = .44B
F< 3, 10)= .15747265
F( 3, 11)= .93718466E-01
F(4,3)= .56718564E-01
F(4,4)= .36997634
F(4,9)=.6955166E-01
F(4:10)= . 24778143
F(4, 11)= . 2559720e
F(日, 1)= .25770986E-101
F( G, 2)= .66349273
F( B, G)= .17794836
F(E, Sこ)= .13E78794
F& 9, 3)= .7944E779
F(G, 4)= .31471245
F(9, )}=.9597672E-0
F& 10, 3)= .19こ392こ6
F(10,4)= .53370762
F(10, 10)= . 27391014
Figure 4－4．Program Output for Steady－State Solution
```

```
F< 11, 3:= .1E556642
F(11,4)= .63374333
F<11, 11)= . 3556%71
F< 1こ, 1こ)= .16354773E-01
F<1E, 15%= . Se13ET31E-01
F& 1E, 16:= .TEO173SEE-01
F(1\Xi, 17)= .1E57E056
F< 1E, 1B)= .F416145
Fe 13, 13)= .E100Eこ37
F< 13, 15)= . ᄏEFPrez7E-01
F< 15, 15)= .E300001E-01
F& 13, 17)= .563
F& 13, 1%)= .51E-01
F& 14, 14:= .4609569ЭE-01
FC 14, 15)= . こ7055こ`
F&14,16)= .4E05555
F< 14, 17)= .193F507
F& 14, 13)= . F9519676E-01
F; 15, 1こ)= .99407595E-01
F(15: 13)= .30E7457%
F&15, 14:= .456015%7
F& 15, 15:= .7E0<147こE-01
F(15, 16)= . 3ESE00EEE-01
F& 15, 17)= .こう513606E-0ミ
F( 16, 1E)= .1553546%
```



```
F( 1G, 14)= .5`5197\Xi5
F(1E, 15)= . こ53ES34EE-01
F(16, 16)= .47701597E-13
F&17:1こ)= .1405136
F< 17, 130= .7370333%
F& 17, 14)= . 1311665%
```



```
F(17, 17)= .3439553`E-13
F&13, 1こ)= .36736197
FC 13, 13)= . 7063556%E-01
F( 13, 14)= .E0ST97EE-01
FC 13, 1B)= .EEこT046SE-03
F< 1ق, 1)= .こここ05ミスTE-0こ
F& 1F, E`= . 3F03ミ13ЭE-01
F& 1F, 1F)= .417059F6
F<1F, 20)= . Eここ`570%
F(19, こ1)= . 33745463E-01
F<1F, 30)= . 37695097
```



```
F& こ0, 1)= .5764E363E-0E
FCE日, z)= .55513`43E-01
F& 20, 19)= .20355%4
FC 20, 20%= .431E63E6
F& こ0, 21)= .1545831こ
FCEO, 30)= . 30418こ7SE-01
F& E0, S%= . 1371154EE-01
F< E1, 1)= .こ0939こ4E-01
FCE1, O)= .46491957E-01
F<E1, 1%)= .50645708E-01
F< \Xi1, \Xi0)= . こ4757053
F( こ1, こ1)= .56364619
FCE1, 30)= . E60ここ0SSE-01
F(E1, 3`)= . Э`6%7こ47E-01
```

Figure 4－4．Program Output for Steady－State Solution（Cont＇d）

```
F< 30, 19)= .69267999
FC S0, こ0)= .こ145こ4`5
FC 30, 21)= .43377136E-01
Fe 30, 30)= . 1664715E-01
F(30, 3こ)= .3E670736E-01
F( 3こ, 1)= . 32776309
F< 3こ, こ)= . 346465ここE-01
F< ES, O)= . 23S1566S
F< 3E, 19)= . こ061373SE-01
F( SE, E0)= .365`4155E-01
F< SE, こ1)= - -5121704E-01
F& SE, 30:= .3こ6T0735E-01
F( 32, 32)= .1734こ`2
CHECK DH SUMMATIDN RUGLE OF F
\begin{tabular}{cc}
1 & -99999999 \\
0 & 1 \\
1 & 1 \\
1 & 1 \\
0 & 0 \\
0 & 1 \\
TIMECHRO & HCC ENGY ENTR（BTI） \\
0 & 0
\end{tabular}
Nade mo NaIE temf:F
\begin{tabular}{ll}
1 & 77 \\
2 & 77 \\
3 & 77 \\
4 & 77 \\
5 & 77 \\
2 & 77 \\
7 & 77 \\
3 & 77 \\
- & 77 \\
10 & 77 \\
11 & 77 \\
12 & 77 \\
13 & 77 \\
14 & 77 \\
15 & 77 \\
16 & 77 \\
17 & 77 \\
13 & 77 \\
19 & 77 \\
30 & 77 \\
31 & 77 \\
32 & 77 \\
23 & 77 \\
34 & 77 \\
25 & 77 \\
\hline 6 & 77 \\
27 & 652 \\
28 & 77
\end{tabular}
```

Figure 4－4．Program Output for Steady－State Solution（Cont＇d）

|  | MOIE RESIDIE，ETMH |  |  |
| :---: | :---: | :---: | :---: |
| 1 | －． 1957 Э3こTE－01 |  |  |
| $\Xi$ | －． 3311896 E －01 |  |  |
| 3 | －． 4449615 |  |  |
| 4 | ． 14346313 |  |  |
| 5 | －． $35146973 \mathrm{E}-15$ |  |  |
| 6 | 0 |  |  |
| 7 |  |  |  |
| 3 |  |  |  |
| 7 | －1656ET7 |  |  |
| 10 |  |  |  |
| 11 | ． 43413534 E －01 |  |  |
| $1 こ$ | － $359380 \overrightarrow{R E-03}$ |  |  |
| 13 | －50ごこ478E－03 |  |  |
| 14 | ． $47350609 E-03$ |  |  |
| 15 | －．136－4365E－19 |  |  |
| 15 | －ごず， 0 06こE－03 |  |  |
| 17 | －ご36こうすこEーが |  |  |
| 13 | －． 37975456 E －04 |  |  |
| 17 | －ア3うごこらこと－0き |  |  |
| こ0 |  |  |  |
| 31 |  |  |  |
| $\Xi$ | －195こ1ご5Eーロコ |  |  |
| $\Xi こ$ | 0 |  |  |
| 34 | － 1 Э5ミ1こらE－ロこ |  |  |
| $\Xi$ | 0 |  |  |
| 3 | －1ヨらごきことー日き |  |  |
| $\Xi 7$ | 0 |  |  |
| 3 | 0 |  |  |
| 2. | 0 |  |  |
| 31 | －1玉きらいく1Eー或 |  |  |
| 31 | 11006 Eご |  |  |
| 3 O | 2978．137 |  |  |
| $\begin{aligned} & \text { TIME CHE } \\ & 1 \end{aligned}$ |  | $\begin{gathered} \text { ADG Sal ENGY INGETA: } \\ 13983.673 \end{gathered}$ | $\begin{aligned} & \text { EEUR EFF } \\ & \cdot 7 E 709305 \end{aligned}$ |
| NIIE NO | HDLE TEMF，F |  |  |
| 1 | 1351.155 |  |  |
| $\Xi$ | 1413 こアごす |  |  |
| 3 | 14ご心．3アご |  |  |
| 4 | 946．57669 |  |  |
| 5 | 301.31097 |  |  |
| $\dot{7}$ | 36 －1 0603 |  |  |
| 7 | $747.356 \% 6$ |  |  |
| 3 | 351．53\％ |  |  |
| 7 | 74こ． 960 こ |  |  |
| 10 | E94．05167 |  |  |
| 11 | \％3． 5055 |  |  |
| 12 | E60．3ア73こ |  |  |
| 13 | らフミ．1ア0゙1 |  |  |
| 14 | 690．90236 |  |  |
| 15 | 727．10こ35 |  |  |
| 16 | 735．37341 |  |  |
| 17 | 743.53195 |  |  |
| 13 | 746.97315 |  |  |
| 17 | 761.43443 |  |  |
| $\pm 0$ | 773.15372 |  |  |
| 31 | ア34．4150 |  |  |
| ここ | 650．3ご心 |  |  |
| $\Xi$ | 671.7915 |  |  |
| こ4 | 690.67439 |  |  |
| ご | アご．3515こ |  |  |
| 36 | 735．71693 |  |  |
| $こ 7$ | 744.05597 |  |  |
| E | 747．54605 |  |  |
| 37 | 747．54605 |  |  |
| 310 | 735．7145こ |  |  |
| 31 | $ら 5 こ$ |  |  |
| 32 | 77 |  |  |
| ＞ |  |  |  |

Figure 4－4．Program Output for Steady－State Solution（Cont＇d）

PRUN


Figure 4-5. Program Output for Transient Solution
TIME (HR) ACC ENG'Y EXTR (BTU) ACO SQL ENIS'Y IN(BTU) RECYR EFF

| HIDE ND | NODE TEMP, F |
| :---: | :---: |
| 1 | 77 |
| 2 | 77 |
| 3 | 77 |
| 4 | 77 |
| 5 | 77 |
| 6 | 77 |
| 7 | 77 |
| 8 | 77 |
| 9 | 77 |
| 10 | 77 |
| 11 | 77 |
| 12 | 77 |
| 13 | 77 |
| 14 | 77 |
| 15 | 77 |
| 16 | 77 |
| 17 | 77 |
| 18 | 77 |
| 19 | 77 |
| 20 | 77 |
| 21 | 77 |
| 29 | 77 |
| 23 | 77 |
| 24 | 77 |
| 25 | 65 |
| 26 | 27 |
| 29 | 29 |

Figure 4-5. Program Output for Transient Solution (Cont'd)

| $\begin{aligned} & \text { T IME CHR) } \\ & .5 E-01 \end{aligned}$ | ACO ENGY EXTR（BTIN） | $\begin{gathered} \text { ACC SQL ENBY IN (ETU) } \\ 699.13364 \end{gathered}$ | RECUR EFF <br> ． 6.592141 |
| :---: | :---: | :---: | :---: |
| HODE MI | HaIIE TEMF，F |  |  |
| 1 | 116．4551こ |  |  |
| 2 | こ19．65807 |  |  |
| 3 | 384.96397 | ， |  |
| 4 | 131.55163 |  |  |
| 5 | $78.64 E 746$ |  |  |
| 6 | 77.00396 |  |  |
| 7 | 50.461258 |  |  |
| 3 | 856．152e7 |  |  |
| 9 | $719.16 こ 34$ |  |  |
| 10 | 681.11226 |  |  |
| 11 | 669.99844 |  |  |
| 12 | 558.06944 |  |  |
| 13 | 665.94315 |  |  |
| 14 | 630.04375 |  |  |
| 15 | 713.50502 |  |  |
| 16 | 721．45219 |  |  |
| 17 | 7こ9．14673 |  |  |
| 13 | 73こ．1013こ |  |  |
| 19 | 744．54955 |  |  |
| 20 | 756.75453 |  |  |
| こ1 | 763.49033 |  |  |
| ご | 657.60997 |  |  |
| こ | 655．61737 |  |  |
| 24 | 679． 3 92e1 |  |  |
| $こ 5$ | 713.71683 |  |  |
| 26 | 721.75096 |  |  |
| 27 | 729.55602 |  |  |
| 38 | 732.56934 |  |  |
| E9 | 732.56934 |  |  |
| 30 | 97.47731 |  |  |
| 31 | 65こ |  |  |
| 32 | 77 |  |  |

Figure 4－5．Program Output for Transient Solution（Cont＇d）

| $\begin{aligned} & \text { TIME CHP) } \\ & .1 \end{aligned}$ | $\begin{array}{r} \text { ACO ENGY EXTR (ETGO } \\ 929.53767 \end{array}$ | $\begin{gathered} \text { HEO SQL ENGY IMEBTU } \\ 1398.3673 \end{gathered}$ | $\begin{aligned} & \text { FELYR EFF } \\ & .66476647 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| MOIE MD | HOIE TEMP：F |  |  |
| 1 | 197.88085 |  |  |
| $\Xi$ | 343．63176 |  |  |
| 3 | 645.53393 |  |  |
| 4 | 186.20760 |  |  |
| 5 | 84．67996E |  |  |
| 6 | 77.068024 |  |  |
| 7 | 91． 203672 |  |  |
| 9 | 856.88251 |  |  |
| 9 | 720.43958 |  |  |
| 10 | $681.6 さ 148$ |  |  |
| 11 | 670.29776 |  |  |
| 12 | 658.16585 |  |  |
| 13 | 656.17894 |  |  |
| 14 | 680．55こ1こ |  |  |
| 15 | 714.05444 |  |  |
| 16 | アここ． 00545 |  |  |
| 17 | 729.70544 |  |  |
| 15 | 73 T .66374 |  |  |
| 19 | 745.13544 |  |  |
| 20 | 757.34004 |  |  |
| E1 | 769.06984 |  |  |
| E2 | 657．70285 |  |  |
| E3 | 665.85153 |  |  |
| E4 | 630.34053 |  |  |
| 35 | 714.26765 |  |  |
| 26 | 72こ． 3058 |  |  |
| 27 | 730.11714 |  |  |
| 29 | 733.13487 |  |  |
| 29 | 733.13487 |  |  |
| 30 | 113.52528 |  |  |
| 31 | $653$ |  |  |
| 3 S | 75 |  |  |

Figure 4－5．Program Output for Transient Solution（Cont＇d）

## SECTION 5

## REFERENCES

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interpolation subroutine





A-10

| NAME | DESCRIPTION | DIMENSION | UNITS |
| :---: | :---: | :---: | :---: |
| A | View factor parameter | -- | -- |
| *ADJ | ADJ ( $I, J$ ) is conduction/convection identifier between $i^{\text {th }}$ and $j^{\text {th }}$ nodes such that | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | -- |
|  | $\operatorname{ADJ}(\mathrm{I}, \mathrm{J})=1$ means that nodes $\mathrm{i}, \mathbf{j}$, are exchanging heat by conduction. |  |  |
|  | $\operatorname{ADJ}(\mathrm{I}, \mathrm{J})=2$ means that nodes $\mathrm{i}, \mathrm{j}$ are exchanging heat by convection |  |  |
| *AMBNN | The number of nodes representing ambient air | -- | -- |
| *ASOL | ASOL (i) is the $i^{\text {th }}$ node surface area between nodes $i, j$ for radiation heat transfer only. | (NN) vector | $f t^{2}$ |
| *ASRF | $\operatorname{ASRF}(i, j)$ is the surface area between nodes $i, j$ for conduction/convection heat transfer only. | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | $\mathrm{ft}^{2}$ |
|  | $\begin{aligned} & =\quad 0 \text { diagonally } \\ & =\quad 0 \text { for } 2 \text { non-adjacent nodes } \end{aligned}$ |  |  |
|  | ASRF is only given if the problem contains conduction or convection. |  |  |
| B | View factor parameter | -- | -- |
| CAPV | CAPV(i) is the thermal capacitance for the $i^{\text {th }}$ node. | (NN) <br> vector | Btu/ ${ }^{\circ} \mathrm{F}$ |
| CLOCK | Main program time counter. Clock is incremented after each interval. | Scalar | hr |
| CLOCK1 | Time counter for interpolation subroutine. | Scalar | hr |
| CONDC | $\operatorname{CONDC}(\mathbf{i}, \mathbf{j})$ is the conductance matrix between nodes $\mathbf{i , j}$ for conduction heat transfer | ( NNxNN ) | $\mathrm{Btu} / \mathrm{hr}{ }^{\circ} \mathrm{F}$ |
| *CONDUC | Flag for conduction heat transfer in solution | -- | -- |

[^0]| NAME | DESCRIPTION | DIMENSION | UNITS |
| :---: | :---: | :---: | :---: |
| CONVC | $\operatorname{CONVC}(i, j)$ is the conductance for nodes $i, j$ exchanging heat by convection | (NNxNN) | $\mathrm{Btu} / \mathrm{hr}^{\circ} \mathrm{F}$ |
| *CONVEC | Flag for convection heat transfer in solution $\begin{array}{rlrl} \text { CONVEC } & =0 & & \text { No convection } \\ & =1 & & \text { Solution includes con- } \\ & & \text { vection } \end{array}$ | -- | -- |
| DDF | ```Disc-to-disc view vector for axi- symmetric configuration factor subroutine``` | (4) vector | -- |
| *DEN | $\operatorname{DEN}(i)$ is the density of the $i^{\text {th }}$ node material | (NN) <br> vector | $1 \mathrm{~b} / \mathrm{ft}^{3}$ |
| DTAU | Minimum time step $(\Delta \tau)$ for transient solution. Also used as the time step for quasi-steady state solution. | Scalar | hr |
| DTAUV | DTAUV (i) is the minimum time step of the $i^{\text {th }}$ node, calculated from stability subroutine. Applicable to transient solution only. | $\begin{aligned} & \text { (NN) } \\ & \text { vector } \end{aligned}$ | hr |
| DTEM | DTEM(i) is the $i^{\text {th }}$ node new temperature at $(\tau+\Delta \tau)$ | (NN) vector | ${ }^{\circ} \mathrm{R}$ |
| *DTOUT | Time interval to print out results. | Scalar | hr |
| EFF | Accumulated receiver efficiency from starting time till time ( $\tau$ ). | -- | -- |
| *EIR | $\operatorname{EIR}(i)$ is the infra-red emittance of the $i^{\text {th }}$ node surface. | $\begin{aligned} & \text { (NN) } \\ & \text { vector } \end{aligned}$ | -- |
| ENEX | Accumulated energy extracted from the receiver, from starting time till time ( $\tau$ ) | Scalar | Btu |
| ENEXIN | Accumulated energy extracted from the receiver, used for interpolation subroutine. | Scalar | Btu |
| ENEX1 | Accumulated energy extracted from the receiver at time $\left(\tau_{1}\right)$ | Scalar | Btu |


| NAME | DESCRIPTION | DIMENSION | UNITS |
| :---: | :---: | :---: | :---: |
| EQVL | Equivalent length for view factor calculation. | Scalar | ft |
| *ESOL | ESOL(i) is the emittance of the $i^{\text {th }}$ node in the solar radiation band | (NN) vector | -- |
| $\begin{aligned} & \mathrm{E} 1, \mathrm{E} 2, \mathrm{E} 3 \\ & \mathrm{E4}, \mathrm{E}, \mathrm{E} \end{aligned}$ | Parameters for view factor subroutine at $\mathrm{v}=2$. | -- | -- |
| *F | $F(i, j)$ is the view (or configuration) factor between nodes i,j that "see" each other. | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | -- |
| FCF | FCF(i,j) is total view factor matrix ( $\hat{F}$ ) between nodes $\mathbf{i}, \mathbf{j}$ for the infrared radiation exchange. | (NNxNN) matrix | -- |
| FCFST | FCFST(i,j) total view factor matrix ( $\mathrm{F}^{*}$ ) for solar radiation exchange between nodes $\mathbf{i}, \mathbf{j}$. | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | -- |
| FLO | FLO( $i, j$ ) is the fluid conductance between nodes $\mathbf{i , j}$. | (NNxNN) <br> matrix | $\mathrm{Btu} / \mathrm{hr}{ }^{\circ} \mathrm{F}$ |
| *FLOW | Flag for fluid flow in solution ```Flow = 0 No fluid flow = 1 Fluid energy is considered in solution``` | -- | -- |
| FLUX | FLUX(i), direct solar flux incident on $i^{\text {th }}$ node. | (NN) <br> vector | Btu/hr ft ${ }^{2}$ |
| *FNDIN | Number of node which represents the fluid inlet section of receiver | -- | -- |
| *FNDOT | Number of node which represents the fluid outlet section of receiver. | -- | -- |
| *FTIME | Final clock or run time | Scalar | hr |

*Variables that must be given by the user as input data.

| NAME | DESCRIPTION | DIMENSION | UNITS |
| :---: | :---: | :---: | :---: |
| FU | $\mathrm{FU}(\mathrm{i})$ is the sum of view factor from node (i) to all other nodes in enclosure $=\Sigma F(i, j)$ | (NN) vector | -- |
| F1 | Transfer matrix between excitation and response vectors for IR radiation | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | -- |
| F1S | Transfer matrix between excitation and response vectors for solar radiation exchange | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | -- |
| HEAP | Heat Energy Analysis Program title | -- | -- |
| *HCONV | HCONV ( $i, j$ ) is the convective heat transfer coefficient between adjacent nodes $i, j$ exchanging heat by convection | (NNxNN) matrix | $\mathrm{Btu} / \mathrm{hr} \mathrm{ft}{ }^{2} \mathrm{o}_{\mathrm{F}}$ |
| *HRIN | $\operatorname{HRIN}(\mathrm{k})$ is the time in hr corresponding to the $k^{\text {th }}$ input transient data. | (NDIN) | hr |
| *HROUT | Starting time for printing out program results | Scalar | hr |
| I, II, II | Index | -- | -- |
| IDN | Identity matrix (Kronecker delta of size NN) | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | -- |
| IFLUX | $\operatorname{IFLUX}(i, k)$ is the input solar flux incident on the $i^{\text {th }}$ node at the $k^{\text {th }}$ time interval | (NNxNDIN) | Btu/hr ft ${ }^{2}$ |
| INR | Inner radius for view factor calculation | Scalar | ft |
| INTER | Flag for interpolation subroutine | Scalar | ft |
| IREXC | $\operatorname{IREXC}(i, j)$ is the infra-red radiation exchange coefficient between nodes i,j | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | $\mathrm{ft}{ }^{2}$ |
| IREXC1 | Special matrix formed from IREXC | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | $f t^{2}$ |

*Variables that must be given by the user as input data.

| NAME | DESCRIPTION | DIMENSION | UNITS |
| :---: | :---: | :---: | :---: |
| J, J1, JJ | Index | -- | -- |
| JAC | Jacobian matrix for steady state solution iteration | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | Btu $/{ }^{\circ} \mathrm{F}$ |
|  | $\operatorname{JAC}(i, j)=\frac{\partial \operatorname{RES}(i)}{\partial \operatorname{TEM}(j)}$ |  |  |
| K, KK | Index | -- | -- |
| *KCOND | KCOND(i), conductivity of $i^{\text {th }}$ node material | (NN) <br> vector | Btu/hr $\mathrm{ft}^{\circ} \mathrm{F}$ |
| KD | Kroencker delta (identity matrix) | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | -- |
| KOST | Parameter for view factor subroutine | -- | -- |
| L, LL | Index | -- | -- |
| LIREXC | Linearized matrix coefficient IREXC to suit the stability subroutine | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | Btu $/ \mathrm{hr}{ }^{\circ} \mathrm{F}$ |
| *LNGTH | $\operatorname{LNGTH}(\mathbf{i}, \mathbf{j})$ is the distance between nodes $i, j$ exchanging heat by conduction measured from center of node $i$ to separating surface between i,j | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | $f t$ |
| LR | Parameter for view factor subroutine | Scalar | ft |
| *MASFLO | $\operatorname{MASFLO}(i, j)$ is the flow direction identifier between nodes i,j | (NNxNN) <br> matrix | -- |
|  | $\begin{aligned} =0 & \text { No fluid exchange } \\ =1 & \text { Flow is from node j to } \\ & \text { node i for a single } \\ & \text { fluid loop } \end{aligned}$ |  |  |
|  | $\begin{aligned} & =\text { Mass flow rate from node } j \\ & \text { to node } i \text { for more than } \\ & \text { one fluid loop } \end{aligned}$ |  |  |

[^1]| NAME | DESCRIPTION | DIMENSION | UNITS |
| :---: | :---: | :---: | :---: |
| MINDTU | Minimum time interval below which nodes are considered zero-capacity. For transient solutions. | Scalar | hr |
| *MFM | The Mass flow rate (assuming one fluid circuit in the receiver) at the $k^{\text {th }}$ time interval. For more than one fluid loop, MFM is taken as 1 . | (NDIN) vector | 1b/hr |
| *MNIT | Maximum number of iterations in Newton-Raphson method. | -- | -- |
| N | Index | -- | -- |
| *NDIN | Number of data points in time varying input or dimension of HRIN vector. | -- | -- |
| NE | Number of equilibrium nodes $N E=N N-S S$ | -- | -- |
| *NN | Total number of nodes | -- | -- |
| OTR | Parameter for view factor subroutine | Scalar | $f t$ |
| PHII | Parameter for view factor subroutine. | Scalar | radian |
| PHIJ | Parameter for view factor subroutine. | Scalar | radian |
| PI | The number $\pi=3.1416$ | -- | -- |
| *POWR | POWR(i), internal heat generated in $i^{\text {th }}$ node, by electrical resistance, nuclear reaction, etc. | (NN) vector | Btu/hr |
| *k | Radial coordinates for node surfaces exchanging radiation. | $(2 * \mathrm{NN})$ <br> vector | ft |
| RES | RES(i), residual of $i^{\text {th }}$ node, or sum of heat input to $i^{\text {th }}$ node (for quasi-steady state solution) | (NN) vector | Btu/hr |

*Variables that must be given by the user as input data.

| NAME | DESCRIPTION | DIMENSION | UNITS |
| :---: | :---: | :---: | :---: |
| RH | Radius of higher disc (or ring) forming the surface of the axisymmetric node in DDF view factor. | (4) vector | ft |
| RHO | RHO (i), spectral reflectivity of $i^{\text {th }}$ node in the infrared radiation spectrum | (NN) <br> vector | -- |
| RHOST | RHOST(i), spectral reflectivity of $i^{\text {th }}$ node in the solar radiation band | (NN) <br> vector | -- |
| RL | Radius of lower disc (or ring) forming the surface of the axisymmetric node in DDF view factor. | (4) <br> vector | ft |
| RR | Parameter for view factor subroutine. | Scalar | ft |
| *S | Height between higher disc and lower disc in DDF view factor. | (4) <br> vector | ft |
| *SAIRR | Flag for solar and infrared radiation existence in solution. | -- | -- |
|  | $\begin{aligned} \text { SAIRR }= & 0 \text { no radiation } \\ = & 1 \text { solar \& IR } \\ & \text { exchange are present. } \end{aligned}$ |  |  |
| SEIN | Accumulated solar energy incident on receiver from start to time ( $\tau$ ). | Scalar | Btu |
| SEININ | Accumulated solar energy incident on receiver used for interpolation subroutine. | Scalar | Btu |
| SEIN1 | Accumulated solar energy incident on receiver from start up to ( $\tau_{1}$ ). | Scalar | Btu |
| *SIGMA | Stefan-Boltzman radiation constant $1.714 \times 10^{-9}$ | Scalar | $\left.f t^{204} R\right)$ |
| SOLCON | SOLCON(i), net solar radiation exchange by $i^{\text {th }}$ node. | (NN) <br> vector | Btu/hr |
| *SORSIN | $\begin{aligned} & \text { Source/sink node identifier } \\ & \begin{aligned} \text { SORSIN(i) } & =0 \text { equilibrium node } \\ & =1 \text { source/sink node } \end{aligned} \end{aligned}$ | (NN) <br> vector | -- |

[^2]| NAME | DESCRIPTION | DIMENSION | UNITS |
| :---: | :---: | :---: | :---: |
| *SPHT | SPHT(i), specific heat of $i^{\text {th }}$ node material. | (NN) <br> vector | $B t u / 1 b^{\circ} \mathrm{F}$ |
| *SS | Number of source/sink nodes | -- | -- |
| SSK | Steady state index to stop temperature calculation when input data no longer changes. | -- | -- |
| **SST | Flag for transient and steady state solutions identifier. | -- | -- |
|  | $\begin{aligned} \mathrm{SST} & =1 & \text { steady state } \\ & =0 & \text { transient } \end{aligned}$ |  |  |
| **SSTEM | Source/sink temperature profile | $\begin{aligned} & \text { (SSxNDIN) } \\ & \text { matrix } \end{aligned}$ | ${ }^{\circ} \mathrm{R}$ |
| T | Index | -- | -- |
| *TEM | TEM(i) is the temperature of $i^{\text {th }}$ node | (NN) <br> vector | ${ }^{\circ} \mathrm{R}$ |
| TEMIN | Nodal temperature vector used for interpolation subroutine | (NN) <br> vector | ${ }^{\circ} \mathrm{R}$ |
| TEMNEW | TEMNEW(i), updated temperature for $i^{\text {th }}$ node at $(\tau+\Delta \tau)$. | (NN) <br> vector | ${ }^{\circ} \mathrm{R}$ |
| TEM1 | Nodal temperature vector at end of time $\left(\tau_{1}\right)$. | (NN) vector | ${ }^{\circ} \mathrm{R}$ |
| UNM | Matrix with all elements $=1$ | $\begin{aligned} & \text { (NNxNN) } \\ & \text { matrix } \end{aligned}$ | -- |
| UNV | Vector with all elements $=1$ | $\begin{aligned} & \text { (NN) } \\ & \text { vector } \end{aligned}$ | -- |
| V | $V(i, j)$ is a radiation exchange flag between nodes $\mathbf{i , j}$ | (NNxNN) | -- |
|  | $\begin{aligned} V(i, j)= & 0 \text { nodes } i, j \text { do not "see" ea } \\ = & 1 \text { interior cone surface (i) } \\ = & 2 \text { interior cone surface }(j) \\ = & 3 \text { exterior cone surface ( } 1 \text { asmetric node } F \text { values } \\ = & 4 \text { be entered by the user. } \\ & \text { using summation rule } \end{aligned}$ | ch other <br> to <br> to <br> to <br> ed |  |


| NAME | DESCRIPTION | DIMENSION | UNITS |
| :---: | :---: | :---: | :---: |
| *VOL | VOL(i), volume of $i^{\text {th }}$ node | (NN) <br> vector | $f t^{3}$ |
| *VPRNT | Flag for view factor matrix printing | -- | -- |
| X | Parameter vector for DDF calculation $\mathrm{x}=1+\left[\frac{\mathrm{RH}}{\mathrm{RL}}\right]^{2}+\left[\frac{\mathrm{S}}{\mathrm{RL}}\right]^{2}$ | (4) <br> vector | -- |
| XX | Parameter for radiation subroutine | Scalar | ft |
| *Z | Axial coordinates of lower and higher discs forming node (for axisymmetric view factor subroutine) | (2XNN) vector | $f t$ |
| ZCN | ```Zero-capacity node flag. ZCN is 1 for a zero-capacity node; else ZCN(i) is 0.``` | (NN) <br> vector | -- |
| ZCN1 | Zero-capacity nodes identifier in transient solution. | -- | -- |

[^3]

Flag for solar-infrared radiation heat transfer

```
SAIRR = 0 if problem does not include nodes
                                    exchanging solar & infrared
                                    radiation
else SAIRR = 1
```

| CONVEC |  |
| :---: | :--- |

Flag for conduction heat transfer

$$
\left.\begin{array}{ll}
\text { CONDUC }=0 & \text { if problem does not include nodes } \\
& \text { exchanging heat by conduction }
\end{array}\right] \begin{aligned}
& \text { CONDUC }=1
\end{aligned}
$$



Flag for thermal convection heat transfer

| CONVEC $=0$ | if problem does not include nodes <br> exchanging heat by convection |
| :--- | :--- |
| else | CONVEC $=1$ |



Flag for flow convection

| FLOW $=0 \quad$ | if problem does not include <br>  <br>  <br> fluid nodes flowing from one <br> node to another |
| :--- | :--- |
| else $\quad$FLOW $=1$ |  |


| SST |  |
| :---: | :---: |

Flag for transient/steady state solutions
SST = $1 \quad$ For steady state solution
SST $=0 \quad$ For transient solution


Number of node representing the ambient air


Flag for printing the view factor matrix
VPRNT $=1$ the program will print the view factor matrix
else VPRNT $=0$, No printing


Final clock or run time of program simulation, in hours


Minimum time interval ( $\Delta \tau$ ) in hours below which nodes are considered zerocapacity nodes. MINDTU is only required in transient solutions (SST $=0$ ). 10/3600 could be used as a first trial.


Total number of nodes in the problem including source/sink nodes and equilibrium nodes


Number of data elements entered in HRIN table, or dimension of HRIN vector


Number of source/sink nodes in problem. These nodes can gain or lose any amount of heat without affecting their temperature


Maximum number of Newton-Raphson iterations required. Usually not less than 6 is recommended. Accuracy of computations may not be affected very much with increasing MNIT.


Starting time (in hours) at which program results are printed


Time interval (in hours) for printing program results


Number of node representing the fluid inlet section. If no fluid circuit is present, enter zero


Number of node representing the fluid outlet section. If no fluid circuit is present, enter zero

Table C-1. SORSIN

| NODE | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Z$ $\bar{\sim}$ 0 0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Source/sink node identifier. SORSIN shall be either zero or 1 according to the following:

$$
\begin{array}{ll}
\text { SORSIN }=0 & \text { For an equilibrium mode } \\
\text { SORSIN }=1 & \text { For a source/sink node }
\end{array}
$$

Nodes that represent ambient air, constant temperature or a phase changing process are examples.

Table C-2. TEM

| NODE | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 른 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Initial temperature nodes in ${ }^{\circ} \mathrm{F}$. The temperature vector is used as an initial estimate for iteration in steady-state solutions or as an initial value for transient solutions. Default value is $77^{\circ} \mathrm{F}$ for all the nodes. If the initial temperature of certain node(s) are different from $77^{\circ} \mathrm{F}$, the user shall enter the new value(s).

Table C-3. EIR

| NODE | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

The semi-grey emissivity of each node in the infrared region extending approximately from a wavelength of 4 microns ( 4000 \&) to infinity. For nodes where the emissivity is unknown, too small, or does not exchange heat by radiation, a value of zero shall be entered.

Tab1e C-4. ESOL

| NODE | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

The semi-grey emissivity of each node in the short wave (solar) region below 4 microns ( $4000 \AA$ ) wavelength. For nodes where the emissivity is unknown, too small, or does not exchange heat by radiation, a value of zero shall be entered.

Table C-5. POWR

| NODE | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Internal power generated in each node in $\mathrm{Btu} / \mathrm{hr}$, due to chemical reaction, nuclear excitation or heat generated by electrical resistances. Default value is zero for each element.

Table C-6. SPHT

| NODE | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 곤 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

The specific heat of each node material is in Btu/lb ${ }^{\circ} \mathrm{F}$. Default value is zero for each element.

Tab1e C-7. DEN

| NODE | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Z |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Z |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| ( |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

The density of each node material in $1 \mathrm{~b} / \mathrm{ft}^{3}$. DEN is only needed for transient solutions. For steady state solutions, DEN table shall be left blank. Default value is zero for each element.

Table C-8. KCOND


The thermal conductivity of node material, in Btu/hr ft ${ }^{\circ} \mathrm{F}$. Default value is zero for all nodes.

Table C-9. VOL

| NODE | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| O |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

The volume of each node material in $\mathrm{ft}^{3}$. VOL is needed for transient solutions only. Default value for each element is zero. Source/sink nodes shall be assigned an arbitrary volume, e.g., $1 \mathrm{ft}^{3}$.

Table C-10. ASOL

| NODE | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Surface area in $\mathrm{ft}^{2}$ of a node subject to heat transfer by radiation with other neighboring nodes, or a node receiving direct solar radiation. The ASOL will be internally computed by HEAP when the axisymmetric view factor subroutine is used. Otherwise, the user shall enter the values of ASOL. For nodes that do not exchange heat by radiation completely, a zero value shall be entered.

Table C-11. View Factor Flag [V]


Table C-12. View Factor Matrix [F]


View factor $F(i, j)$ between nodes (i) and ( $j$ ) is only entered whenever the flag $V$ (i,j) is 3 ; the case of two nodes where the view factor is not determined by HEAP subroutines. Only elements in the hatched area need to be entered. Non-applicable cases shall be left blank.

Table C-13. ADJ


ADJ is a "flag" matrix to identify the type of heat transfer between adjacent and non-adjacent nodes. $A D J$ is an integer that can be as follows:

B1ank or 0 For non-adjacent nodes
1 For two adjacent nodes exchanging heat by conduction
2 For two adjacent nodes exchanging heat by convection
Only half of the matrix needs to be entered (the hatched area) because of symmetry. Diagonal elements shall be left blank. Default value is zero for each element.

Table C-14. ASRF


Contact surface area between two adjacent nodes (in $f t^{2}$ ) that exchange heat by conduction $(A D J=1)$ or convection ( $A D J=2$ ). Default value is zero. Non-applicable boxes shall be left blank. Only half of the matrix (shaded area) shall be entered with blank diagonal elements.

Table C-15. LNGTH


The heat transfer average path length between two adjacent nodes exchanging heat by conduction. LNGTH is entered in ft for nodes with ADJ $=1$ only. LNGTH ( $i, j$ ) is measured from the center of node (i) to separating surface between nodes (i) and ( $j$ ).



Coefficient of heat transfer by convection in Btu/hr ft ${ }^{\circ} \mathrm{F}$. HCONV is entered for nodes exchanging heat by convection only (ADJ =2). On1y half of the matrix, excluding the diagonal elements, shall be entered. Non-applicable boxes shall be left blank. Default value is zero for all elements.

Table C-17. MASFLO


MASFLO ( $i, j$ ) is an identifier for the direction of flow between nodes (i) and ( $j$ ) if a single fluid loop is present in the receiver. MASFLO (i,j) is also an identifier for both the direction and mass flow rate if the receiver includes more than one fluid loop. For a single fluid loop, MASFLO ( $i, j$ ) is entered as 1 if the flow direction is from node (j) to node (i) and zero if the flow direction is from node (i) to node (j). For a single fluid loop, MFM (Table C-18) will be the mass flow rate in $1 \mathrm{lb} / \mathrm{hr}$. For receivers with more than one fluid loop, the actual mass flow rate in $1 b / h r$ will be entered in MASFLO (i,j) if the fluid flow direction is from node ( $j$ ) to node (i), and zero otherwise. In the latter case, MFM shall be entered as 1 .

Table C-18. HRIN, SSTEM and MFM

| HRIN | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 24 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| SSTEM <br> $(1)$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| SSTEM <br> $(2)$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| MFM |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

HRIN Time domain data, giving the sequence of clock times (not necessarily of equal intervals) when some variables change or are not to be re-specified by the user.

SSTEM Source/sink temperature variations with time domain, entered in ${ }^{\circ} \mathrm{F}$. SSTEM (1) gives the temperature of the ambient node, and SSTEM (2) gives the temperature of the inlet-to-receiver fluid node. Both the ambient node and inlet-to-receiver node are considered source/sink nodes here, but could be changed in the program structure.

Mass flow multiplier in $1 \mathrm{~b} / \mathrm{hr}$. This is the mass flow rate of the working fluid inside the single receiver loop. Variations of MFM with time are entered according to the number of data points in the time domain. For receivers with more than one loop, the number 1 should be entered as a flow baseline and variations of flow rates with time should be entered as the ratio of new flow rate to the base line. Tables C-17 and C-18 should be completed simultaneously. For receivers with more than one fluid loop, the mass flow rate of each loop is assumed to vary simultaneously with time so that their ratios remain unchanged.

Table C-19. IFLUX


Solar radiation intensity in Btu/hr ft ${ }^{2}$. Variations of IFLUX with time intervals are provided in the tables. The values from 1 to 24 in the HRIN row are time intervals.

Table C-20. Nodal Coordinates

|  | RADIAL <br>  <br>  <br>  <br> NODERDINATES |  | $r_{1}$ | $r_{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  | AXIAL <br> COORDINATES |  |
| 2 |  |  |  | $Z_{2}$ |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |
| 8 |  |  |  |  |
| 9 |  |  |  |  |
| 10 |  |  |  |  |
| 11 |  |  |  |  |
| 12 |  |  |  |  |
| 13 |  |  |  |  |
| 14 |  |  |  |  |
| 15 |  |  |  |  |
| 16 |  |  |  |  |
| 17 |  |  |  |  |
| 18 |  |  |  |  |
| 19 |  |  |  |  |
| 20 |  |  |  |  |

This nodal coordinates table is needed only when the view factor matrix is calculated using the axisymmetric node subroutine. Each axisymmetric node (whether it is a conical or cylindrical ring, or a planar disc) is defined by four coordinates: $\left(r_{1}, z_{1}\right)$ for the bottom boundary, and ( $r_{2}, z_{2}$ ) for the top boundary where $Z_{2}>Z_{1}$. The radial distance $r$ is measured from an arbitrary datum. For nodes that are not exchanging heat by radiation, the coordinates shall be left blank. Default value is zero for each coordinate. All values are entered in $f t$.


SURFACE NORMAL
INTERIOR NODE (PLANE RING)
FACING DOWNWARD


INTERIOR NODE (PLANE RING) FACING UPWARD


```
10N IMPIIT USIMG EHFF & O% + ENTER ZERD FDF TPANSIENT ANFL'SIS
    ZF ENTEF 1 FOF DUASI STEHDY STATE DHE:#*:SST
1こ0 IMFUIT USING EHAF (13) + ENTER NUMBER DF NDNES:%':NN
```



```
1EO INFIIT ISIMIE IHFF (1SO + ENTER ND DF TIME IIEFENDENT IIATA FOIINTS:**NIIN
```



```
    ELSE EHTEP 1::*:SHIPR
EOG IHPUT ISING EHAFG1S% + EHTEF ZEPD IF THEFE IS ND EDNIUIETIDN
    ELSE ENTEF 1::*:EDMIMIO
EこO IHFIIT USIHE EHAFG1SO+`ENTER ZEFD IF THFFE IS ND EOHUEGTIDA
    ELSE EMTEF 1:**:IDNVEI:
E40 INPUTT USINE EHAFG1马, + EMTEF ZERD IF. THERE IS MD FLUIII FLDM
    ELSE ENTER 1:#*:FLDL
```



```
80 ELDEK=0,SIGMA=1.714E-0G
300:
IIMEHSIDM AHII IHITIALIZE ALL AREFYE
```





``` LHETH © HH，NH：KCOHI CHN
```








```
380 TEMCI \(=77\) FDF：\(I=1\) TD HH
400：THE FDLLDUINF SFAEE IS FOF EHTERTHE TATA
EHEEK THE FRDIRRAM MARHIHL
```

```
4E0 HP[]ITT=0. [ITDIIT=1,FNIIIH=31, FMENNH=3E,FHTDT=;`
440 mHIT=E,FTIME=1
4EO MIHIITIL=1%O
4E0 SOFSJH!S1, SDFSIN&SE=1
50% TEM!日%=65%
```








```
                1.E4,1.E4,1.E4,.1E,1.E4,.E4
```


























 $=0$
TO 13
1145



$$
\begin{aligned}
& \begin{array}{l}
15,25), A S R F(15,33)=023 \\
436, A R F(17,27), A S R F(17,32)=.06372, \\
152, A S R F(19.26), A S R F(19,32)=.05236,
\end{array} \\
& \begin{array}{l}
(21,32)=.03436, \\
=.1296
\end{array}
\end{aligned}
$$







IFLU《 301$)=30294$
140 STEM 1,1$\rangle=77,3 \mathrm{STEM}(2,1)=65 \mathrm{e}$



1500

3OMO ：EAIGULATIOH DF THE YIE，FAGTORS FMF FXISVMMETPIC HONES


3040 FRE $I=1$ TD HN
goto FDE $1=1$ TC Hin

3100 IF $\because$ © $, ~ I:=3$ THFN $10 \square$ TO 3580
31 EO IF AESGUG，Jid＝e THEN 90 TO 3320
$3140 \mathrm{FH}(1)=\mathrm{F}(\mathrm{E}+1-1$ ，FH（E）＝F（E）， $\mathrm{FH}(3)=\mathrm{F}(\mathrm{E}+\mathrm{O}, \mathrm{FH}(4)=\mathrm{P}(\mathrm{e}+1-1)$

 $5(4)=2(-1-1)-2(-1-1)$
 FDF $L L=1$ TD 4
 IF FL（LL）：0 FDE $L L=1$ TD 4



3500 150 TO 3580










 （E＋LFO）：

350 IF FiI1，11）（0 THEH F（I 1,11$)=0$
$35 S 0$ HE KT ．

36ごI FDF $k=1$ TD HN

SEC日 IF FCI，K © THEN FCI，K）＝0

3700 HEST K
ETEO HEKT I


30 BO FDE $\mathrm{I}=1 \mathrm{TD} \mathrm{NM}$
3001 FOF $\quad 1=1$ ТП HN
3910 IF F\＆I，$\because=0$ THEN Gロ Tत 3940

3940 THERT． 1
SEO FFETHT
Ben rext I
GOUO FRINT EHETK ON SIMMATIDH FILLE OF F：
GEO FFIHT F $\rightarrow$ INY，
F4！：EOMFLETE THE SEDOMI HAl＿F DF THE METEIEES
FII I－HEFF－HCOM

$390!$
4000 ：GDHEET IHITIAI TEMF IN LIEG．F TD IEG．AESDLITE
4 HEO TEM $=$ TEM $+460 \rightarrow 11+4$ ，
4040 ：
4 OEO LDMFIITE F GIFIIMFLEX MATFILES FDF INFFAFETI FAIIHTIDH ANII SDLAF

$4100 \mathrm{FHD}=\mathrm{COH}+\mathrm{V}-E I F$ ， $\mathrm{FHOST}=\mathrm{GHW}-E S D L$ ）


$41 E 0 \mathrm{FEF}=\mathrm{F}$－INUGF1，FEFST＝F－INWGF1S，
41BO！IETEFMIME INFFARED EXEHANGE EDEFFIEIENT
 FOF $\quad 1=1$ TD NH


```
4-4!!
```



```
4EGI゙! EDMFUTE GOHIIITTIDH EDEFFIGIENTS FOF AIIAREHT SDLIII NOIES
```



```
                                    ELDHI&I%LNETH&I,I%,% IF ALICI,J%=1 FDF I=1 TD HH
                                    FDF: I=1 TD MN
```



```
4%4!
```




```
4401F SOT=1 THEN ELDRK=HFIHBK
44E| IF KYHIIN THEN GO TD 44E゙自
```





```
        FHEN:THE{4O: SEIH:THE:GO:EFF
```



```
                EF:TAE:1ご: TEM &&&-4EO FDF &K=1 TD HN
45חG IF ELDEK=HFOUT THEH HFOIIT=HFDIIT + [ITDUT
4EEO IF HFOGIT ELDOR+IITHUI FRHT HFDIITSLDEK
        THEH TEM1=TEM, LLDI&1=RLDEK, IHTEF=1, EHEN1=FHEQ, SEIH1=SEIH
454:
                    EDMFIITE EHARIGE IH TEMFEFRTUIFE AT EAIGH HOLE IM&ING THE
                    TINE IHTEFUHL IITA|I EHTI GALEIILATE HEM TEMF
```



```
            SK`H[IIH THEH GD Tח SGOO
45G: TFFHNSTENT SIILITIORH
4GOIF IOHI=0 THEH ED TD 4E40
```



```
4GM FES= IFESEI* TEM MILL TEM MILL TEM MILL TEM: +FOMF+SDLEDH+
```



```
4EG日 TEMTE|=TEM+![TH|&FES IIW EAFW
4EBG GD TD 4GEG IF ZOHI=0
4?OITITEM=TEM
4FO FDF IJ=1 TD NHIT
```



```
        &FLD+SDM[MI+I:OH,S: ITEM,
```



```
        GOHWE:I, F FOF I=1 TD HN-1 FDF I=1 TO H+N
4FSG FOF T=1 TD ttt
```




```
4%40 JHO:I,I;=1 IF TENCI!=0
4EG HEXT I
4SOITITEM=[ITEM-IHWUIASO&FESO
4%0% HEOT ! !
4GEO FDF I=1TO NH
4F4 TEM:I: IITEM:I, IF ZIN:T:=1
```



```
4GOI HEYT I
SBOD GLDEK=ELDEK+IITHU!
SNG0 EO TO SFGO
El|i| ! STEATI% ETHTE OMLITIOR*
5060 FDF I_I=1 TD MrAIT
```






```
5tEg FOF I=1 TD FH|
5140 FESIS=0 IF OFEIN!I:=1
```



```
51G抽:I-J:=1 IF -DFSIHEI:=1
SGOM THFOT J
SE=0 TEM=TEM- IH4, INIO &FES,
5ご4! HFQT !1
SGGFES=:TFENE1* TEM MILL TEM MILL TEM M|| TFMY +FOMF+GOLEDH+GFLD+
```




```
    FOF KR=1 TD HN
```



```
53こ0!
```




```
        CTFM&FHTIOT: -TEMSFHTITH:\)
53sn IF SST=1 THEN EMEX=ENEX+RES (FNIINO
```



```
54E0 EFF=ENE%SEIN IF SEIN:% I
5440 IF INTEF=1 THEN GOSUE 55EO
54%01 IF ELDEK&=FTIME THEN FD TD 44EO
5 4 8 0 ~ E N T I ~
SEGO!:INHEAF INTEFPRLATIDN SIERDIITINE FDF FRINT DUIT
S5E0 TEMIN=TEM1+ HFOUT-ELDEK1) GLLDCK-CLDCK1)*(TEM-TEM1)
```



```
550 SEININ=SEINI + (HFDUT-ELDEK1) GCLQCK-LLDCK1: *SEIN-SEIN1,
55GO EFF=ENEQINSSEINTN IF SEIMIN:O
```



```
            ACC SDL ENG'Y INGETBO:TAESSO: FECWF EFF HFOUT:TAE\19:
        ENEXIN:TAE (4): SEININ:THEGO):EFF
```



```
    FOF KK=1 TQ NH
5640 HFOUT=HEDUT+IITDIIT, INTER= II
5 0 6 0 ~ F E T I J R N
56.0% !
5700:
STE0: TIME LIEFEMIENT UHFIGELES FHII STAEIIITY SIEFROITIHE
```



```
STEO 2OHI=0
500 2OHCI:=0 FOE J=1 TD NHt
5S0# TF SHIFF=0 THFN EO TO 5980
```



```
540 !
```




```
ESDL MIIL FIIBS:
```



```
    FDF I=1 TO NN FDF: }=1=1\mathrm{ TO HN
SGEO LIFESG=LIFENO-&GIFENC+UNMM MUL KTO
5940!
FGGO: DETEFMINE EONVELTIDN ANII FLDU MATFIEFS
5980 IF COHYEC=O THEN GO TD E04%
```



```
                                    FQF" =1=1 TD HN
```



```
6040 IF FLOW=0 THFH BO TO E100
```



```
G0E0 FI D=FID-(GFI ח||NM, MIIN KII
5100 50 TO E=S0 IF SST=1
G120:
G140: EOMFIITE MTH TIME INTEFUAL IITAII FDF STAFLE TFANEIENT SDLITIDH
```



```
                    FDE: I=1 TD NN
```



```
                FOE NK=1 TD NH
Eこ00 ITAUI=1.
```






```
            ST=1 THEM TITAUI=1
600k=k+1
GO FETIIFN
```


[^0]:    *Variables that must be given by the user as input data.

[^1]:    *Variable that must be given by the user as input data.

[^2]:    *Variables that must be given by the user as input data.

[^3]:    *Variables that must be given by the user as input data.

