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Parabolic Dish Systems Development

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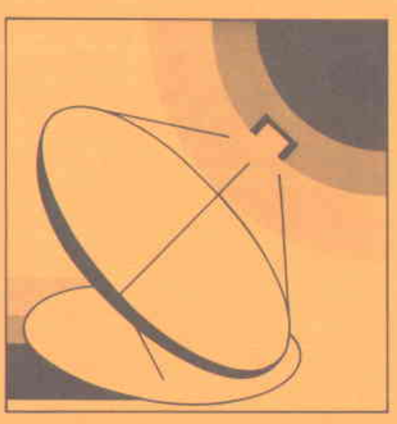

Prepared for
U.S. Department of Energy
Through an Agreement with
National Aeronautics and Space Administration
by
Jet Propulsion Laboratory
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Pasadena, California

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Demonstration of Multiattribute Decision Analysis Applied to Small Solar Thermal Electric Power Plants

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A. Feinberg
R. E. Brooks

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ABSTRACT

An activity within the Solar Thermal Power Systems Project of the Jet Propulsion Laboratory is the design and analysis of selected technologies. Multiattribute decision analysis is a general methodology appropriate for evaluating and ranking alternative solar thermal electric power systems. This Report describes a demonstration of the Keeney and Raiffa formulation of multiattribute decision analysis as applied to a number of alternative small (1 to 10 MWe) solar thermal electric power systems. This Report considers the feasibility of the methodology, rather than the ranking of alternative solar thermal electric power systems, because only preliminary system data was available at the time the study was undertaken.

Fourteen interviews with knowledgeable energy system representatives were conducted in 1979 to assess their preferences for attributes of small solar thermal electric power systems. Because only one interview, of less than three hours in duration, was possible with each person, it was necessary to abbreviate parts of the multiattribute decision analysis methodology. The Report concludes that, with some further modification, it is feasible to apply this methodology to the evaluation and ranking of small solar thermal electric power systems.

ACKNOWLEDGMENT

The specific methodology for this Report was prepared by Abe Feinberg and Ralph F. Miles, Jr. of the Jet Propulsion Laboratory, with suggestions incorporated by Robert E. Brooks of TERA, Inc. of Los Angeles, California. The 14 interviews were conducted and Appendix A was prepared by Robert E. Brooks. The analysis and documentation of this Report was prepared by Ralph F. Miles, Jr. At the time that the major effort was done for this Report, Thomas J. Kuehn (presently on leave from the Jet Propulsion Laboratory) was the Task Manager, Applications Analysis and Development, Point-Focusing Thermal and Electric Applications Project. The authors of this Report wish to thank the 14 representatives who were interviewed for their time and effort.

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SECTION I

A. PURPOSE AND CONTENT

In 1979, the Solar Thermal Power Systems Project of the Jet Propulsion Laboratory was examining several different technology alternatives for small solar thermal electric power applications. This report demonstrates the multiattribute decision analysis methodology of the two volumes References 1-1 and 1-2. See Ref. 1-3 for a comparative assessment in terms of the levelized busbar energy cost. Specifically, this report presents:

(1) The multiattribute decision analysis methodology for determining a JPL preference ranking and to interview representatives of interest groups to determine their preference ranking.

(2) The multiattribute decision analysis value model.

(3) The nine alternative solar thermal electric power systems assessed in the multiattribute decision analysis, described generically and in terms of the attributes of the value model.

(4) The JPL utility function.

(5) The data obtained from interviews of representatives of interest groups.

(6) An analysis of both the JPL preference and the preferences of the representatives of the interest groups.

(7) Conclusions concerning the applicability of this multiattribute decision analysis methodology to the evaluation of small solar thermal electric power systems.

Every multiattribute decision analysis, whatever the specific methodology, generally requires two kinds of models. One is a model that is representative of the systems under consideration. The other is a model of the value structure of the persons or interest groups whose preferences are being assessed. The system model must describe the systems to be considered in terms of the criteria and attributes that appear in the value model. Criteria are goals or aspects of preference about the systems. An attribute is a measure that quantifies the degree to which the corresponding criterion is met. A set of attribute "states" characterizes a given system. The specific multiattribute decision analysis methodology employed determines how value numbers are to be assigned to the attributes (called "attribute utility function values") and how the attribute utility function values are to be combined algebraically to determine value numbers (called "alternative utility function values" or "system utility function values") that permit a rank ordering in preference of the set of alternative systems under consideration.

Within this report Section II describes the multiattribute decision analysis value model and its eight attributes that were used to determine preference rankings for the nine alternative solar thermal electric power systems. Section III describes the nine alternative systems, both in terms of their general engineering characteristics, but more specifically in terms of the attribute states of each alternative system. It is these attribute states that were assessed in terms of attribute utility function values, and the attribute utility function values, in turn, combined algebraically into system utility function values by which the nine systems were preferentially ranked. Section IV presents the multiattribute decision analysis methodology by which the interviews were conducted, and by which preferences for sets of attribute states for each of the nine alternative systems were aggregated to form a preference ranking for the nine systems. Section V presents the JPL utility function as determined from interviews with knowledgeable JPL managers. Section VI presents the utility functions for representatives of various interest groups that were interviewed. It is from these utility functions that the preferences for the nine alternative systems could be determined. Section VII presents the analysis of the interview data obtained from interviewing both the JPL managers and the representatives of the interest groups. A preference ranking for the nine alternative systems was determined, both for JPL and for each of the representatives of the interest groups. No sensitivity analysis was performed, because the document only represents a demonstration of the methodology, and not a ranking of existing or proposed systems. Finally, Section VIII presents conclusions concerning the applicability of multiattribute decision analysis to the evaluation and ranking of small solar thermal electric power plants.

B. NOTATION

$x \triangleq$ An "alternative" or "system" under consideration. For this Report, a system is characterized by a set of 8 attributes, $x = (x_1,\ldots,x_8)$.

$x_i \triangleq$ The ith attribute characterizing a system. An attribute is a measure that quantifies the degree to which a corresponding criterion for evaluating a system is met. The least-preferred attribute state is indicated by x_i^o. The most-preferred attribute state is indicated by x_i^*.

$u(x) \triangleq$ The utility function value of system "x". Utility functions have the property that systems with greater utility function values are more preferred. The least-preferred system that can be constructed from the x_i's, $x^o = (x_1^o,\ldots,x_8^o)$, has a utility function value of $u(x^o) = 0.0$. The most-preferred system that can be constructed from the x_i's, $x^* = (x_1^*,\ldots,x_8^*)$, has a utility function value of $u(x^*) = 1.0$.

$u_i(x_i) \triangleq$ The attribute utility function value of the ith attribute in the state x_i. The attribute utility function values are scaled from $u_i(x_i^o) = 0.0$ to $u_i(x_i^*) = 1.0$.

(x_i, x_j) $\triangleq$ A two-attribute space, x_i and x_j $(i \neq j)$, in which indifference curves and trade-offs can be assessed.

$(x_i, \bar{x}_i^o)$ $\triangleq$ A system for which the i<u>th</u> attribute assumes the state x_i and all other attributes are at their least-preferred state.

k_i $\triangleq$ The scaling constant (or weighting factor) for the i<u>th</u> attribute. k_i quantifies the importance of the i<u>th</u> attribute. $k_i = (x_i^*, \bar{x}_i^o)$ and can assume a value $0 \leqq k_i < 1.0$.

k $\triangleq$ The master scaling constant that appears in the multiattribute utility function. k is uniquely determined by the k_i's. $-1 < k < +\infty$.

$A \succ B$ $\triangleq$ Alternative A is preferred to alternative B.

$A \sim B$ $\triangleq$ Alternative A and B are equally preferred (indifference).

SECTION II

THE MULTIATTRIBUTE DECISION ANALYSIS VALUE MODEL

A. THE VALUE MODEL

The value model used in the multiattribute decision analysis interviews was modified slightly from the extensive discussion given in Ref. 1-2. The hierarchy of criteria and attributes is given in Figure 2-1. The value model has eight attributes (formerly there were seven, as safety and environmental impacts previously had been combined). The value model has six attributes (x_1 through x_6) measured on "objective" scales and two attributes (x_7 and x_8) measured on "subjective" scales. An attribute is said to be measurable on an "objective" scale when there exists a commonly understood and easily quantifiable measure for the states of the attributes. When no such scale exists, the attribute is said to be measured on a "subjective" scale. The eight attributes are considered to be complete enough to distinguish in preference and to properly rank-order all nine of the alternative solar thermal electric power systems under consideration. The attribute definitions are specified so as to minimize redundancy or double-counting. The two subjective attributes are both multiattributed in character.

B. THE ATTRIBUTES

1. First Year Busbar Energy Cost (x_1)

The First Year Busbar Energy Costs (x_1) for each of the alternative systems were derived from engineering analysis of each system and the "USES" economic model (Reference 2-1) for calculating the cost of energy from utility-owned solar electric systems. The attribute range was from 50 to 125 mills/kWh, with the upper end of the scale restricted so that all systems with a cost greater than 125 mills/kWh were still assigned the value of 125 mills/kWh. This was done because costs greater than 125 mills/kWh were perceived by the interviewees as having virtually no attribute utility function value. Since this attribute was used as the reference attribute by which most other

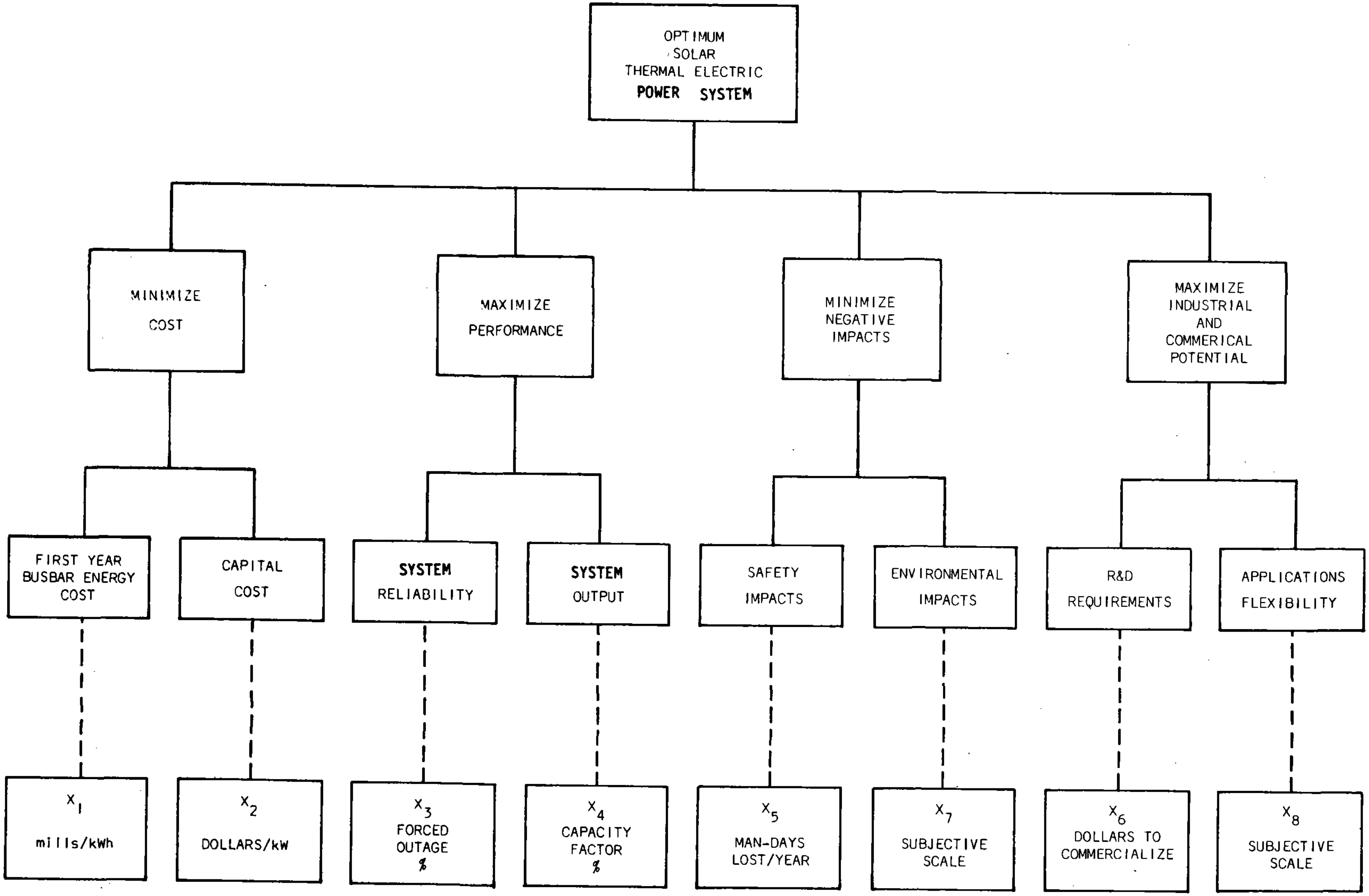


Figure 2-1. Hierachy of Criteria and Attributes for Ranking Small Solar Thermal Electric Power Systems

attribute scaling constants were measured, it was extremely important that the least-preferred state of this reference attribute (x_1^o = 125 miles/kWh) be comprehensible to the interviewee in making trade-offs with other attributes for determining their scaling constants (k_2 - k_8). First year busbar energy cost was used as the attribute measure rather than the "levelized energy cost" measure discussed in Ref. 1-2 for a similar reason. First year busbar energy cost is lower (about a factor of 2) than levelized energy cost and was more comprehensible to the interviewees.

2. Capital Cost (x_2)

The capital cost attribute (x_2) was measured in $/kW$_e$. The cost was determined as the sum of all costs associated with the design, construction, and bringing the system to an operational state. The system output was taken as its "rated" output. The fact that these systems have a "sun-following" power profile was accounted for in the attribute for capacity factor. The attribute capital cost was included in the value model not as it affects the first year busbar energy cost (x_1), for this would be double-counting, but only as it affects the balance of costs between "first costs" and "operation and maintenance costs," and the degree of difficulty, if any, in raising the required initial capital investment. The capital cost attribute was measured over the range of 1,500 $/kW$_e$ to 3,000 $/kW$_e$.

3. System Reliability (x_3)

The system reliability attribute (x_3) was measured in terms of forced outage percentage. Regularly scheduled maintenance was not assumed to be part of forced outage. A system was assumed to be "down" when its power output was zero. Thus modularized systems were assumed to be "on-line" even when they were producing only a fraction of their rated power due to a system problem. The "importance" of system reliability depends both upon its intended purpose and the degree to which alternative sources of power could be provided. In assessing the importance of system reliability, one must consider that these systems may be constructed to operate for a variety of applications. The system may be backed-up by the balance of a utility company's output generation or purchased power capability, or the system may be electrically isolated from a power grid and may be backed-up by fossil fuel generators providing total or only partial back-up. The system reliability attribute was measured over the range of 0% to 25% forced outage.

4. System Output (x_4)

The system output (x_4) was measured by the capacity factor in %. The formula for calculating x_4 was:

$$x_4 = 100\% \frac{(\text{Watt-hours produced/year})}{(8760 \text{ hours/year})(\text{Rated Output})} . \qquad (2\text{-}1)$$

The capacity factor percentage was measured over the range of 20% to 35%.

5. Safety Impacts (x_5)

The safety impacts attribute (x_5) was measured on a scale of man-days-lost/year. The safety impacts attribute was determined for each of the nine alternative systems by a 5-step process.

(1) Conduct a generic systems failure analysis.

(2) Determine the human interfaces with the failures.

(3) Identify potential hazards as a result of the human interfaces.

(4) Determine the degree of exposure to all of the hazards.

(5) Determine the safety impacts attribute state (man-days-lost/year) for each of the systems.

The safety impacts attribute was measured over the range of 0 man-days-lost/year to 60 man-days-lost/year.

6. R&D Requirements (x_6)

The R&D requirements attribute (x_6) was measured in dollars of R&D funds required for commercialization. Each of the following two questions were asked concerning each of the systems:

(1) What degree of technology development is required to complete the R&D stage?

(2) Are any new fabrication or manufacturing methods required for commercial production?

The R&D requirements cost for each system was taken to be the sum of the following four costs:

(1) R&D completion.

(2) Manufacturing development.

(3) Production demonstration.

(4) Demonstration plants.

The R&D requirements attribute was measured over the range from $\$200 \times 10^6$ to $\$600 \times 10^6$.

7. Environmental Impacts (x_7)

The environmental impacts attribute was measured on a subjective scale of 0 (least-preferred) to 10 (most-preferred). This attribute was a combination of subattributes, which were:

(1) Land requirements.

(2) Air quality impact.

(3) Water quality impact.

(4) Water use.

(5) Noise

(6) Aesthetics.

The least-preferred systems ($x_7^o = 0$) use large amounts of land and water and will present potential air quality and water quality concerns. The general public will view the systems in this category as unslightly and undesirable when placed near urban areas or within frequently viewed natural areas. Systems in this category will have environmental impacts similar to an oil tank farm of comparable size.

Systems with an attribute state value of $x_7 = 3$ cover large amounts of land and require extensive site preparation. These systems may generate considerable public concern over the extent of the land required relative to the energy generated and may be viewed as unsightly when placed near urban areas. Systems in this category will have environmental impacts similar to a light industrial manufacturing facility or a truck repair and maintenance yard covering a similarly sized area of land.

Systems with an attribute state value of $x_7 = 7$ are more land intensive and require more land disruption (vegetation removal and grading) than those in the "most-preferred" category ($x_7^* = 10$). These systems typically use larger amounts of water than the "most-preferred" category, present potential water quality problems, and the likelihood of public concern regarding the aesthetics of the facility is greater. Systems in this category will have environmental impacts similar to a light industrial manufacturing facility utilizing a similarly sized amount of land.

The most-preferred systems ($x_7^* = 10$) will result in environmental impacts that will be primarily confined to land disruption within the site boundary. Gaseous and liquid effluents are minimal and water use is relatively low. Systems in this category will have environmental impacts similar to a covered water reservoir requiring a similar amount of land.

8. Applications Flexibility (x_8)

The applications flexibility attribute (x_8) measured the degree of matching between system factors and plant requirements on a subjective scale between 0 (least-preferred) and 10 (most-preferred).

This attribute was multiattribute in character. The system subattributes considered were system output, hybrid flexibility, and modularity. The system requirements considered were construction time, reliability, transportability, adaptability to site land area size, shape, and contour, use of fuels that are readily available, plant availability, electric load, thermal load, and utilization of operations and maintenance personnel.

The least-preferred systems ($x_8^o = 0$) produce only electricity and no usable thermal energy. In hybrid configuration they can utilize only liquid or gaseous fuels and require a relatively long burning time to reach an operational capacity factor. These systems are generally considered to be custom-made fixed installations and are land-shape constrained.

The most-preferred systems ($x_8^* = 10$) produce both electricity and high temperature thermal energy. In hybrid configurations they can utilize solid as well as liquid or gaseous fuels and are able to reach an operational capacity factor in a relatively short time. These systems are highly modular, are mass produced, and use a small land area.

SECTION III

A. INTRODUCTION

The concepts defining a solar thermal electric power system are presented in this Section. The nine systems used in the multiattribute decision analysis are described generically, and in terms of the attributes of the value model. Since only preliminary system designs were available at the time that the analysis for this report was performed, no conclusions can be made concerning the preferences of the interviewees for the current designs. In the last part of this Section and in the following Sections the systems are identified as System I to System IX. This numbering scheme bears no correspondence to the order in which the systems are described generically in this Section. Subsection B: "System Concepts" is taken verbatim from Ref. 1-3.

B. SYSTEM CONCEPTS

A solar thermal electric power system consists of collector, power conversion, energy transport, and energy storage subsystems (see Figure 3-1). The solar collectors considered in this study consist of a concentrator and receiver. The concentrator, using mirrors or lenses, collects sunlight and focuses it at the receiver. The receiver, a specially-designed heat exchanger, absorbs the solar flux and converts it to thermal energy. The power conversion subsystem, which consists of a heat engine and electrical generator, then converts the thermal energy into electricity. Storage subsystems are used for storing excess energy for later use.

The two major collector designs currently being examined are the central receiver and distributed receiver. Central receiver systems comprise a large field of sun-tracking mirrors (heliostats), which focus sunlight on a centralized receiver. Distributed receiver systems consist of a field of many smaller concentrator/receiver modules. There is a trade-off between these two collector system designs to be considered between the savings resulting from the mass production of many small concentrator/receiver modules and the economy of scale provided by large central receivers. A further dimension in

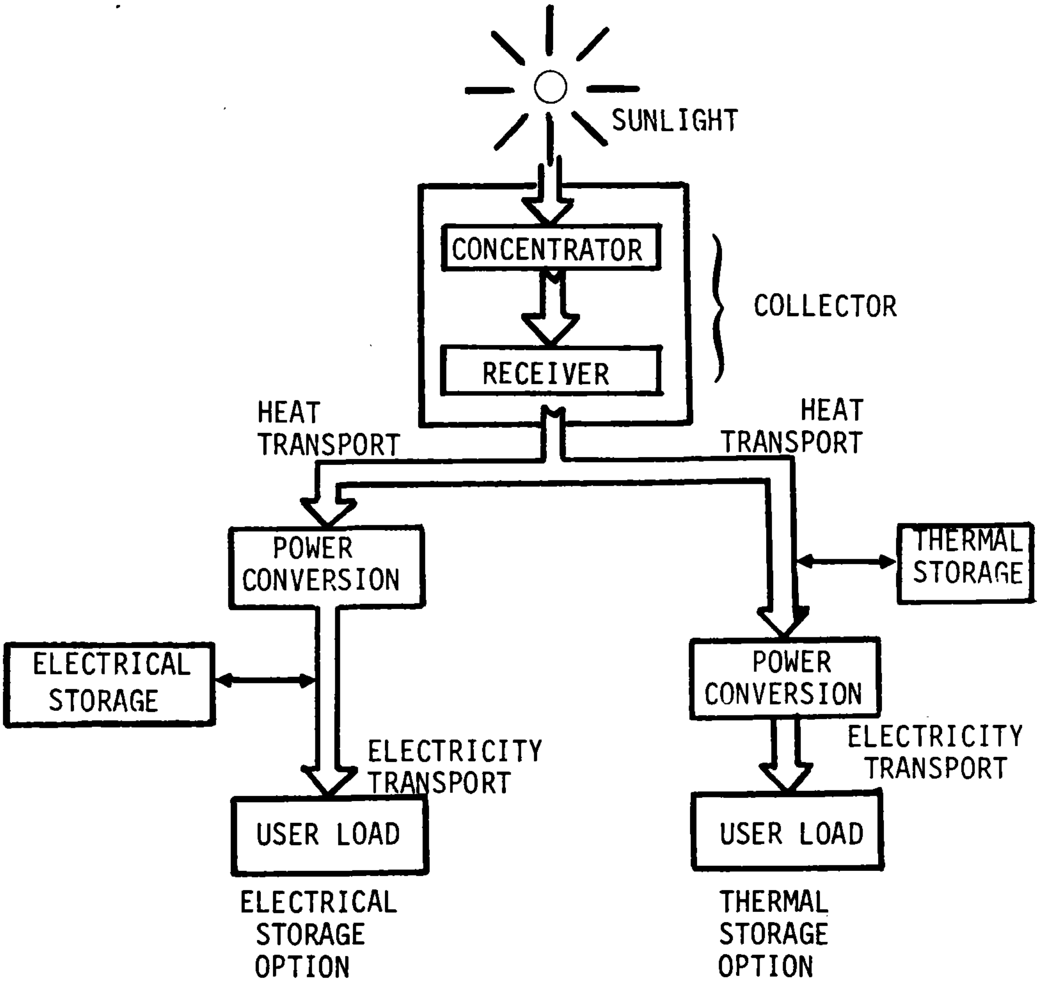


Figure 3-1. Solar Thermal Electric Power System

which collector designs may be distinguished is the type of sun-tracking mechanism employed. Collectors may be fixed (non-tracking), one-axis tracking, or two-axis tracking. The tracking capability may be included in either the concentrator or the receiver. Fixed collectors are usually flat-plate or low-concentration devices, which produce low collector operating temperatures (50 to 250°C) and low system efficiencies (2 to 10%). One-axis systems employ higher concentration ratios and linear receivers for higher temperatures (150 to 425°C) and higher system efficiencies (10 to 18%). Two-axis collectors with point-focusing capabilities can provide high temperatures (425 to 1100°C) as well as high system efficiencies (15 to 30% or better). A second trade-off exists between the higher cost, complexity, and higher performance of the two-axis systems, and the lower cost, relative simplicity, and lesser performance of the one-axis or non-tracking systems.

The power conversion subsystem may be either centrally located or distributed in the collector field. In central conversion, thermal energy from the receiver is converted into electricity at a nearby large, central heat engine/generator, while distributed power conversion is accomplished with many smaller heat engine/generators dispersed within the collector field. Distributed power conversion is only feasible with distributed receiver systems. In this study it was assumed that the point-focusing distributed receiver systems, which use distributed conversion, have the engine/generator mounted onto a module with the receiver near the concentrator focal point. There is a trade-off between the cost reduction potential of mass producing many small units versus the economy of scale realized by one large unit.

Solar thermal power systems may also differ from one another with respect to the type of thermodynamic conversion cycle employed. The conversion cycles most often considered are Rankine, Brayton, and Stirling engines. Although the Rankine-cycle engines studied are limited to lower temperatures (250 to 500°C) and have lower efficiencies (15 to 40%) than the Brayton or Stirling engines, the Rankine systems are commercially available and future cost/performance estimates are fairly certain. The Rankine-cycle engines considered in this study were applied to central power conversion systems with a capacity of 1 to 10 MWe with either distributed or central receiver systems.

Efficiencies for Brayton-cycle engines (25 to 45%) are potentially better than those for Rankine systems because of higher temperature capabilities (750°C or more) and differences in the thermodynamic cycle. The Brayton-cycle engines, however, require higher temperature receivers as well as additional development. Although large central Brayton-cycle engines could be used in distributed receiver systems, current development is focused on small dish-mounted engines. Stirling-cycle engines seem to offer higher performance potential than the Brayton engine when operating at the same temperature but would require more frequent major overhauls. It seems that the Brayton and Stirling engines are best suited to point-focusing distributed systems in which their small size and high temperature needs are well matched.

Energy storage for a solar power plant can be accomplished by storing the thermal energy received by the collector field or by utilizing electrical storage. The latter is most applicable to distributed energy conversion systems that utilize Brayton or Stirling engines. All other solar thermal power plant concepts were assumed to store thermal energy prior to conversion into electrical energy. Based on operational reliability and technological maturity, dual media thermal storage subsystems (composed of salt and rock for high and medium temperature systems, and oil and rock for low temperature systems) were chosen for this study. The electrical storage system assumed was a redox system. This system, which uses an iron and chromium electrolytic solution, is under development at NASA Lewis Research Center (LeRC) for DOE.

C. THE SYSTEMS

Figure 3-2 presents a morphological diagram of the small thermal thermal electric power systems evaluated in this Report. Even though configurations other than the nine systems shown can be synthesized, they were not considered because they are either sufficiently represented by those systems evaluated, or they have a clearly identifiable cost disadvantage. The system abbreviations, as shown in Figure 3-2 are defined as follows:

Collector/Engine Concept	Abbreviation
Low Concentration Non-Tracking/Central Rankine Engine	LCNT
Line-Focusing Distributed Receiver, Tracking Concentrator/Central Rankine Engine	LFDR-TC
Line-Focusing Distributed Receiver, Tracking Receiver/Central Rankine Engine	LFDR-TR
Line-Focusing Central Receiver/Central Rankine Engine	LFCR
Fixed-Mirror Distributed Focus/Central Rankine Engine	FMDF
Point-Focusing Central Receiver/Central Rankine Engine	PFCR
Point-Focusing Distributed Receiver/Central Rankine Engine	PFDR/R
Point-Focusing Distributed Receiver/Brayton Engine	PFDR/B
Point-Focusing Distributed Receiver/Stirling Engine	PFDR/S

The system attribute states are given in Figure 3-3. The attributes of the value model were defined and discussed in Section II.

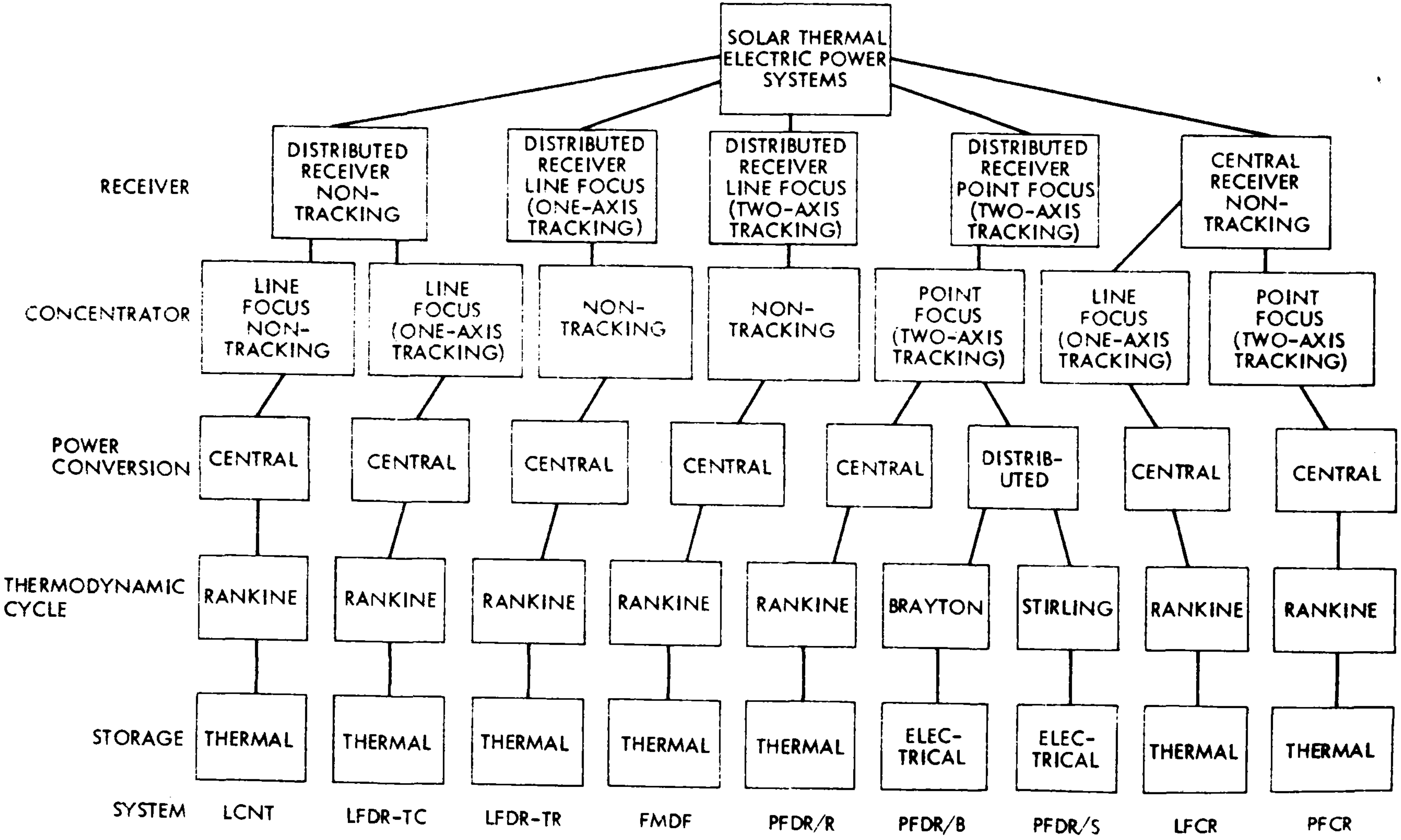


Figure 3-2. Morphology of Solar Thermal Power Plant Technology Options

	Attribute States							
	x_1 Energy Cost	x_2 Capital Cost	x_3 Forced Outage	x_4 Capacity Factor	x_5 Safety	x_6 R&D	x_7 Environment	x_8 Applications
System	mills/ kWh	$/kW	%	%	Man-days Lost/Year	10^6 $	0 - 10	0 - 10
I	99	2540	1.4	30	2.46	288	8.6	9.0
II	76	1854	11.8	28	13.6	231	8.1	5.8
III	59	1578	0	32	2.11	360	8.6	4.8
IV	69	1810	0	33	2.11	541	8.6	4.8
V	88	2140	13.6	28	16.0	341	6.1	4.1
VI	125	2280	25.0	21	35.2	325	5.8	4.3
VII	167	2760	1.4	19	0.82	327	4.0	3.44
VIII	149	2912	6.6	21	9.91	357	3.9	2.54
IX	86	1816	1.4	24	2.68	339	6.8	3.54

Figure 3-3. System Attribute States

SECTION IV

THE MULTIATTRIBUTE DECISION ANALYSIS METHODOLOGY

A. INTRODUCTION

This Section discusses the theory of the multiattribute decision analysis methodology used in this report and the interview process used to obtain the required data. The multiattribute decision analysis methodology differs slightly from some of the simplifying assumptions made in Ref. 1-2 after early interviews revealed that three simplifying assumptions were inappropriate:

(1) The attribute utility function for first year busbar energy cost (x_1) could not be approximated by a function with constant risk aversion. Also, it was convenient to graph the attributes environmental impact (x_7) and applications flexibility (x_8).

(2) Utility and preferential independence were occasionally violated. Thus it was necessary to make utility independence and preferential independence checks.

(3) Some attribute scaling constants assessed indirectly through tradeoff curves were mathematically inconsistent with the reference attribute scaling constants. These inconsistencies were resolved by direct assessment of these attribute scaling constants.

Other than these three modifications, the theory and the interview process is the same as that discussed in Ref. 1-2.

B. THE THEORY

The multiattribute decision analysis theory used is that of Keeney and Raiffa (Reference 4-1). The application of the Keeney and Raiffa theory requires that the following assumptions be valid:

(1) The multiattribute decision analysis value model must be decomposible into a set of attributes that are complete in

that they cover all the important aspects of the problem, operational in that they are measurable on scales that are meaningful to the decision maker, and nonredundant in that two attributes do not measure the same aspect of the problem, thus resulting in double counting.

(2) The von Neumann and Morgenstern axioms of decision making for an individual decision maker (Reference 4-2) are applicable, both for the systems and for the states of the attributes. Elementary expositions of these axioms are given in Hadley (Reference 4-3) and Luce and Raiffa (Reference 4-4). An intermediate exposition is given in DeGroot (Reference 4-5). An advanced exposition is given in Fishburn (Reference 4-6).

(3) Certain independence conditions exist among the attributes that permit the decision maker to consider the relative value of the states of a specific attribute (construct an attribute utility function) independent of the states of the other attributes, and to consider pair-wise tradeoffs between attributes independent of the states of the other attributes.

If the three assumptions listed above are satisfied, it can be proved that the Keeney and Raiffa multiattribute utility function is a valid mathematical representation of the value model of the decision maker. The form of the Keeney and Raiffa multiattribute utility function is such that if,

$$\sum_{i=1}^{8} k_i \neq 1.0 \qquad (4\text{-}1)$$

then,

$$u(x) = \frac{1}{k}\left\{\prod_{i=1}^{8}\left[1 + kk_i\ u_i(x_i)\right] - 1\right\} \tag{4-2}$$

where the master scaling constant "k" is determined from:

$$1 + k = \prod_{i=1}^{8}\left[1 + kk_i\right] . \tag{4-3}$$

If, on the other hand,

$$\sum_{i=1}^{8} k_i = 1.0 \tag{4-4}$$

then,

$$u(x) = \sum_{i=1}^{8} k_i\ u_i(x_i) . \tag{4-5}$$

Appendix D presents a computer program for solving the Keeney and Raiffa multiattribute utility function. It is often referred to as the "Keeney Multiplicative Utility Function" as the proof was first derived by Keeney (Reference 4-7). The notation for these equations was presented in Section I.

Equation (4-2) can hardly be said to be very transparent. Equation (4-2) can best be understood when it is expanded out into individual terms and then recombined into the form:

$$u(x) = \sum_{i=1}^{8} k_i \; u_i(x_i) + \{\text{interactive terms}\}. \qquad (4\text{-}6)$$

The set of interactive terms contain all possible products of the attribute utility functions. Each term contains a power of the master scaling constant k. If the attribute scaling constants k_i $(i = 1,\ldots,8)$ sum to 1.0 as in Equation (4-4), then the only solution to Equation (4-3) in the applicable region $(-1 < k < +\infty)$ is $k = 0$, and all of the interactive terms of Equation (4-6) drop out, resulting in the additive form of Equation (4-5).

For two attributes, Equations (4-2) and (4-3) reduce to:

$$u(x_1,x_2) = k_1 \; u_1(x_1) + k_2 \; u_2(x_2) + (1 - k_1 - k_2) \; u_1(x_1) \; u_2(x_2). \qquad (4\text{-}7)$$

Here it is obvious that the product term $u_1(x_1) \; u_2(x_2)$ enters only when

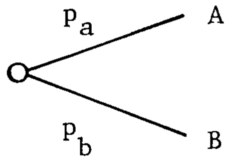
$$k_1 + k_2 \neq 1.0 \, . \qquad (4\text{-}8)$$

There are different axiomatic formulations resulting in the Keeney and Raiffa multiattribute utility function that would require different sets of questions for the interview process. The questions asked in the interview process of this report are necessary and sufficient to test in an approximate manner the validity of the Keeney and Raiffa multiattribute utility function and to assess all the necessary data. The assessed data determine the form of the attribute utility functions $u_i(x_i)$ and the scaling constants k_i for the attributes.

Three definitions need to be introduced at this point:

Definition 1: A Gamble

A "gamble" is a chance situation which yields Outcome A with probability p_a and Outcome B with probability p_b such that $p_a + p_b = 1.0$. Gambles may be more complicated, with many outcomes with their associated probabilities all summing to 1.0. Only two-outcome gambles will be considered in this Report. An outcome may be a specific attribute state or a specific system. Gambles are frequently called "lotteries" in the literature. The graphical notation for the two-outcome gamble described above is:



Gambles such as these are used to quantify preferences on a cardinal scale called a utility function.

Definition 2: Utility Independence

An attribute is "utility independent" when preferences for gambles on the states of the attribute do not depend on the states of the other attributes, given that the states of the other attributes are held constant for all outcomes of all gambles. This condition permits the assessment of an attribute utility function $u_i(x_i)$ that is independent of the states of the other attributes.

Definition 3: Preferential Independence

A pair of attributes x_i and x_j are "preferentially independent" if preferences in the (x_i,x_j) trade-off space do not depend on the states of the other attributes, given that the other attributes are held constant. This condition permits the assessment of two-attribute trade-off curves independent of the states of the other attributes.

The interview procedure of this Report is based on the following axiomatic formulation of the Keeney and Raiffa multiattribute utility function:

Axiom 1. One attribute, call it the reference attribute x_1, is utility independent of the other attributes.

Axiom 2. All pairs of attributes of the form (x_1,x_i), $i = 2,\ldots,8$, are preferentially independent of the other attributes.

These two axioms are a mathematically precise statement of the third assumption given earlier for the validity of the Keeney and Raiffa multiattribute utility function. It can also be proved that Axioms 1 and 2 imply that all the attributes satisfy utility independence. This permits the construction of utility functions $u_i(x_i)$, $i = 2,\ldots,8$, without the requirement to test each for utility independence.

This axiomatic formulation of the Keeney and Raiffa multiattribute utility function thus requires of the interview process:

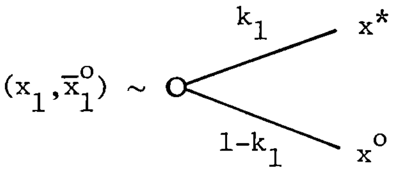
(1) Data to construct the 8 attribute utility functions.

(2) Validation that the reference attribute x_1 satisfies utility independence.

(3) The assessment of one indifference relationship in each of the 7 two-attribute trade-off spaces (x_1,x_i), $i = 2,\ldots,8$, to determine the scaling constants k_i, $i = 2,\ldots,8$, as a function of k_1.

(4) Validation that the two-attribute trade-off spaces (x_1,x_i), $i = 2,\ldots,8$, satisfy preferential independence.

(5) One lottery to determine the scaling constant k_1 for the reference attribute. The scaling constant can be assessed by means of the indifference relationship:



(4-9)

since $k_1 = u(x_1,\bar{x}_i^o)$, $u(x^*) = 1.0$, $u(x^o) = 0.0$, and the utility function equation corresponding to Equation (4-9) is:

$$u(x_1,\bar{x}_1^o) = k_1\, u(x^*) + (1-k_1)\, u(x^o). \qquad (4\text{-}10)$$

With the attribute utility functions $u_i(x_i)$ and the scaling constants k_i derived from the interview data, and the form of the multi-attributed utility function validated, it is then possible to substitute in attribute states for each of the systems as determined by engineering and economic analysis at JPL. This is done in Section V to determine the preferences for several knowledgeable persons at JPL and in Section VI for representatives of various interest groups.

An important point should be made with respect to the attribute scaling constants and the associated attribute state ranges. The preference ordering for the alternative systems will not be affected by the state ranges selected for the attributes. The scaling constants, however, will be affected by the selected attribute state ranges. From a practical standpoint, it is most desirable to restrict the attribute state ranges as much as is possible, but theoretically it will make no difference. Different attribute scaling constants will result and different utility function values will be calculated for the alternative systems, but the preference ordering for the alternative systems will remain the same.

For monotonic preferences, increases in the attribute state range will increase the associated attribute scaling constant. Thus what intuitively appears to be the most "important" attribute may not have the largest attribute scaling constant because of a small attribute state range.

The attribute state ranges must be sufficiently large to encompass the attribute states of all of the alternative systems. Making the attribute state ranges much larger than this will only decrease the accuracy of the preference assessment, because of the increased difficulty the decision maker will have in understanding and expressing his preferences over greater attribute state ranges.

The remainder of this Section is devoted to a discussion of the interview process for obtaining the required preference data.

C. THE INTERVIEW PROCESS

The most difficult aspect of the interview process was to compress it into a time span of two hours. This was about the limit of the span of attention for the interviewees, and only one interview was possible for most interviewees. In what follows every attempt was made to shorten the interview process while retaining as much of the theoretical rigor as possible. The graphical material used in the interviews is presented in Appendix A. Most of the references to figures will be to those in Appendix A.

The interviews started with a discussion of preferences and uncertainty, and the concept of a gamble (Figures A-1 through A-3). Expected value was explained (Figure A-4), and the fact was discussed that the certainty equivalent (what one would be willing to received "for sure" in lieu of the gamble) to a gamble may not be the expected value of the gamble (Figure A-5). The word "gamble" was used instead of the word "lottery" which often appears in the literature. The interviewees more quickly comprehended the word "gamble" and the associated concept than they did the word "lottery." The attributes of the value model were presented (Figure A-6) and the questions for the relevant data began.

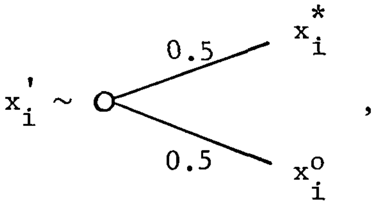
The first data assessed was for the purpose of constructing the attribute utility function for "first year busbar energy cost" (x_1). The attribute x_1 was used as the reference attribute. Data were assessed on several 50/50 gambles as discussed in Keeney and Raiffa (Reference 4-1) using the diagrams of Figures A-7 through A-10. The attribute state for which the interviewee was indifferent between the "gamble" and receiving the "sure thing" was assessed. In Figure A-7 it was pointed out that the interviewee would prefer the most-preferred attribute state ($x_1 = 50$ mills/kWh) as a "sure thing" to a gamble. On the other hand, the gamble would be preferred to the least-preferred attribute state ($x_1 = 125$ mills/kWh). Thus between the most-preferred and the least-preferred attribute states there would be an attribute state or at least a range of states for which the interviewee would be indifferent to the gamble. Questions were asked such as "Would you prefer 60 mills/kWh for sure or the gamble?" Questions similar to this one but with different "for sure" states were asked until the interviewee had "homed in" on his indifference point. As the questions proceeded, the responses were marked on the vertical scale in the right of the diagram so the interviewee could observe his progress toward the indifference point. When the indifference point was reached, the questions for that diagram were terminated and the indifference point was recorded. This indifference point had a utility function value of $u_1(x_1) = 0.5$. The same procedure was repeated for other 50/50 gambles for x_1, as shown in Figures A-8 through A-10. The indifference point of Figure A-7 was inserted into the gamble of Figure A-8 to determine x_1 for $u_1(x_1) = 0.75$, and into the gamble of Figure A-9 to determine x_1 for $u_1(x_1) = 0.25$. The indifference point of Figure A-9 was inserted into the gamble of A-10 to determine x_1 for $u_1(x_1) = 0.125$. With these assessed points, and the fact that $u_1(x_1^o) = 0.0$ and $u_1(x_1^*) = 1.0$, the utility function for x_1 could be plotted.

A utility independence test was later made for x_1 by means of Figure A-18, after the other attributes had been fully explained. Where utility independence was violated, a sensitivity analysis was done to insure that the violation did not change the preference ordering of the alternative systems. Utility independence was tested by assessing the variation of x_1' for $u_1(x_1') = 0.5$ as the states of the other attributes were varied from their least-preferred states to their most preferred states.

A similar procedure was used for the next five attributes $(x_2 - x_6)$ to assess their attribute utility functions. For these attributes the assumption was made that the attribute utility functions were of the form:

$$u_i(x_i) = a_i + b_i\, e^{c_i x_i} \quad \text{or} \quad u_i(x_i) = a_i + b_i\, x_i\ . \qquad (4\text{-}11)$$

All of these attributes could be expected to yield monotonic attribute utility functions. As long as the state ranges of these attributes were sufficiently restricted so that no breakpoints or regions occurred where preferences were altered such as to change the sign of the second derivative of the utility function, then Equation (4-11) should be a reasonable approximation. Utility functions of the form of Equation (4-11) exhibit a property called "constant risk aversion." See See Keeney and Raiffa (Reference 4-1) for an extended discussion of this property. Since $u_i(x_i^o) = 0.0$ and $u_i(x_i^*) = 1.0$, only one gamble assessment was required to determine the entire function. With x_i' determined such that $u_i(x_i') = 0.5$, which was assessed from the indifference relation,



(4-12)

the three constants for $u_i(x_i) = a_i + b_i\, e^{c_i x_i}$ could be calculated from:

$$a_i = -b_i\, e^{c_i x_i^o} \tag{4-13}$$

$$b_i = \frac{1}{e^{c_i x_i^*} - e^{c_i x_i^o}} \tag{4-14}$$

$$0 = e^{c_i x_i^*} - 2e^{c_i x_i'} + e^{c_i x_i^o} \, . \tag{4-15}$$

If $x_i' = (1/2)(x_i^o + x_i^*)$, then the utility function was of the form $u_i'(x_i) = a_i + b_i x_i$, where:

$$a_i = -b_i\, x_i^o \tag{4-16}$$

$$b_i = \frac{1}{x_i^* - x_i^o} \, . \tag{4-17}$$

An HP-97 computer program for solving these equations is presented in Appendix B.

Because of the subjective nature and the multiattribute complexity of the environmental impacts attribute (x_7) and the applications flexibility attribute (x_8), the limited time available for the interview process precluded assessing an attribute utility function for these two attributes. The "best case" (most-preferred state) and "worst case" (least-preferred state) for both x_7 and x_8 were explained as shown in Figure A-16 and Figure A-17. Although JPL attribute utility functions were used in the analysis for x_7 and x_8, later in the interview process scaling constant k_7 for x_7 and k_8 for x_8 were assessed.

Next, data was assessed for determining the attribute scaling constants k_2 through k_8. This was done with the graphical aids of

Figures A-19 through A-25. It was first noted that on these diagrams points B and C were generally preferred to point A. Also point D was generally preferred to points B and C. Thus one can define $A = (x_1^o, x_i^o)$, $B = (x_1^o, x_i^*)$, $C = (x_1^*, x_i^o)$, and $D = (x_1^*, x_i^*)$. If C was preferred to B (this meant that $k_i < k_1$) then on the line between A and C there must be a point $B' = (x_1^i, x_i^o)$ which was indifferent to B. The indifference point B' was found by a set of "homing in" questions similar to those used for the attribute utility functions. When B' was identified as the indifference point to B, the attribute state x_1^i was recorded. If instead, B was preferred to C (this meant that $k_i > k_1$), then on the line between A and B there must be a point $C' = (x_1^o, x_i^1)$ which was indifferent to C. The indifference point C' was found in a manner described for B', and the attribute state x_i^1 was recorded.

For scaling constants $k_i, i = 2,\ldots,8$, where k_i is less than k_1,

$$k_i = k_1 u_1(x_1^i) \qquad (4\text{-}18)$$

and for scaling constants greater than k_1,

$$k_i = \frac{k_1}{u_i(x_i^1)} . \qquad (4\text{-}19)$$

It is important to note that x_1^i and x_i^1 are considerably different even though they appear (only one of them) as a component for a point in the same trade-off diagram. They are not even the same attribute. See Appendix C for the proof of Equations (4-18) and (4-19).

Preferential independence for all trade-off pairs (x_1, x_i), $i = 2,\ldots,8$, was tested by assessing the independence of x_1^i (or x_i^1) by varying all attribute states other than x_1 and x_i from their least-preferred states to their most-preferred states and noting if x_1^i (or x_i^1) varied. If preferential independence is violated, then a sensitivity analysis should be performed to ensure that the violation does not affect the preference ordering of the alternative systems.

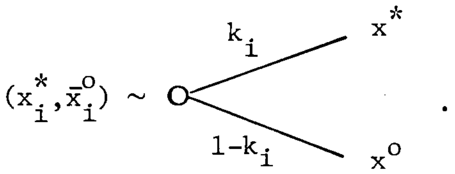
If, during the questioning for data for the attribute utility functions or for the scaling constants, the interviewee asked what states were to be assumed for the other attributes, the interviewee was told to assume "nominal" states. Any violation of utility independence or preferential independence would, of course, be observed during the independence checks.

The scaling constant k_1 for the attribute "first year busbar energy cost" (x_1) was assessed with the use of Figure A-26. A value for the probability "p" was found by "homing in" on a probability where the interviewee was indifferent between the "reference system" and the gamble. The reference system had the attribute x_1 at its most-preferred state and all other attributes at their least-preferred states. The gamble yielded with probability "p" a system with all attributes at their most-preferred states and with probability "1-p" a system with all attributes at their least-preferred state. The scaling constant k_1 equals p as was shown in Equations (4-9) and (4-10).

This was the most difficult assessment of the interview process, because all of the attributes were varied. An attempt to make the decision more transparent was made by the following argument,

> "Assume you have the reference system for sure, which you can have. If instead, you elect to pick the gamble, then if you win the gamble what you gain is that all the attributes other than x_1 are changed from their least-preferred to their most-preferred states. If you lose the gamble, then your loss is that x_1 is changed from its most-preferred state to its least-preferred state."

Finally, direct gamble assessments were made for some k_i greater than k_1 (the scaling constant for the reference attribute x_1). These assessments were of the form:



$$(x_i^*,\bar{x}_i^o) \sim \begin{matrix} & \overset{k_i}{\nearrow} & x^* \\ \circ & & \\ & \underset{1-k_i}{\searrow} & x^o \end{matrix} \quad . \qquad (4\text{-}20)$$

These gamble assessments were made primarily to preclude an incorrect trade-off assessment for x_i^1 which could result in a calculated $k_i > 1$, which would be inadmissible in the multiattribute utility function. Unless this inadmissible condition occurred, the k_i's were derived from the x_i^1 values, and not from the gambles. The two values for k_i do, of course, provide a test of the consistency of the methodology.

This completed the interview process. The interview data was then transferred to Worksheet No. 1 shown in Figure 4-1. The spaces marked with the letter (a) were filled in with the data from Figures A-7 through A-15 for the attribute utility functions. If utility independence was violated for x_1, then the space marked (a'/a") was filled in with the indifference point for all other attributes at their least-preferred states (a') and the indifference point for all other attributes at their most-preferred states (a"). The space marked (b/b'/b") was filled in with the data from the indifference trade-offs of Figures A-19 through A-25 when the indifference point relative to B lay on the A to C line. The assessed value for x_1^i was entered in the space marked b. If preference indifference was violated, (b') corresponded to the indifference point with other attributes at their least-preferred states and (b") to the indifference point with other attributes at their most-preferred states. The space marked (c/c'/c") was filled in with the data from the indifference trade-offs of Figures A-19 through A-25 when the indifference point relative to C lay on the A to B line. The assessed value for x_i^1 was entered in the space marked (c). If preference independence was violated, (c') corresponded to the indifference point with other attributes at their least-preferred states and (c") to the indifference point with other attributes at their most-preferred states.

The value assessed for k_1 was entered in the space marked d. If direct gamble assessments were made of other k_i's, $i \neq 1$, they were entered in the spaces marked f'. This completed the direct translation of the interview data onto Worksheet No. 1 as shown in Figure 4-1.

D. INTERVIEW DATA ANALYSIS

The interview data analysis started by completing Worksheet No. 1 shown in Figure 4-1. Attribute utility functions were constructed using

Interview: **Position and Organization:**

Date:

x_i	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	a a'/a"	d	—	—
x_2	Capital Cost $/kWh	1500 - 3000	a	f f'	b/b'/b" e	c/c'/c" e
x_3	Reliability Forced Outage %	0 - 25	a	f f'	b/b'/b" e	c/c'/c" e
x_4	System Output Capacity Factor %	20 - 35	a	f f'	b/b'/b" e	c/c'/c" e
x_5	Safety Man-Days-Lost/Year	0 - 60	a	f f'	b/b'/b" e	c/c'/c" e
x_6	R&D Requirements 10^6 $	200 - 600	a	f f'	b/b'/b" e	c/c'/c" e
x_7	Environment Subjective	0 - 10	—	f f'	b/b'/b" e	c/c'/c" e
x_8	Applications Subjective	0 - 10	—	f f'	b/b'/b" e	c/c'/c" e

$\Sigma k_i = g$

$k = g$

x_1	a	a	a	a				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:

Figure 4-1. Worksheet No. 1 for the Interview Data Analysis the data marked a on the worksheet. The attribute utility function for x_1, first year busbar energy cost, was plotted graphically. The attribute utility functions for attributes x_2 through x_6 were computed by means of the computer program of Appendix B. The JPL attribute utility functions of Figures 5-1 and 5-2 were used for x_7 and x_8. Using this information, entries e were made. Then, using the formulas of Equations (4-18) or (4-19), the k_i's were calculated and entries f were made. With all the k_i's calculated, one then made entries (g) by use of the computer program of Appendix D. This completed the data analysis for Worksheet No. 1.

With all the k_i's calculated and all the attribute utility functions known, one then completed Worksheet No. 2 of Figure 4-2 and calculated system utilities and rank orderings in preference. The entries (f) were the k_i's. Entry (g) was the master scaling constant k as calculated by the computer program of Appendix D. The attribute utility function values for each of the nine alternative systems were either taken from the graphs for $u_1(x_1)$, $u_7(x_7)$, or $u_8(x_8)$, or computed for the other attributes by the computer program of Appendix B, and were entered in the spaces marked h.

The computer program of Appendix D then used the (f), (g) and (h) entries to compute system utility function values, which were entered in the spaces marked m in the worksheet. By inspection, one then assigned a rank ordering in preference for all nine alternative systems using the fact that systems with larger utility function values were more preferred. The rank ordering numbers were entered in the spaces marked n. This completed the analysis of the interview data, except for sensitivity analyses that might be made to insure that violations of utility independence did not affect the rank ordering in preference for the alternative systems.

Interview: Position and Organization:

Date:

System Number	x_1 $u_1(x_1)$ x_1 RANK	x_2 $u_2(x_2)$ x_2 RANK	x_3 $u_3(x_3)$ x_3 RANK	x_4 $u_4(x_4)$ x_4 RANK	x_5 $u_5(x_5)$ x_5 RANK	x_6 $u_6(x_6)$ x_6 RANK	x_7 $u_7(x_7)$ x_7 RANK	x_8 $u_8(x_8)$ x_8 RANK	$u(x)$ RANK
I	99	2540	1.4	30	2.46	288	8.6	9.0	
	h	h	h	h	h	h	.835	.685	m
	6	7	3	3	4	2	1	1	n
II	76	1854	11.8	28	13.6	231	8.1	5.8	
	h	h	h	h	h	h	.790	.330	m
	3	4	7	4	7	1	4	2	n
III	59	1578	0	32	2.11	360	8.6	4.8	
	h	h	h	h	h	h	.835	.270	m
	1	1	1	2	2	8	1	3	n
IV	69	1810	0	33	2.11	541	8.6	4.8	
	h	h	h	h	h	h	.835	.270	m
	2	2	1	1	2	9	1	3	n
V	88	2140	13.6	28	16.0	341	6.1	4.1	
	h	h	h	h	h	h	.620	.225	m
	5	5	8	4	8	6	6	6	n
VI	125	2280	25.0	21	35.2	325	5.8	4.3	
	h	h	h	h	h	h	.595	.238	m
	7	6	9	7	9	3	7	5	n
VII	167	2760	1.4	19	0.82	327	4.0	3.44	
	h	h	h	h	h	h	.435	.185	m
	9	8	3	9	1	4	8	8	n
VIII	149	2912	6.6	21	9.91	357	3.9	2.54	
	h	h	h	h	h	h	.430	.127	m
	8	9	6	7	6	7	9	9	n
IX	86	1816	1.4	24	2.68	339	6.8	3.54	
	h	h	h	h	h	h	.680	.188	m
	4	3	3	6	5	5	5	7	n
k_i, k	f	f	f	f	f	f	f	f	g

Figure 4-2. Worksheet No. 2 for the Interview Data Analysis

SECTION V

THE JPL UTILITY FUNCTION

A. INTRODUCTION

The JPL Utility Function was derived from interviews with four JPL managers knowledgeable of energy systems analysis and of the alternative systems being considered in this Report. The four JPL managers constitute the hierarchy of management responsible for this Report.

B. THE INTERVIEWS

The interviews with the four JPL managers constitute the first four of the 14 interviews. These interviews were conducted with the following managers:

Interview No. 1:

Task Manager, Applications Analysis and Development, Point-Focusing Thermal and Electric Applications Project, Jet Propulsion Laboratory, Pasadena, California.

Interview No. 2:

Technical Manager, Point-Focusing Thermal and Electric Applications Project, Jet Propulsion Laboratory, Pasadena, California.

Interview No. 3:

Project Manager, Solar Thermal Power Systems Project, Jet Propulsion Laboratory, Pasadena, California.

Interview No. 4:

Manager, Solar Energy Program, Jet Propulsion Laboratory, Pasadena, California.

C. ANALYSIS OF THE INTERVIEWS

The data for the four JPL managers is presented on Worksheet No. 1 as shown in Figures E-1 through E-4. The attribute utility functions for environmental impacts (x_7) and applications flexibility (x_8) were not determined in any of the interviews, but were assessed by two JPL experts, as shown in Figures 5-1 and 5-2. These two attribute utility functions were used in the analysis of all 14 interviews.

Three of the four JPL managers exhibited minor violations of utility independence or preferential independence, but the independence violations did not affect the rank ordering in preference for the nine alternative systems. First year busbar energy cost (x_1) was the most important attribute for all four JPL managers, with the attribute scaling constant for x_1 ranging from k_1 = 0.55 to k_1 = 0.97. One JPL manager (Interview No. 3) exhibited constant risk-proneness over essentially the entire attribute range of x_1. The other three JPL managers were risk averse from 50 mills/kWh to 65 mills/kWh and risk-prone above 80 mills/kWh. This can be interpreted to mean that at low x_1, these latter JPL managers were somewhat unwilling to risk the prospect of a higher x_1 in return for a chance at an even lower x_1. However, at high values for x_1, where the systems might not prove to be economically viable for a large number of applications, these JPL managers were more willing to gamble for the prospect of a lower value for x_1. Other than these statements, there are no obvious common elements of preference among the four JPL managers.

D. THE JPL UTILITY FUNCTION

The JPL utility function was derived by averaging the assessment data of the four JPL managers. It is important to emphasize that this averaging does not represent a consensus of the four JPL managers. The four JPL managers were not brought together to work out a consensus value model. There is no particular theoretical justification for the averaging process. Use of median data could be as easily justified.

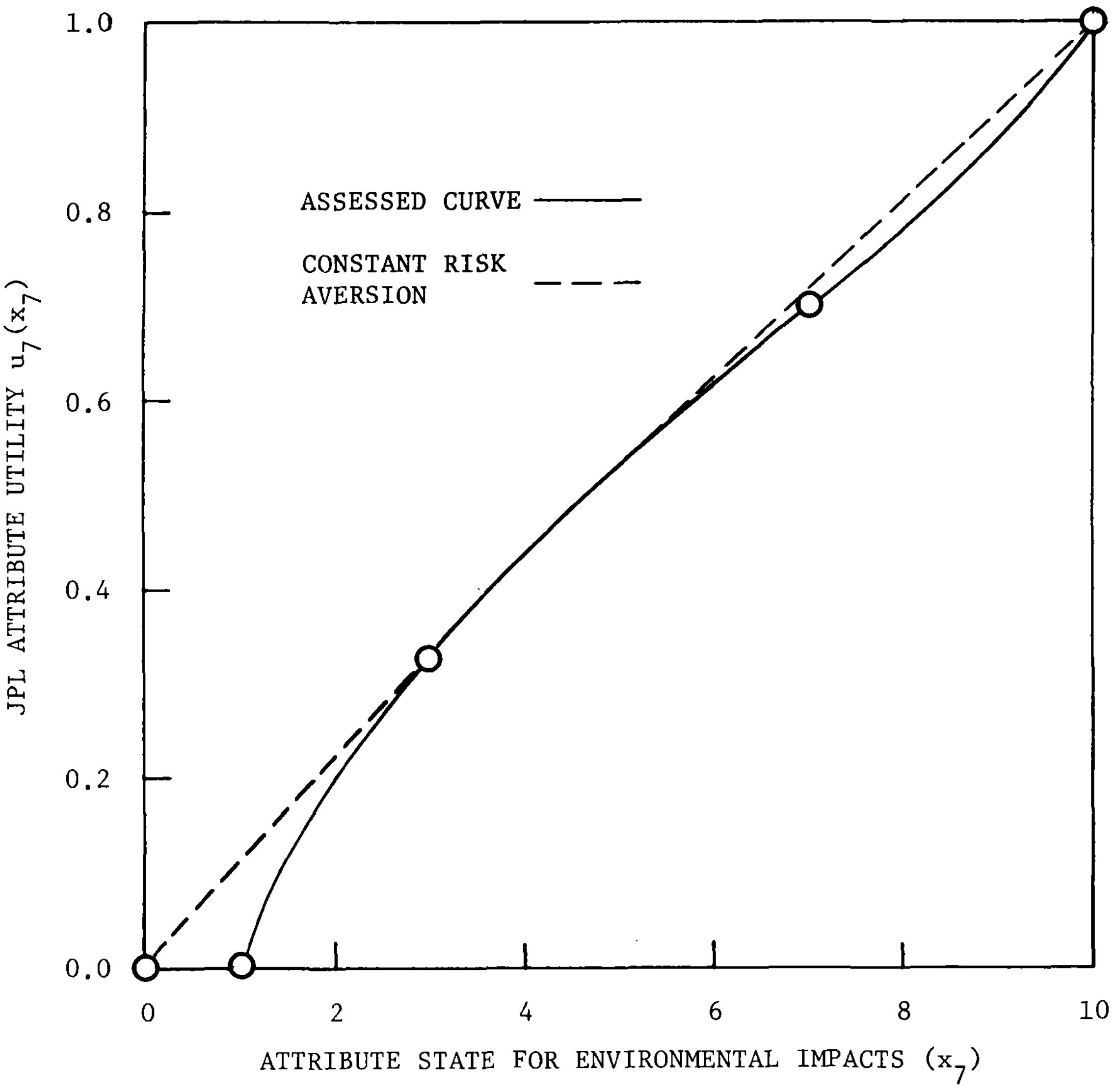


Figure 5-1. JPL Attribute Utility Function for Environmental Impacts (x_7)

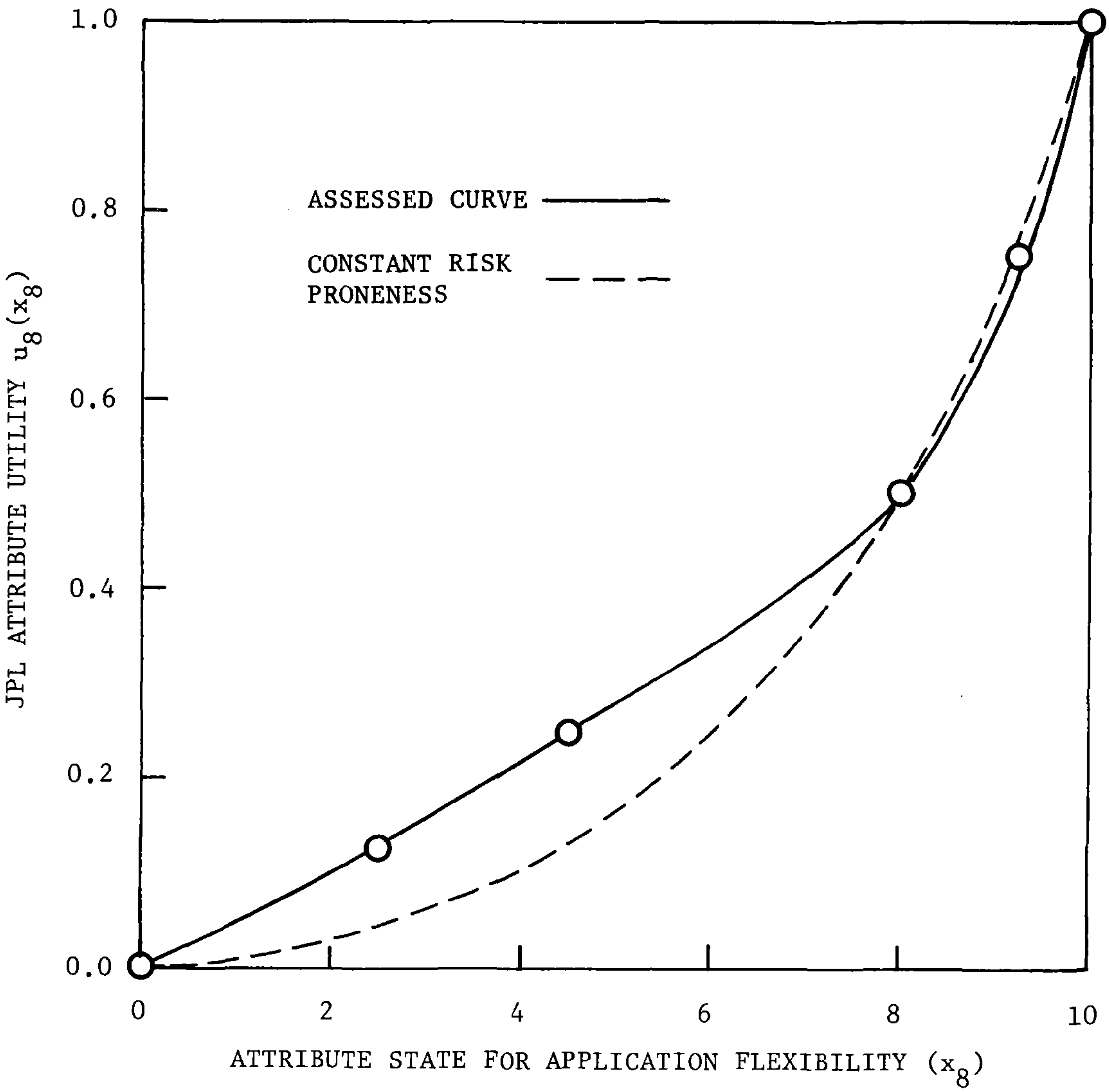


Figure 5-2. JPL Attribute Utility Function for Applications Flexibility (x_8)

The averaged data for the JPL Utility Function is shown in Figure 5-3 for first year busbar energy cost (x_1) and in the Worksheet No. 1 data shown in Figure 5-4. The attribute utility function for first year busbar energy cost (x_1) is risk averse from 50 mills/kWh to 75 mills/kWh and risk prone from 75 mills/kWh to 125 mills/kWh.

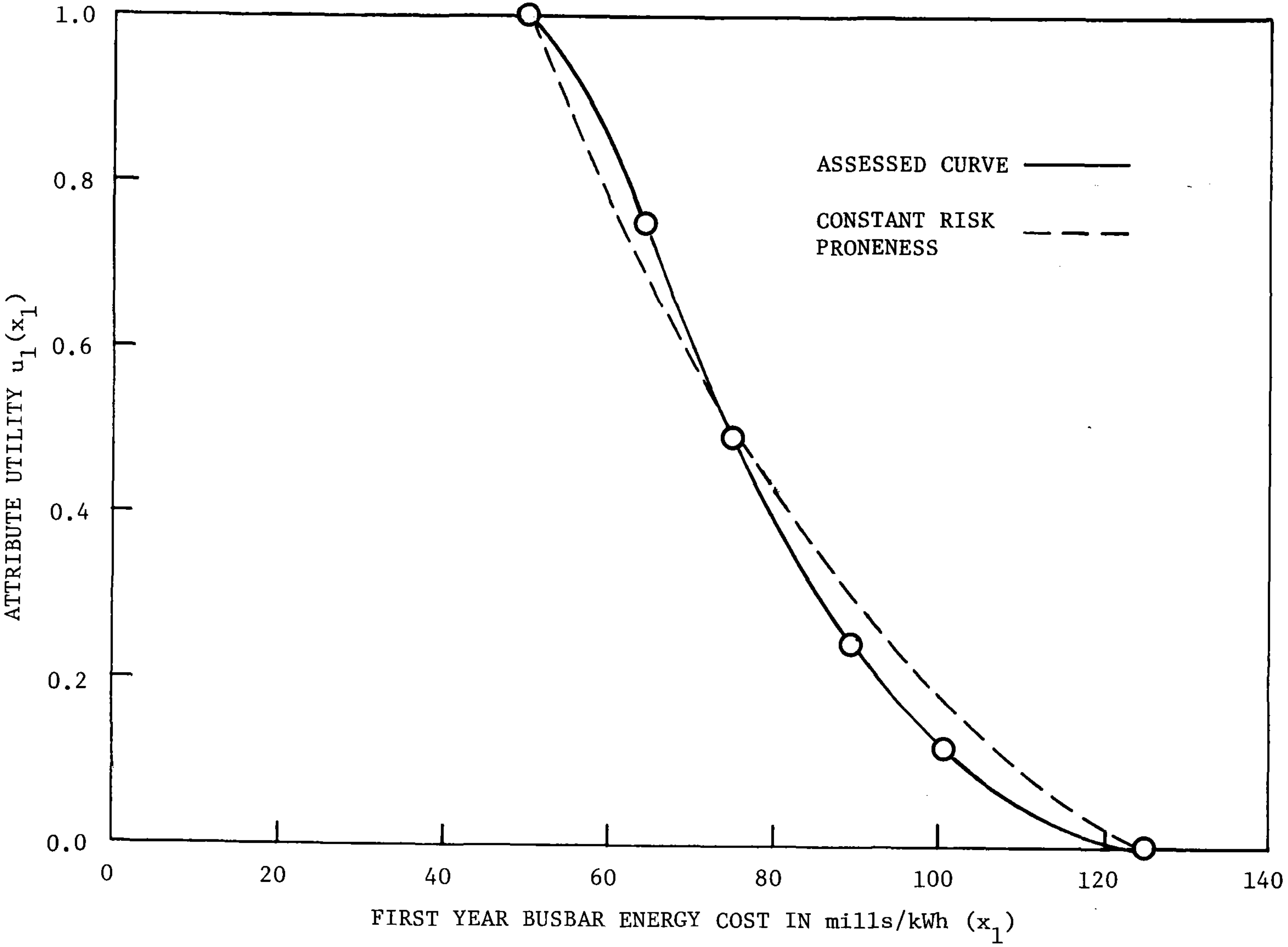


Figure 5-3. Attribute Utility Function for First Year Busbar Energy Cost (x_1) for the JPL Utility Function

x_i	Attribute	Range	x_i'	k_i
x_1	First Year Cost mills/kWh	50 - 125	See Graph	0.805
x_2	Capital Cost $/kW	1500 - 3000	2062	0.186
x_3	Reliability % Forced Outage	0 - 25	13.2	0.189
x_4	Plant Output % Capacity Factor	20 - 35	28.9	0.240
x_5	Safety Man-days-lost/year	0 - 60	31.2	0.152
x_6	R&D 10^6 $	200 - 600	387.5	0.218
x_7	Environment Subjective	0 - 10	See Graph	0.113
x_8	Applications Subjective	0 - 10	See Graph	0.264

$\Sigma k_i = 2.167$

$k = -0.9420$

Figure 5-4. Worksheet No. 1 for the JPL Utility Function

SECTION VI

THE INTERVIEWS WITH REPRESENTATIVES OF INTEREST GROUPS

A. INTRODUCTION

This Section discusses the data obtained in Interviews No. 5 through No. 14 (the first four interviews were discussed in Section V). The data is presented in Figures E-1 through E-14 of Appendix E. These 10 interviews could be classified as 5 with members of public or municipal utility companies (Interviews No. 5 - No. 8) or an associated organization (Interview No. 9), three interviews with energy system experts (Interviews No. 10 - No. 12), one interview with an environmental advocate (Interview No. 13), and one interview with a state government energy decision maker (Interview No. 14).

B. THE INTERVIEWS

Interview No. 5:

Manager of R&D Programs, A Public Utility Company, Arizona.

Interview No. 6:

Power System Engineer, A Municipal Utility Company, Pasadena, California.

Interview No. 7:

Manager of Resource Development, A Municipal Utility Company, Los Angeles, California.

Interview No. 8:

Supervising Research Engineer, A Public Utility Company, Southern California.

Interview No. 9:

Director of Energy Research, American Public Power Association, Washington, D.C.

Interview No. 10:

Manager of Planning and Assessment, Northeast Solar Energy Center, Cambridge, Massachusetts.

Interview No. 11:

Professor of Nuclear Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Interview No. 12:

President, Transportation and Economic Research Associates, Arlington, Virginia.

Interview No. 13:

Past President, Sierra Club.

Interview No. 14:

Commissioner, Energy Resources Conservation and Development Commission, State of California.

Interview No. 5 displayed essentially constant risk proneness over the entire range of first year busbar energy cost from 50 mills/kWh to 125 mills/kWh. The most important attribute was reliability ($k_3 = 0.74$) with first year busbar energy cost second ($k_1 = 0.67$). Plant output (Capacity factor) was not considered to be a significant attribute ($k_4 = 0$). Utility independence was violated for attribute x_1. Preferential independence was violated for attribute pairs (x_1,x_2), x_1,x_3), (x_1,x_5), and (x_1,x_7).

Interview No. 6 displayed risk aversion for first year busbar energy cost from 50 mills/kWh to 90 mills/kWh and was essentially risk neutral above 90 mills/kWh. The most important attribute was reliability ($k_3 = 0.78$), with first year busbar energy cost second ($k_1 = 0.50$). Interview No. 2 displayed a reversal of the usual preference for capital cost, with high capital cost (x_2) preferred to low capital cost. It should be noted that attribute x_2 is defined <u>not</u> to include the effect of capital cost on first year busbar energy cost (x_1), but only as capital cost is a factor in the balance between first costs and operating and maintenance costs, and in the difficulty in raising the initial capital investment. This preference reversal was also included in the gamble in the indifference relation for k_1 by modifying Figure A-26. Utility independence was violated for attribute x_1. Preferential independence was satisfied.

Interview No. 7 displayed risk aversion over almost the entire attribute range of first year busbar energy cost (x_1). The most important attribute was safety (x_5), but the value assessed for $u_5(x_5^1)$ was inconsistent with k_1. The formula of Equation (4-19) would have resulted in $k_5 > 1.0$, so the value assessed directly by means of the indifference relation of Equation (4-20) was used ($k_5 = 0.70$). First year busbar energy cost (x_1) was ranked second ($k_1 = 0.5$). Utility independence was satisfied for attribute x_1. Preferential independence was violated for attribute pairs (x_1,x_2), (x_1,x_3), (x_1,x_4), and (x_1,x_6).

Interview No. 8 displayed constant risk aversion from 50 mills/kWh to 90 mills/kWh for first year busbar energy cost (x_1). At 100 mills/kWh the utility function dropped vertically, and insufficient data was collected to graph well the utility function from 100 mills/kWh to 125 mills/kWh. The most important attribute was first year busbar energy cost ($k_1 = 0.70$), with capital cost ranked second ($k_2 = 0.616$). Utility independence and preferential independence were satisfied.

Interview No. 9 displayed risk aversion from 50 mills/kWh to 75 mills/kWh and risk proneness from 75 mills/kWh to 125 mills/kWh for first year busbar energy cost (x_1). Reliability was ranked as the most important attribute ($k_3 = 0.418$), with first year busbar energy cost ranked fifth ($k_1 = 0.167$). Utility independence was satisfied for x_1. Preferential independence was violated by the attribute pair (x_1,x_2), which was not a significant factor ($k_2 = 0.002$). Since applications flexibility (x_8) was assessed as more important than x_1, and no intermediate states for x_8 were defined, it was necessary to assess $u_8(x_8^1)$ directly ($u_8(x_8^1) = 0.60$). There were gross discrepancies for the scaling constants k_3, k_5, k_6, and k_7 when they were assessed by means of k_1 and $u_i(x_i^1)$ in Equation (4-19) as compared with the direct assessment by means of the indifference relation of Equation (4-20).

Interview No. 10 displayed essentially constant risk aversion for first year busbar energy cost (x_1) from 50 mills/kWh to 95 mills/kWh, and was risk neutral from 105 mills/kWh to 125 mills/kWh. First year busbar energy cost (x_1) was the most important attribute ($k_1 = 0.90$) with applications flexibility (x_8) ranked second ($k_8 = 0.518$). Utility independence and preferential independence were satisfied.

Interview No. 11 displayed constant risk proneness over the entire attribute range for first year busbar energy cost (x_1). First year busbar energy cost (x_1) was the most important attribute ($k_1 = 0.70$) with capital cost ranked second ($k_2 = 0.217$). Utility independence was satisfied for attribute x_1. Preferential independence was violated for attribute pairs (x_1,x_2), (x_1,x_7), and (x_1,x_8). Plant output (x_4) had to be excluded from the analysis (k_4 was set to $k_4 = 0$) because of a gross violation of utility independence. A complete preference reversal occurred between setting the other attributes at their least-preferred states $(x_4,\bar{x}_4^o)$ and setting the other attributes at their most-preferred states $(x_4,\bar{x}_4^*)$. The rationale was that if the cost of energy was to be low (x_1^*), then high capacity factors (x_4) were preferred. Contrariwise, if the cost of energy was to be high (x_1^o), then low capacity factors were preferred, because Interviewee No. 11 preferred that society not use much of this source of energy. Safety (x_5) was originally assessed as more important than the attribute x_1 ($k_5 > k_1$), but the value assessed for $u_5(x_5^1)$ was inconsistent with k_1 ($k_1 = 0.70$ and $u_5(x_5^1) = 0.50$), and the use of Equation (4-19) would have resulted in a value of $k_5 > 1.0$. Thus k_5 was directly assessed by means of the indifference relation of Equation (4-20), where k_5 was determined to be less than k_1 ($k_5 = 0.333$ and $k_1 = 0.70$). This latter value of $k_5 = 0.333$ was used in the analysis. Interviewee No. 11 had difficulty in assessing k_5 because of a concern that the least-preferred attribute state for safety ($x_5^o = 60$ man-days-lost/year for a 5 MWe rated plant) would be viewed as unacceptably high by society. If the number of solar plants are scaled up to equal the annual energy output of a typical large power plant (1000 MegaWatts average output), then (for a solar plant capacity factor of 0.25)

800 solar plants would be required for a worst-case injury rate equivalent to 131 man-years-lost/year.

Interview No. 12 was essentially risk neutral over the entire attribute range for first year busbar energy cost (x_1). Reliability (x_3) was the most important attribute ($k_3 = 0.71$) with first year busbar energy cost (x_1) ranked fourth ($k_1 = 0.25$). Once again, the scaling constants (for k_3, k_5, and k_8) as determined by k_1 and $u_i(x_i^1)$ in Equation (4-19) were inconsistent with the scaling constants as directly assessed by means of the indifference relation of Equation (4-20). Utility independence was violated for attribute x_1. Preferential independence was violated by attribute pairs (x_1,x_2) and (x_1,x_3).

Interview No. 13 displayed essentially constant risk proneness over the entire attribute range for first year busbar energy cost (x_1). First year busbar energy cost (x_1) was the most important attribute ($k_1 = 0.75$) with environmental impacts (x_7) ranked second ($x_7 = 0.375$). No importance was placed on capital cost ($k_2 = 0$) or plant output ($k_4 = 0$). Safety was not considered to be an important attribute ($k_5 = 0.009$). Utility independence and preferential independence were satisfied.

Interview No. 14 displayed essentially constant risk proneness over the entire attribute range for first year busbar energy cost (x_1). Applications flexibility (x_8) was the most important attribute ($k_8 = 0.056$) with first year busbar energy cost (x_1) ranked second ($k_1 = 0.022$). As in previous interviews, the scaling constant for attribute x_8 as determined by Equation (4-19) was inconsistent ($k_8 = 0.056$ vs. $k_8 = 0.200$) with the direct assessment by means of the indifference relation of Equation (4-20). There was insufficient time in the interview to verify utility independence and preferential independence. It was necessary to directly assess $u_8(x_8^1)$, as intermediate states for applications flexibility (x_8) were not defined.

SECTION VII

THE ANALYSIS

A. INTRODUCTION

This Section describes the analysis of the interview data in generating system utility function values for each of the interviews for the nine alternative systems. It establishes a preference ranking for each of the 14 interviews and JPL preference ranking. It aggregates the rankings by several collective choice rules.

B. CALCULATION OF THE SYSTEM UTILITY FUNCTION VALUES

System utility function values were calculated for each of the nine systems for each of the 14 interviews and for the JPL utility function using the Keeney and Raiffa multiattribute utility function of Equations (4-2) and (4-5). The attribute utility function values for each of the eight attributes and the system utility function values are displayed in Worksheets No. 2 which are presented in Appendix F. Worksheet No. 2 for the JPL utility function is displayed in Figure 7-1 of this Section. The attribute utility function values for first year busbar energy cost (x_1) were taken from each of the $u_1(x_1)$ graphs for each of the 14 interviews as presented in Appendix F and from Figure 5-3 for the JPL utility function. The attribute utility function values for reliability (x_3) through R&D requirements (x_6) were all calculated from Equation (4-11) and the data presented in the respective Worksheet No. 1. All of the attribute data for environmental impacts (x_7) and applications flexibility (x_8) were taken from the JPL attribute utility functions of Figures 5-1 and 5-2. The scaling constants for the multiattribute utility function were all taken from the respective Worksheet No. 1, and also displayed at the bottom of Worksheet No. 2. With the system utility function values calculated and displayed on Worksheets No. 2, the nine systems can be rank ordered in preference, with larger system utility function values more preferred.

System No.	x_1 $u_1(x_1)$ x_1 Rank	x_2 $u_2(x_2)$ x_2 Rank	x_3 $u_3(x_3)$ x_3 Rank	x_4 $u_4(x_4)$ x_4 Rank	x_5 $u_5(x_5)$ x_5 Rank	x_6 $u_6(x_6)$ x_6 Rank	x_7 $u_7(x_7)$ x_7 Rank	x_8 $u_8(x_8)$ x_8 Rank	$u(x)$ System Rank
I	99 0.140 6	2540 0.205 7	1.4 0.950 3	30 0.421 3	2.46 0.962 4	288 0.758 2	8.6 0.835 1	9.0 0.685 1	0.683 5
II	76 0.475 3	1854 0.663 4	11.8 0.556 7	28 0.562 4	13.6 0.787 7	231 0.913 1	8.1 0.790 4	5.8 0.330 2	0.775 3
III	59 0.865 1	1578 0.918 1	0 1.0 1	32 0.265 2	2.11 0.968 2	360 0.570 8	8.6 0.835 1	4.8 0.270 3	0.908 1
IV	69 0.620 2	1810 0.700 2	0 1.0 1	33 0.181 1	2.11 0.968 2	541 0.132 9	8.6 0.835 9	4.8 0.270 3	0.784 2
V	88 0.270 5	2140 0.445 5	13.6 0.484 8	28 0.562 4	16.0 0.749 8	341 0.619 6	6.1 0.620 6	4.1 0.225 6	0.635 6
VI	125 0.000 7	2280 0.353 6	25.0 0.000 9	21 0.954 7	35.2 0.433 9	325 0.660 3	5.8 0.595 7	4.3 0.238 5	0.503 9
VII	167 0.000 9	2760 0.099 8	1.4 0.950 3	19 1.000 9	0.82 0.987 1	327 0.655 4	4.0 0.435 8	3.44 0.185 8	0.608 7
VIII	149 0.000 8	2912 0.034 9	6.6 0.757 6	21 0.954 7	9.91 0.846 6	357 0.577 7	3.9 0.430 9	2.54 0.127 9	0.549 8
IX	86 0.300 4	1816 0.695 3	1.4 0.950 3	24 0.803 6	2.68 0.959 5	339 0.624 5	6.8 0.680 5	3.54 0.188 7	0.739 4
	0.805	0.186	0.189	0.240	0.152	0.218	0.113	0.264	-0.9420

Figure 7-1. Worksheet No. 2 for the JPL Utility Function

C. THE JPL RANKING

The JPL utility function is derived from the data of Figure 5-4 and the graphs of Figures 5-1 through 5-3. Worksheet No. 2 for the JPL attribute and system utility function values is displayed in Figure 7-1, along with preference rankings for the nine alternative systems. Figure 7-2 shows the rank orderings and the system utility function values for the four JPL managers and for the JPL utility function. It should be stressed once again that the JPL utility function is derived from averaging over the parameters of the utility functions of the four managers, and does not represent a consensus of the managers.

D. THE SYSTEM RANKINGS ESTABLISHED BY EACH OF THE 14 INTERVIEWS

The data of Worksheets No. 2 of Appendix E for each of the 14 interviews are summarized in Figure 7-3: "System Rankings by Interviews and by Attributes x_1 and x_2." The rankings of Figure 7-3 show that System III was ranked first by the decision analysis of the interviews (except for Interview No. 9 where a tie essentially existed for first) and by the x_1 and x_2 attributes. System IV was ranked second by all but three interviews, and never lower than fourth. System VI was never ranked higher than seventh.

E. SYSTEM RANKINGS ESTABLISHED BY COLLECTIVE CHOICE RULES

Three collective choice rules (rules for aggregating individual preferences into a group preference) were applied to the interview data as shown in Figure 7-4. The three collective choice rules were the Majority Decision Rule, the Borda Rule, and the Additive Utility Rule. There is no theoretically compelling reason to use the results of any of these collective choice rules, or to use the preferences of the 14 representatives interviewed, but they do provide information for Project decision making.

Probably the most well known collective choice rule based on the ordinal ranking of the individuals is the Majority Decision Rule (Reference 7-1). The axiomatic counterpart of the Majority Decision

System No.	Interviews				JPL Utility Function	Attribute* x_1	Attribute* x_2
	1	2	3	4			
	Rank (Utility)	Rank (Utility)	Rank (Utility)	Rank (Utility)	Rank (Utility)	Rank (mills/kWh)	Rank ($/kW)
I	4 (0.726)	4 (0.821)	5 (0.787)	6 (0.346)	5 (0.683)	6 (99)	7 (2540)
II	3 (0.781)	3 (0.829)	3 (0.847)	3 (0.536)	3 (0.775)	3 (76)	4 (1854)
III	1 (0.917)	1 (0.902)	1 (0.940)	1 (0.883)	1 (0.908)	1 (59)	1 (1578)
IV	2 (0.824)	2 (0.833)	2 (0.875)	2 (0.621)	2 (0.784)	2 (69)	2 (1810)
V	5 (0.631)	7 (0.697)	6 (0.752)	5 (0.366)	6 (0.635)	5 (88)	5 (2140)
VI	7 (0.457)	9 (0.466)	9 (0.558)	7 (0.260)	9 (0.503)	7 (125)	6 (2280)
VII	8 (0.438)	6 (0.718)	7 (0.697)	8 (0.251)	7 (0.608)	9 (167)	8 (2760)
VIII	9 (0.406)	8 (0.649)	8 (0.641)	9 (0.248)	8 (0.549)	8 (149)	9 (2912)
IX	6 (0.607)	5 (0.802)	4 (0.838)	4 (0.478)	4 (0.739)	4 (86)	4 (1816)

*x_1 = First year busbar energy cost.

x_2 = Capital cost.

Figure 7-2. System Rankings for the JPL Managers and for the JPL Utility Function

System No.	Interviews														Attributes*	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	x_1	x_2
	Rank Utility														Rank State	
I	4 0.726	4 0.821	5 0.787	6 0.346	4 0.886	4 0.945	4 0.891	6 0.789	1 0.850	5 0.744	5 0.636	3 0.936	4 0.688	2 0.262	6 99	7 2540
II	3 0.781	3 0.829	3 0.847	3 0.536	5 0.808	6 0.875	5 0.879	2 0.923	6 0.717	3 0.813	3 0.728	7 0.815	3 0.728	4 0.208	3 76	4 1854
III	1 0.917	1 0.902	1 0.940	1 0.883	1 0.982	1 0.975	1 0.952	1 0.962	2 0.848	1 0.902	1 0.888	1 0.953	1 0.894	1 0.280	1 59	1 1578
IV	2 0.824	2 0.833	2 0.875	2 0.621	2 0.956	2 0.968	2 0.926	4 0.899	4 0.772	2 0.845	2 0.806	2 0.942	2 0.806	3 0.212	2 69	2 1810
V	5 0.631	7 0.697	6 0.752	5 0.366	7 0.690	8 0.817	6 0.806	5 0.834	8 0.588	6 0.723	6 0.620	8 0.751	6 0.595	6 0.114	5 88	5 2140
VI	7 0.457	9 0.466	9 0.558	7 0.260	9 0.411	9 0.469	9 0.559	8 0.658	9 0.363	9 0.339	9 0.355	9 0.432	9 0.323	8 0.062	7 125	6 2280
VII	8 0.438	6 0.718	7 0.697	8 0.251	6 0.804	5 0.928	7 0.800	7 0.715	5 0.768	8 0.388	7 0.487	5 0.888	7 0.437	7 0.081	9 167	8 2760
VIII	9 0.406	8 0.649	8 0.641	9 0.248	8 0.640	7 0.867	8 0.744	9 0.630	7 0.658	7 0.434	8 0.400	6 0.819	8 0.385	9 0.049	8 149	9 2912
IX	6 0.607	5 0.802	4 0.838	4 0.478	3 0.904	3 0.953	3 0.910	3 0.907	3 0.778	4 0.779	4 0.721	4 0.931	5 0.682	5 0.176	4 86	3 1816
	JPL				Utility Companies				Utility Association	Energy Experts			Sierra Club	California Energy Commissioner		

*x_1 = First year busbar energy cost in mills/kWh.

x_2 = Capital cost in \$/kW.

Figure 7-3. System Rankings by Interviews and by Attributes x_1 and x_2

System No.	Ranking					
	Visual Inspection Rank	Majority Decision Rank	Borda Rule Rank (Count)	Additive Utility Rank (Utility)	Attribute* x_1 Rank (mills/kWh)	Attribute* x_2 Rank ($/kW)
I	3-5	4	5 (57)	5 (0.736)	6 (99)	7 (2540)
II	3-5	3	3.5 (56)	3 (0.749)	3 (76)	4 (1854)
III	1	1	1 (15)	1 (0.877)	1 (59)	1 (1578)
IV	2	2	2 (33)	2 (0.806)	2 (69)	2 (1810)
V	6-7	6	6 (89)	6 (0.642)	5 (88)	5 (2140)
VI	8-9	9	9 (120)	9 (0.408)	7 (125)	6 (2280)
VII	6-7	7	7 (93)	7 (0.600)	9 (167)	8 (2760)
VIII	8-9	8	8 (111)	8 (0.541)	8 (149)	9 (2912)
IX	3-5	5	3.5 (56)	4 (0.748)	4 (86)	3 (1816)

*x_1 = First year busbar energy cost.

x_2 = Capital cost.

Figure 7-4. System Rankings by the Collective Choice Rules and by Attributes x_1 and x_2

Rule was first developed by May (Reference 7-2). According to this rule, an alternative with a simple majority over each of the other alternatives should be the collective choice. This rule requires that the alternatives be compared pairwise in order to determine the majority winner in each case.

The Borda Rule (References 7-3 and 7-4) or the Rank Sum Rule in the slightly modified form used here requires the calculation of the sum of the ordinal ranks for each alternative, with the alternative receiving the lowest rank sum being most preferred. Young (Reference 7-5) has stated four axioms that are necessary and sufficient for any collective choice rule to be equivalent to the Borda Rule.

The modern formulation of the Additive Utility Rule is that of Harsanyi (Reference 7-6). The Additive Utility Rule averages (here the weights are assumed equal) the utility function values assigned to each alternative, with the highest average utility function value being most preferred.

SECTION VIII

CONCLUSIONS

A. INTRODUCTION

The conclusions to this Report are made in three parts. The first part is concerned with the results of the decision analysis with respect to the nine alternative systems. The second part is concerned with the decision analysis methodology itself. How well could the theory be adhered to in practice, and what problems were encountered. The third part makes recommendations for future applications of this multiattribute decision analysis methodology.

B. THE DECISION ANALYSIS RESULTS

The collective choice rules ranked System III, System IV, and System II first, second, and third in that order. The collective choice rules ranked System VII, System VIII, and System VI seventh, eighth, and ninth in that order. System III was clearly the most-preferred system with 13 interviews ranking it first and a tie for first essentially existing in the other interview.

First year busbar energy cost (x_1) was ranked as the most important attribute by eight of the 14 interviews. Capital cost (x_2) was not ranked first by any interview. The first three rankings by first year busbar energy cost (x_1) corresponded to the first three rankings of the collective choice rules. It should be restated that (1) these results are based on preliminary, not current designs, and (2) the scaling range of an attribute can affect its apparent importance.

C. THE MULTIATTRIBUTE DECISION ANALYSIS METHODOLOGY

Keeney and Raiffa did not intend for their methodology to be applied in a single two-hour interview to alternatives of this complexity. They intended for the methodology to be applied in an iterative manner, with the decision analyst feeding back the implications of the stated preferences of the interviewee, with opportunity for the interviewee to reconsider the implications and to alter his stated preferences if so desired. Specifically, internal inconsistencies would be noted, as would results inconsistent with the general philosophy of the interviewee. It is through this iterative process that the interviewee would come to understand better his own preferences, and to state them more correctly.

In the application of the Keeney and Raiffa methodology for this Report, the opportunity for this iterative process just did not exist. All of the representatives interviewed were either decision makers or worked closely with decision makers, and as a result were extremely pressed for time--even more so because of the current national energy--crisis.

In spite of the severe time constraint, it was possible to complete all of the interviews, with the sole exception of Interview No. 14, where insufficient time was available for the utility independence and the preferential independence tests. The interviews took from two to three hours. The interviewees had no trouble grasping the concepts of indifference relations, gambles, and trade-offs. Part of this can undoubtedly be explained by the fact that managers and decision makers involved in energy issues are more quantitatively oriented than the typical manager.

Nevertheless, some gross inconsistencies appeared in the interview data when the data was subsequently analyzed in detail. It is not known to what extent these inconsistencies could have been resolved with the interviewees, and were therefore artifacts of the single interviews and their time restrictions, or whether the inconsistencies were real, thus raising important theoretical questions.

The only part of the interviews that could be subjected to self-consistency tests were the data for determining scaling constants where $k_i > k_1$. Here the scaling constants were assessed by two independent means. One was by the assessed data which determined k_i through Equation (4-19), and the other was directly by means of Equation (4-20).

Interviews No. 7 and No. 11 produced values for k_1 and $u_5(x_5^1)$ inconsistent with a solution of Equation (4-19) such that $k_5 < 1.0$. Interviews No. 12 and No. 14 produced significantly different k_i's through Equations (4-19) and (4-20)--different as much as a factor of four. The degree to which these inconsistencies could have been resolved through feedback and reassessment by the interviewee remains unknown.

These inconsistencies quantitatively illustrate the necessity for feedback and reassessment in order for a decision analysis to be most valid. It was clear that the interview process was a valuable experience for the interviewee in understanding and articulating his own preference structure. Obviously, changes in preferences during the interviews could partly explain the observed inconsistencies. Fortunately, the rank ordering in preference for the alternative systems were robust with respect to these inconsistencies.

Similarly, a question can be raised as to whether the stated utility independence and preferential independence violations were real or were an artifact of the interviews. Since there were logical rationales for many of the independence violations, especially with respect to the attribute first year busbar energy cost (x_1), it would follow that at least some of the independence violations were real.

D. RECOMMENDATIONS

The first point to be made is that whenever possible, analysis, feedback, and reassessment of preferences should form an integral part of the interview process. Where this is not possible, as in the interviews for this Report, the interview process must be well thought out in advance such that the limited time can be focused on the value model assessment.

Careful consideration must be given to the attribute state ranges used in the preference interviews. Attribute state ranges that are too large with respect to the actual system attribute states make the assessment of preferences more difficult, and most likely less accurate. Assessed attribute state ranges that do not span the actual system attribute state ranges do not permit a ranking of the alternative systems by the methodology.

It is most desirable that the alternative system designs be completed prior to conducting the preference interviews. This will guarantee that the assessed attribute state ranges span the actual system attribute state ranges, without being excessively large. The attribute states of the systems considered in this Report subsequent to the preference interviews evolved out of the assessed attribute state ranges, thus making it not possible to rank the final system designs. There are two factors that mitigate against completing the preference interviews prior to completing the system designs. The first factor is that the schedule may require that the preference interviews and the system designs proceed concurrently. The second factor is that the preliminary results of the preference interviews may actually affect the system designs. While this second factor speaks very positively for the value of the multiattribute decision analysis, it can create an attribute range problem, as encountered in this Report.

The interview times were too long, running up to three hours. Careful consideration needs to be given to the trade-off between theoretical rigor and the time required for an interview. For an example of how the interview time might be shortened, at the expense of some theoretical rigor, see Ref. 8-1.

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B-1. W. E. Grove, Brief Numerical Methods, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1966.

INTRODUCTION TO PREFERENCE ASSESSMENT

IN THIS SESSION WE ARE INTERESTED IN FINDING OUT YOUR PREFERENCES FOR VARIOUS ATTRIBUTES OF ELECTRICAL POWER SYSTEMS, IN PARTICULAR SOLAR THERMAL ELECTRIC POWER SYSTEMS.

WE WILL DO THIS BY LOOKING AT THE SYSTEMS INDIRECTLY THROUGH THE VARIOUS ATTRIBUTES WHICH ALL SUCH SYSTEMS HAVE: FOR EXAMPLE, COST, RELIABILITY, SAFETY, ENVIRONMENTAL IMPACT. BY LOOKING CLOSELY AT YOUR ATTITUDE TOWARD EACH OF THESE ATTRIBUTES, WE CAN QUANTITATIVELY EVALUATE EACH OF THE VARIOUS ALTERNATIVE SOLAR THERMAL ELECTRIC POWER SYSTEMS IN ORDER TO DETERMINE WHICH SYSTEM BEST SATISFIES YOUR PARTICULAR SET OF PREFERENCES.

IN THIS ASSESSMENT OF YOUR PREFERENCES, IT IS IMPORTANT TO RECOGNIZE THAT THERE IS NO ABSOLUTE RIGHT OR WRONG. EACH PERSON IS DIFFERENT. EACH HAS HIS OWN OBJECTIVES AND PREFERENCES. TODAY WE ARE INTERESTED IN ASSESSING YOURS.

NAME________________________ COMPANY________________________
DATE________________________ POSITION________________________
RELEASE________________________

Figure A-1. Page 1 of the Interview Process

UNCERTAINTY

THROUGHOUT THIS ASSESSMENT ONE CONCEPT WILL RECUR FREQUENTLY. THIS IS THE CONCEPT OF UNCERTAINTY. IN ALL DECISIONS WE HAVE TO MAKE IN THE WORLD, WE DEAL WITH CHOICES WHERE THE OUTCOME IS UNCERTAIN. DECISION ANALYSIS IS A LOGICAL FRAMEWORK FOR MAKING RATIONAL CHOICES UNDER SUCH CONDITIONS.

GAMBLES

IN DOING OUR ASSESSMENTS, WE WILL BE ASKING YOU TO MAKE SIMPLE CHOICES BETWEEN TWO DIFFERENT ALTERNATIVES. FREQUENTLY, THE ALTERNATIVES WILL DIFFER IN THE RISK INVOLVED. FOR EXAMPLE, WE MIGHT ASK YOU TO CHOOSE BETWEEN THESE ALTERNATIVES

- RECEIVE A $1000 BONUS
- FLIP A COIN: IF IT COMES UP HEADS, YOU WILL RECEIVE A $2000 BONUS; IF IT COMES UP TAILS YOU WILL RECEIVE NO BONUS.

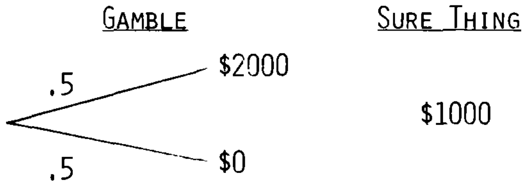
IN THE FIRST ALTERNATIVE YOU HAVE TOTAL CERTAINTY. IT IS A SURE THING THAT YOU WILL RECEIVE A $1000 BONUS. IN THE SECOND ALTERNATIVE YOU HAVE WHAT IS CALLED A GAMBLE (OR LOTTERY)--A CHANCE TO WIN AND A CHANCE TO LOSE. YOU MIGHT WIN $2000 OR YOU MIGHT WIN NOTHING.

Figure A-2. Page 2 of the Interview Process

UNCERTAINTY (Continued)

When comparing the two alternatives, you might look at it this way. Since you can choose the first alternative, you can consider that you already have the $1000 bonus. It is a sure thing. Now, if you take the gamble, you can either win an additional $1000 bonus or you can lose the $1000 bonus you already have.

We can illustrate this choice between the gamble and the sure thing as follows:



In the gamble, we have drawn in the two possible outcomes ($2000 and $0) and for each we have included its probability or chance of occurring. Since we are assuming we are to flip a fair coin, these probabilities are both 1/2.

Figure A-3. Page 3 of the Interview Process

UNCERTAINTY (CONTINUED)

EXPECTED VALUE

NOW ONE THING WE CAN NOTICE RIGHT AWAY ABOUT THIS CHOICE IS THAT IF WE DID IT A LOT OF TIMES, SAY 100 TIMES, THE AVERAGE VALUE OF THE GAMBLE WOULD BE ABOUT $1000. THIS IS BECAUSE ABOUT HALF THE TIME YOU WOULD WIN AND HALF THE TIME YOU'D LOSE, SO ON THE AVERAGE YOU WOULD WIN $1000 PER TRY. THIS IS CALLED THE EXPECTED VALUE OF THE GAMBLE. IT IS COMPUTED BY MULTIPLYING THE OUTCOMES BY THEIR PROBABILITIES AND ADDING THEM TOGETHER. IN THE CASE ABOVE THE EXPECTED VALUE OF THE GAMBLE IS THE SAME AS THE VALUE OF THE SURE THING. SO, IF YOU LOOKED AT THIS CHOICE WHEN YOU WERE GOING TO DO IT OVER AND OVER AGAIN, THERE MIGHT NOT BE A GOOD REASON TO CHOOSE THE SURE THING OVER THE GAMBLE OR VICE VERSA.

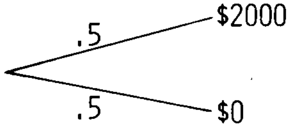
BUT (AND THIS IS THE IMPORTANT THING), THE CHOICE WE ARE PRESENTING YOU GETS TO BE MADE ONLY ONCE. THUS, YOUR PREFERENCES FOR SURE THINGS AS COMPARED TO GAMBLES (LOTTERIES) WILL COME INTO PLAY HERE.

Figure A-4. Page 4 of the Interview Process

UNCERTAINTY (CONTINUED)

SO AS A FIRST EXERCISE IN SUCH AN ASSESSMENT WHICH WOULD YOU CHOOSE: THE GAMBLE OR THE SURE THING? AGAIN THERE IS NO RIGHT OR WRONG ANSWER HERE.

<u>GAMBLE</u>



<u>SURE THING</u>

$1000

ANSWER: ____________________

NOW, SUPPOSE WE CAN CHANGE THE "SURE THING." WE WOULD LIKE TO FIND THE VALUE OF THE SURE THING SUCH THAT YOU WOULD BE EXACTLY INDIFFERENT BETWEEN THE SURE THING AND THE GAMBLE.

LET'S TRY A FEW POSSIBILITIES AND SEE IF WE CAN ZERO IN ON THAT POINT.

INDIFFERENCE PT ____________________

Figure A-5. Page 5 of the Interview Process

ENERGY SYSTEMS ATTRIBUTE

NOW WE ARE GOING TO DO THE SAME KIND OF EXERCISE WITH RESPECT TO ATTRIBUTES OF SOLAR ENERGY SYSTEMS.

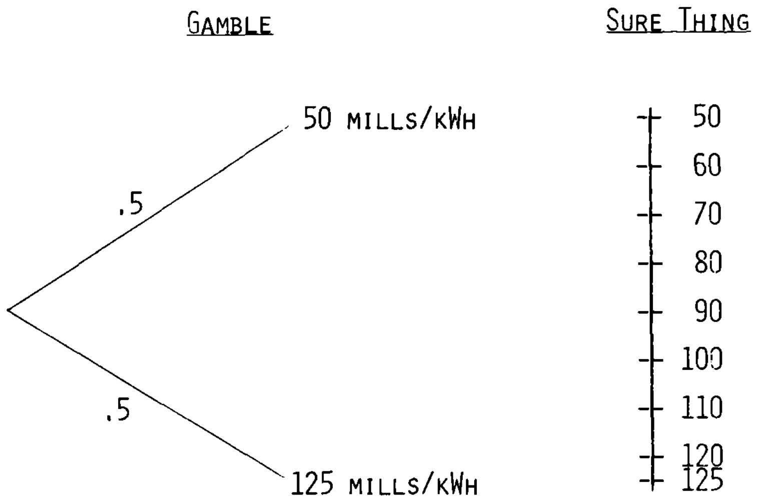
THE EIGHT ATTRIBUTES WE ARE GOING TO LOOK AT ARE:

- FIRST YEAR BUSBAR ENERGY COST
- CAPITAL COST
- PLANT RELIABILITY
- PLANT OUTPUT
- PLANT SAFETY
- R&D REQUIREMENTS
- ENVIRONMENTAL IMPACT
- APPLICATIONS FLEXIBILITY

READY? HERE WE GO.

Figure A-6. Page 6 of the Interview Process

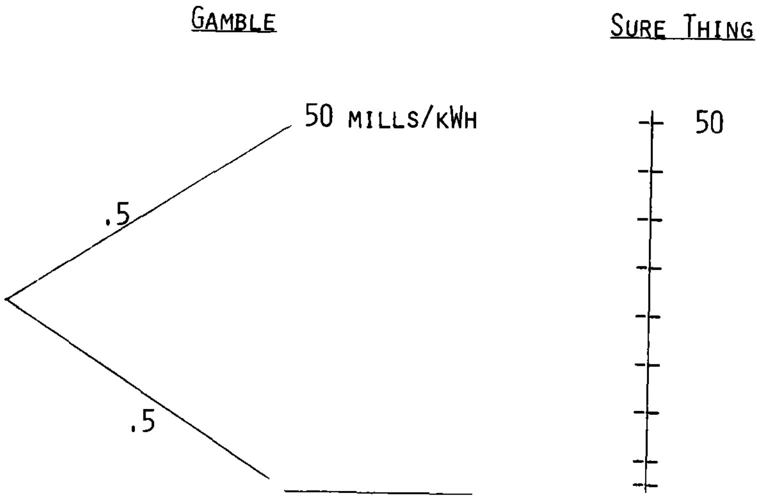
FIRST YEAR BUSBAR ENERGY COST (1978 DOLLARS)
(1)



INDIFFERENCE PT ______________

Figure A-7. Page 7 of the Interview Process

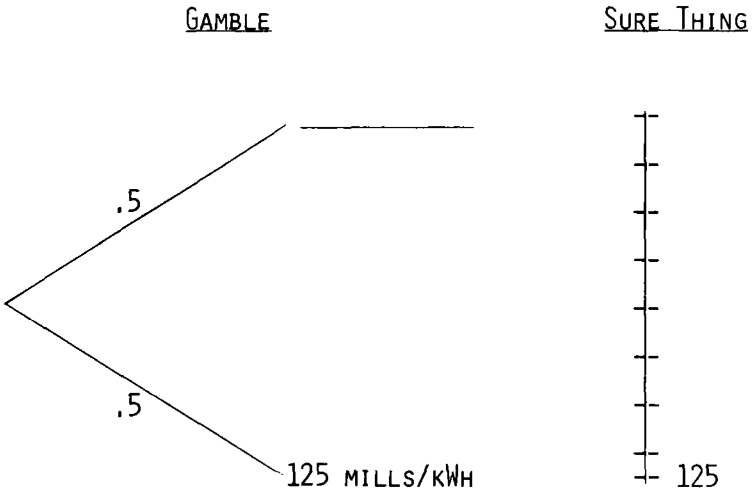
BUSBAR ENERGY COST
(2)



INDIFFERENCE PT ____________

Figure A-8. Page 8 of the Interview Process

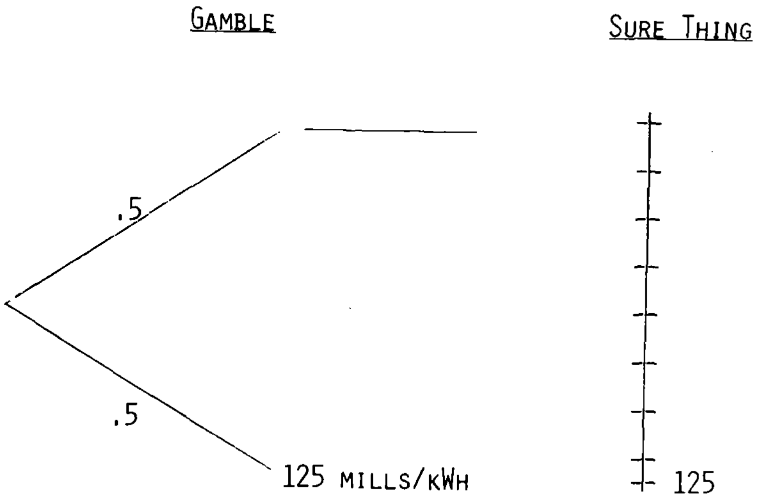
BUSBAR ENERGY COST
(3)



INDIFFERENCE PT ____________________

Figure A-9. Page 9 of the Interview Process

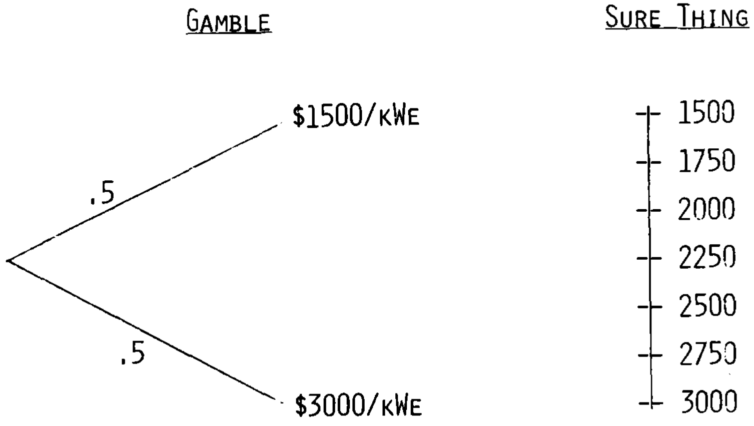
BUSBAR ENERGY COST
(4)



INDIFFERENCE PT ____________________

Figure A-10. Page 10 of the Interview Process

CAPITAL COST (1978 DOLLARS)

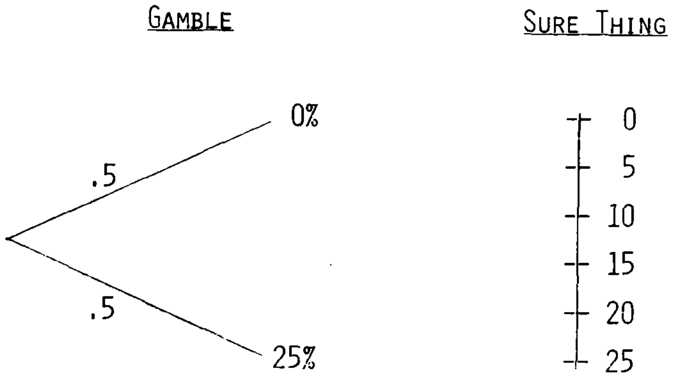


INDIFFERENCE PT ____________________

Figure A-11. Page 11 of the Interview Process

SYSTEM RELIABILITY

FORCED OUTAGE

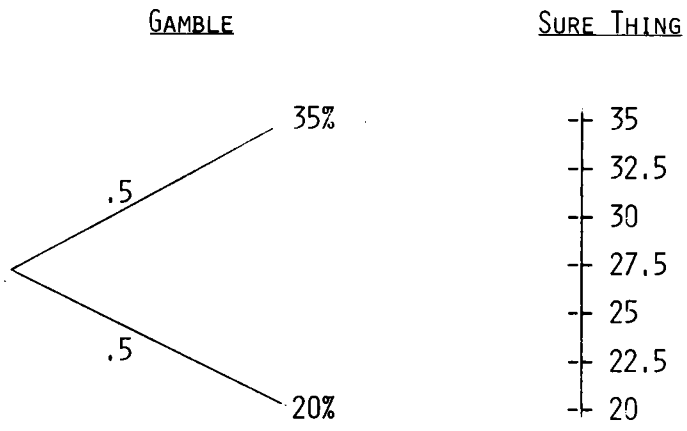


INDIFFERENCE PT ____________________

Figure A-12. Page 12 of the Interview Process

SYSTEM OUTPUT

CAPACITY FACTOR

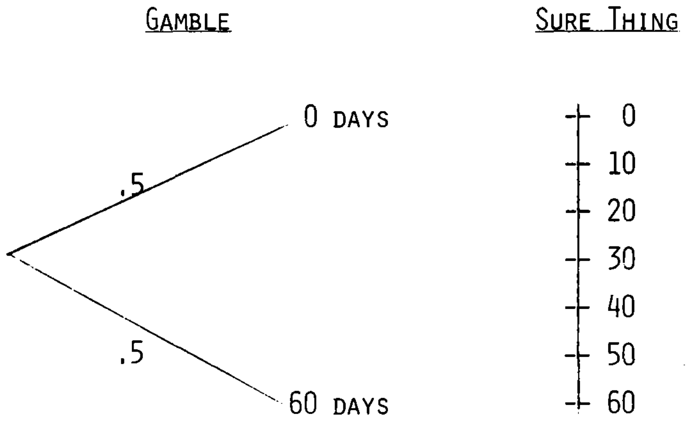


INDIFFERENCE PT ____________________

Figure A-13. Page 13 of the Interview Process

SAFETY

MAN DAYS LOST PER YEAR

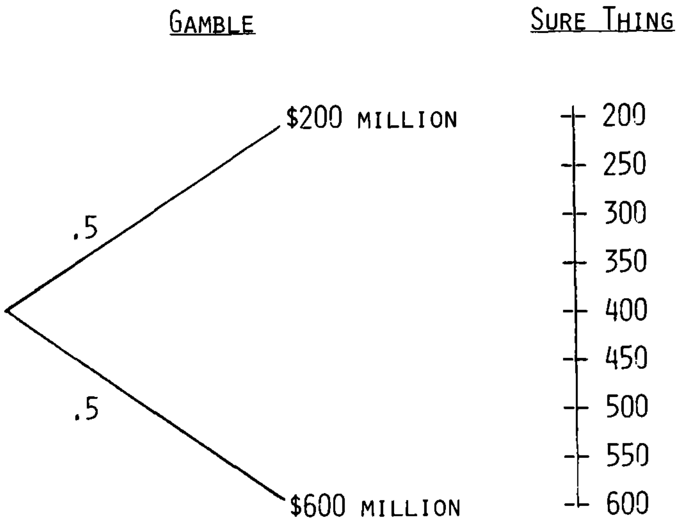


INDIFFERENCE PT ____________________

Figure A-14. Page 14 of the Interview Process

R&D COSTS

<u>1978 DOLLARS</u>



INDIFFERENCE PT ____________________

Figure A-15. Page 15 of the Interview Process

TWO OTHER ATTRIBUTES

ENVIRONMENTAL IMPACT

BEST CASE - MINIMAL ENVIRONMENTAL EFFECTS. SYSTEMS IN THIS CATEGORY WILL RESULT IN ENVIRONMENTAL IMPACTS WHICH WILL BE PRIMARILY CONFINED TO LAND DISRUPTION WITHIN THE SITE BOUNDARY. GASEOUS AND LIQUID EFFLUENTS ARE MINIMAL AND WATER USE IS RELATIVELY LOW. AN EXAMPLE OF A FACILITY WHICH POSES SIMILAR IMPACTS IS A COVERED WATER RESERVOIR REQUIRING A SIMILAR AMOUNT OF LAND.

WORST CASE - MAXIMUM ENVIRONMENTAL EFFECTS. SYSTEMS IN THIS CATEGORY WILL USE LARGE AMOUNTS OF LAND AND WATER AND WILL PRESENT POTENTIAL AIR QUALITY AND WATER QUALITY CONCERNS. THE GENERAL PUBLIC WILL VIEW THE SYSTEMS IN THIS CATEGORY AS UNSIGHTLY AND UNDESIRABLE WHEN PLACED NEAR URBAN AREAS OR WITHIN FREQUENTLY VIEWED NATURAL AREAS. SYSTEMS IN THIS CATEGORY WILL HAVE IMPACTS SIMILAR TO AN OIL TANK STORAGE FARM OF COMPARABLE SIZE.

Figure A-16. Page 16 of the Interview Process

APPLICATIONS FLEXIBILITY

APPLICATIONS FLEXIBILITY REFERS TO THE DEGREE OF MATCHING BETWEEN SYSTEM FACTORS AND PLANT REQUIREMENTS FOR VARIOUS APPLICATIONS. THE SYSTEM FACTORS OF MOST IMPORTANT CONSIDERATION ARE PLANT OUTPUT PRODUCTS (ELECTRICITY AND/OR THERMAL), HYBRID FLEXIBILITY, AND MODULARITY.

BEST CASE - **HIGH FLEXIBILITY**. SYSTEMS IN THIS CATEGORY PRODUCE BOTH ELECTRICITY AND HIGH TEMPERATURE THERMAL ENERGY. IN HYBRID CONFIGURATIONS THEY CAN UTILIZE SOLID AS WELL AS LIQUID AND GASEOUS FUELS AND ARE ABLE TO REACH AN OPERATIONAL CAPACITY FACTOR IN A RELATIVELY SHORT TIME THUS REQUIRING LESS FUEL. THESE SYSTEMS ARE HIGHLY MODULAR BEING MASS PRODUCED AND USING A SMALL LAND AREA.

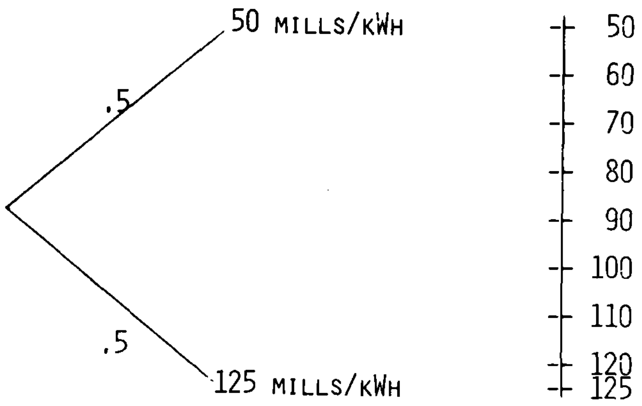
WORST CASE - **LOW FLEXIBILITY**. SYSTEMS IN THIS CATEGORY PRODUCE ONLY ELECTRICITY AND NO USABLE THERMAL ENERGY. IN HYBRID CONFIGURATIONS THEY CAN UTILIZE ONLY LIQUID OR GASEOUS FUELS AND REQUIRE A RELATIVELY LONG BURNING TIME TO REACH AN OPERATIONAL CAPACITY FACTOR. THESE SYSTEMS ARE GENERALLY CONSIDERED CUSTOM-MADE FIXED INSTALLATIONS AND ARE LAND-SHAPE CONSTRAINED.

Figure A-17. Page 17 of the Interview Process

INDEPENDENCE OF ATTRIBUTES

REFERRING AGAIN TO BUSBAR ENERGY COSTS:

WOULD YOU BE MORE (OR LESS) WILLING TO TAKE A GAMBLE ON BUSBAR ENERGY COSTS IF ALL OTHER ATTRIBUTES SHIFTED FROM THEIR WORST TO THEIR BEST VALUES?



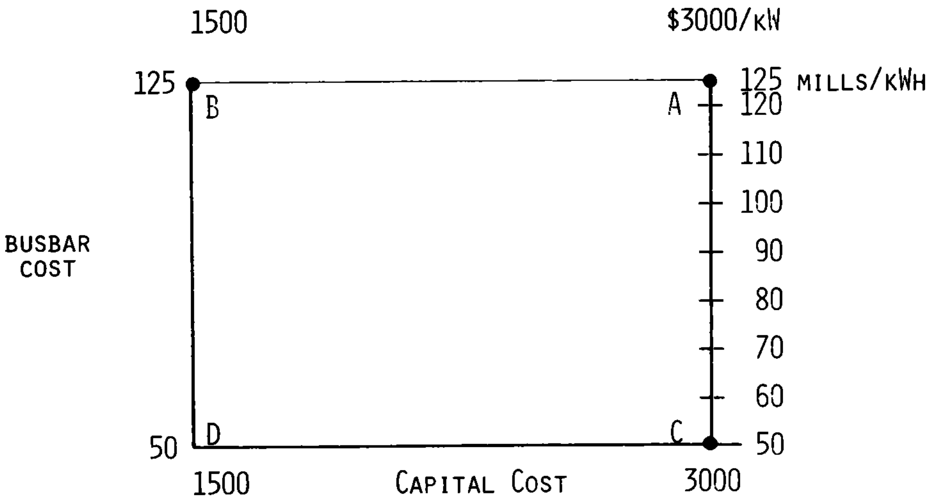
INDIFFERENCE PT ______________

INDIFFERENCE PT FOR WORST CASE OTHER ______________

INDIFFERENCE PT FOR BEST CASE OTHER ______________

Figure A-18. Page 18 of the Interview Process

RELATIVE IMPORTANCE OF CAPITAL COST

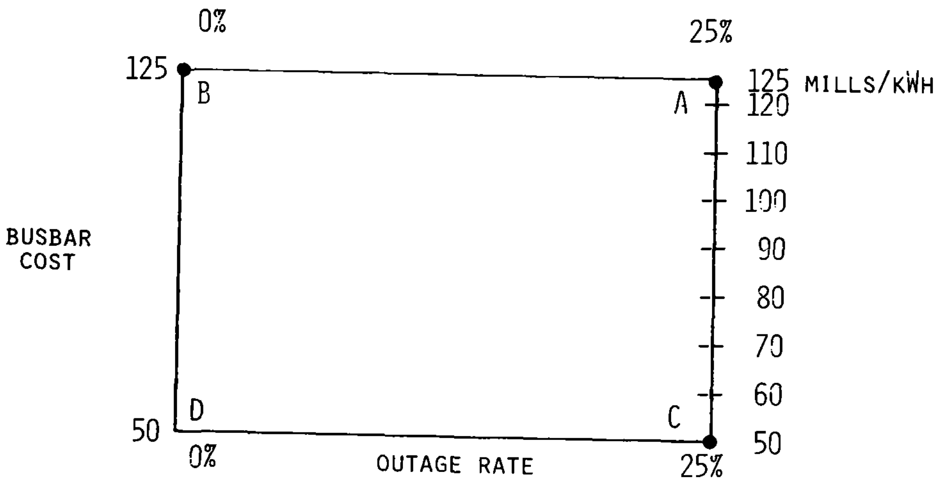


INDIFFERENCE PT (B′) ____________

INDEPENDENCE? ____________

Figure A-19. Page 19 of the Interview Process

RELATIVE IMPORTANCE OF RELIABILITY

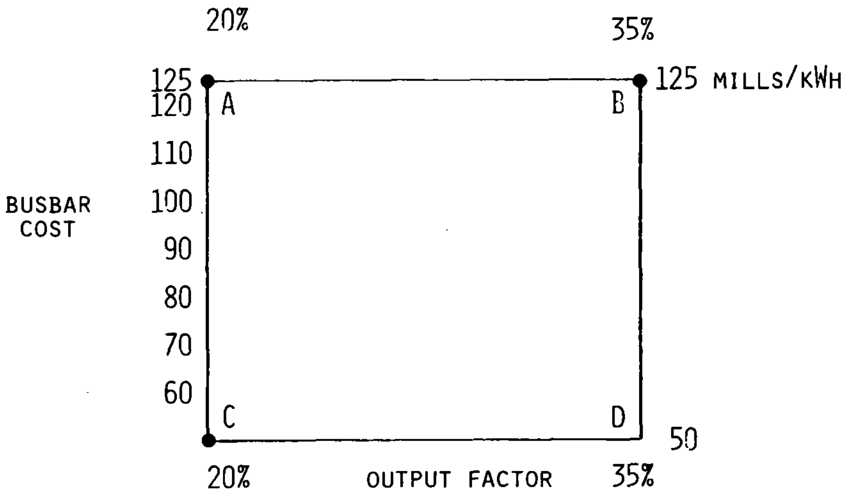


INDIFFERENCE PT (B') ______________

INDEPENDENCE? ________________

Figure A-20. Page 20 of the Interview Process

RELATIVE IMPORTANCE OF SYSTEM OUTPUT

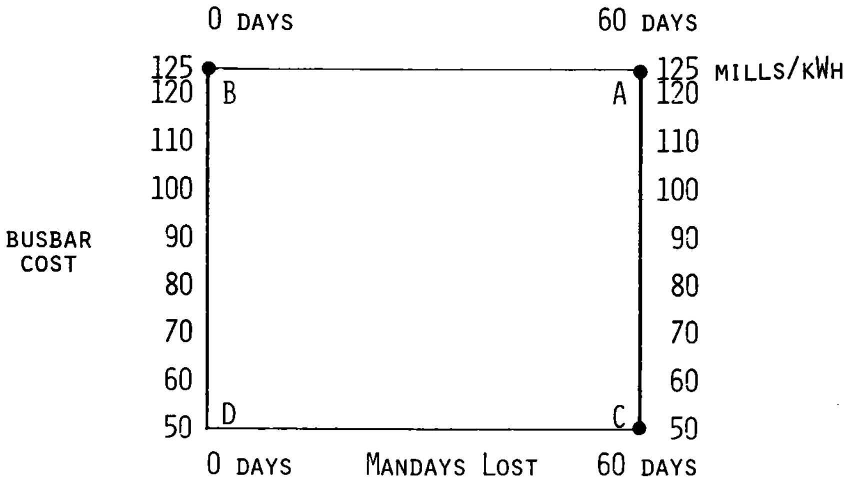


INDIFFERENCE PT (B') ____________

INDEPENDENCE? ______________

Figure A-21. Page 21 of the Interview Process

RELATIVE IMPORTANCE OF SAFETY

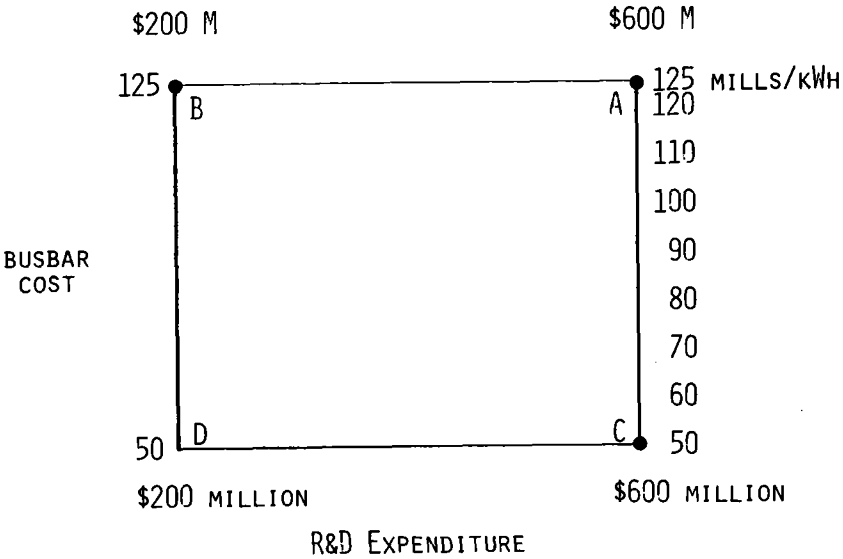


INDIFFERENCE PT (B′) ____________________

INDEPENDENCE? ___________________________

Figure A-22. Page 22 of the Interview Process

RELATIVE IMPORTANCE OF R&D

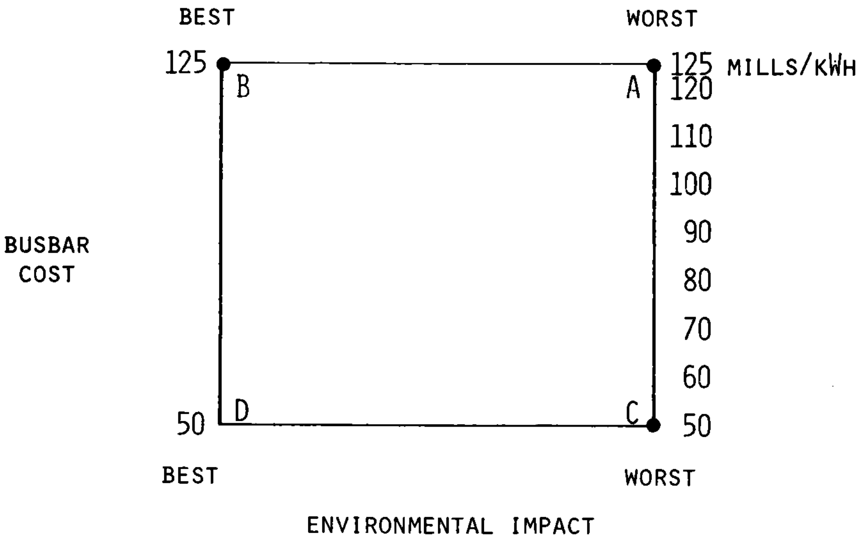


INDIFFERENCE PT (B′) ______________________

INDPENDENCE? ______________________

Figure A-23. Page 23 of the Interview Process

RELATIVE IMPORTANCE OF ENVIRONMENTAL IMPACT

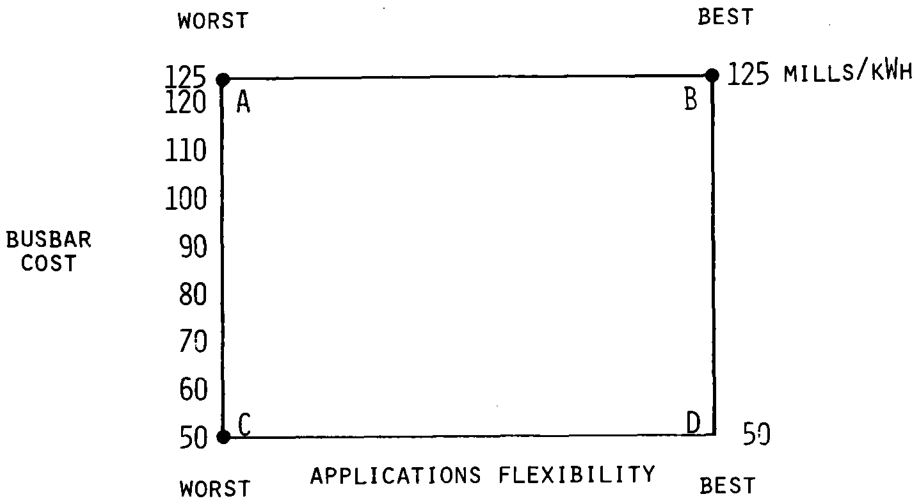


INDIFFERENCE PT (B′) ________________

INDEPENDENCE ________________________

Figure A-24. Page 24 of the Interview Process

RELATIVE IMPORTANCE OF APPLICATIONS FLEXIBILITY



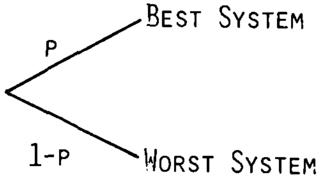
INDIFFERENCE PT (B') ____________________

INDEPENDENCE ____________________________

Figure A-25. Page 25 of the Interview Process

IMPORTANCE OF BUSBAR COSTS

GAMBLE



SURE THING

REFERENCE SYSTEM

P = ____

	BUSBAR COST	CAPITAL COST	OUTAGE RATE	OUTPUT FACTOR	SAFETY (M-D LOST)	R&D	ENVIRN.	APPL. FLEX.
REFERENCE:	50							
		3000	25%	20%	60	$600M	WORST	WORST
BEST:	50	1500	0%	35%	0	$200M	BEST	BEST
WORST:								
	125	3000	25%	20%	60	$600M	WORST	WORST

WHAT DO YOU WIN, IF YOU WIN THE GAMBLE?

WHAT DO YOU LOSE, IF YOU LOSE THE GAMBLE?

Figure A-26. Page 26 of the Interview Process

APPENDIX B

PROGRAM FOR CALCULATING ATTRIBUTE UTILITY FUNCTIONS ON AN HP-97 PROGRAMMABLE CALCULATOR

APPENDIX B

PROGRAM FOR CALCULATING ATTRIBUTE UTILITY FUNCTIONS ON AN HP-97 PROGRAMMABLE CALCULATOR

I. INTRODUCTION

This Appendix presents a program for calculating utility function values of the form $u(x) = a + be^{cx}$ or $u(x) = a + bx$ on a Hewlett Packard HP-97 or HP-67 programmable calculator. The name of the program is UTICA 97: "An HP-97 Program for UTIlity CAlculations of the Form $u(x) = a + be^{cx}$." It calculates utility function values u(x) given only the least-preferred state x^o, the certainty equivalent state x' of a 50/50 gamble between the least-preferred state and the most-preferred state, and the most preferred state x^*. If x^o, x', and x^* are entered, the utility function is assumed to be the exponential form:

$$u(x) = a + be^{cx} \tag{B-1}$$

unless $x' = 1/2(x^o + x^*)$, in which case the utility function is the linear form:

$$u(x) = a + bx \; . \tag{B-2}$$

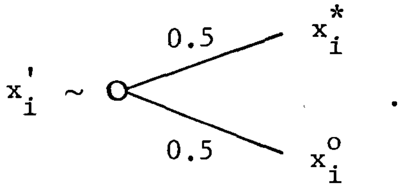
If only x^o and x^* are entered, the utility function is assumed to be linear.

II. THE COMPUTATIONAL EQUATIONS

In the Keeney Multiplicative Utility Function (References 4-1 and 4-7), the assignments of $u_i(x_i^o) = 0.0$ and $u_i(x_i^*) = 1.0$ are required. Only one gamble is required to determine an attribute utility function of the form

$$u(x_i) = a_i + b_i \, e^{c_i x_i} \; . \tag{B-3}$$

The single gamble used in UTICA 97 is a 50/50 gamble between the least-preferred attribute state x_i^o and the most preferred attribute state x_i^*. The gamble is required to determine the certainty equivalent to the lottery x_i', which is used in the computational equations. The indifference diagram is:



(B-4)

The attribute utility function value of x_i' will be:

$$u_i(x_i') = 0.5 \,. \tag{B-5}$$

With x_i' determined, the three constants a_i, b_i, and c_i can be calculated from:

$$a_i = -b_i \, e^{c_i x_i^o} \tag{B-6}$$

$$b_i = \frac{1}{e^{c_i x_i^*} - e^{c_i x_i^o}} \tag{B-7}$$

$$0 = e^{c_i x_i^*} - 2e^{c_i x_i'} + e^{c_i x_i^o} \tag{B-8}$$

The constant c_i must be obtained by using a technique from numerical analysis for finding the roots of an equation. Once the constant c_i is determined, it is then possible to solve for a_i and b_i.

If $x_i' = 1/2(x_i^o + x_i^*)$, then the attribute utility function is linear, where:

$$u_i(x_i) = a_i + b_i x_i \tag{B-9}$$

$$a_i = -b_i x_i^o \tag{B-10}$$

$$b_i = \frac{1}{x_i^* - x_i^o} \tag{B-11}$$

III. THE COMPUTATIONAL PROCEDURE

The computational procedure starts by determining whether the functional form is exponential or linear. If the functional form is linear, then the constants a_i and b_i are readily calculated from x_i^o and x_i^*. If the functional form is exponential, then the procedure is more complicated.

If the functional form is exponential, then c_i is calculated from

$$0 = e^{c_i x_i^*} - 2e^{c_i x_i'} + e^{c_i x_i^o} \tag{B-8}$$

by a numerical analysis technique for finding the roots of an equation. The root is to be obtained from the equation:

$$f(\xi_i) = e^{\xi_i x_i^*} - 2e^{\xi_i x_i'} + e^{\xi_i x_i^o} . \tag{B-12}$$

For the value of ξ_i for which $f(\xi_i) = 0$, $\xi_i = c_i$ if the correct root has been found. There is obviously a root at $\xi_i = 0$ which is incorrect. Thus the shape of $f(\xi_i)$ must be examined to ensure that the numerical analysis technique will converge to the proper root. Table B-1 gives the sign of c_i based on properties of the attribute utility function. Figures B-1 through B-4 give two of the four basic shapes of $f(\xi_i)$. The other shapes are obtained by reflection across the ordinate.

The program finds the correct root by an "interval halving" or "bisection" technique of numerical analysis (Reference B-1). The convergence rate of this technique is slow, but it is possible to assign initial values for the iteration that guarantee the iteration will converge to the proper root. The program first determines the sign of c_i by examining the first derivative of $f(\xi)$. From Figures B-1 through B-4 it can be seen that if $f'(0) < 0$, then $c_i > 0$ and if $f'(0) > 0$ then $c_i < 0$. The initial iteration starts with the interval between $\xi_i = 0$ and ξ_i^o, where ξ_i^o is selected such that $f(\xi_i^o) > 0$. It can be seen from Figures B-1 through B-4 that with the correct sign for ξ_i^o, the root $\xi_i = c_i$ will always lie between $\xi_i = 0$ and ξ_i^o. The initial value ξ_i^o is found (the sign is already known) by trying $|\xi_i^o| = (|x^o| + |x^*|)^{-1}$ and testing to see if $f(\xi_i^o) > 0$. If not, ξ_i^o is doubled and tested again. These trial initial values are selected to ensure that the calculator will not overflow when exponentials are calculated. From Figures B-1 through B-4 it can be seen that an initial value will eventually be found such that $f(\xi_i^o) > 0$. Then the interval bisection technique can be applied to determine c_i. After c_i has been obtained, then b_i and a_i are readily calculated.

Table B-1. Sign of Constant "c_i" Based on Properties of the Utility Function.

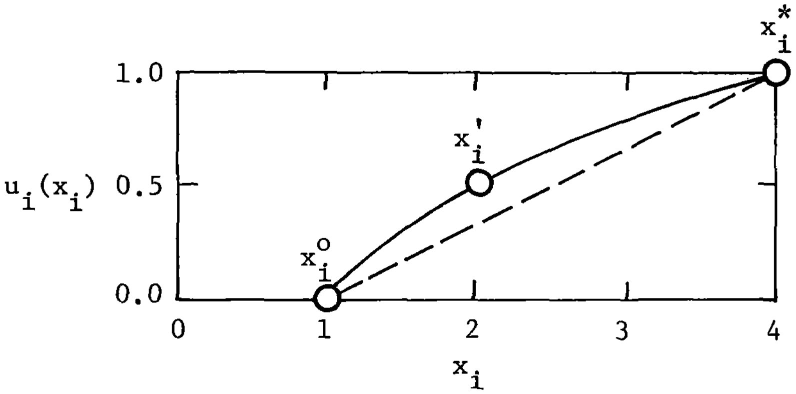
Figure	$u_i(x_i)$	x_i'	Risk Property	c_i
B-1	Increasing	$x_i' < \frac{1}{2}(x_i^o + x_i^*)$	Risk averse	$c_i < 0$
B-2	Increasing	$x_i' > \frac{1}{2}(x^o + x_i^*)$	Risk prone	$c_i > 0$
B-3	Decreasing	$x_i' > \frac{1}{2}(x^o + x_i^*)$	Risk averse	$c_i > 0$
B-4	Decreasing	$x_i' < \frac{1}{2}(x^o + x_i^*)$	Risk prone	$c_i < 0$

$x_i^o = 1$ $\qquad a_i = 1.3090$

$x_i' = 2$ $\qquad b_i = -2.1180$

$x_i^* = 4$ $\qquad c_i = -0.4812$

$$u_i(x_i) = a_i + b_i e^{c_i x_i}$$



$$f(\xi_i) = e^{\xi_i x_i^*} - 2e^{\xi_i x_i'} + e^{\xi_i x_i^o}$$

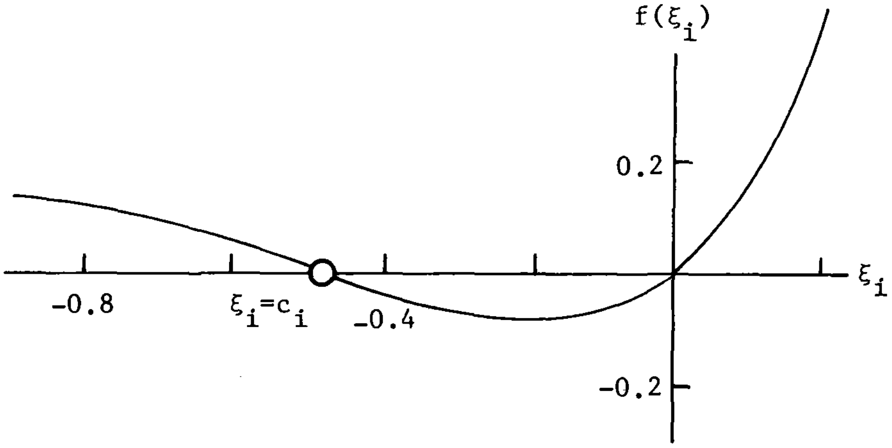


Figure B-1. Utility Function and $f(\xi_i)$ for $u_i(x_i)$ Increasing and Risk Averse.

$x_i^o = 1$

$x_i' = 3$

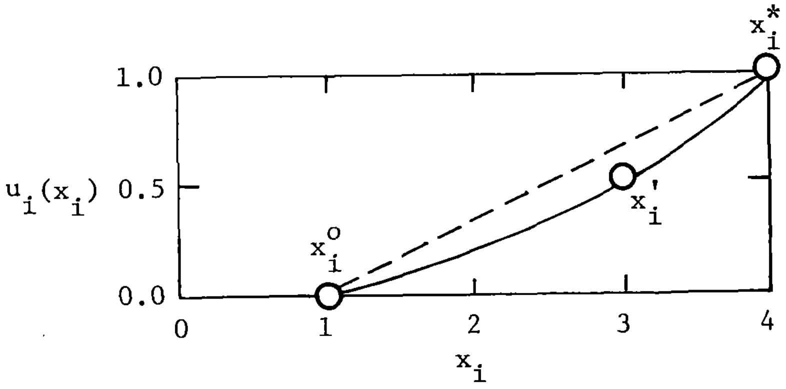
$x_i^* = 4$

$a_i = -0.3090$

$b_i = 0.1910$

$c_i = 0.4812$

$$u_i(x_i) = a_i + b_i e^{c_i x_i}$$



$$f(\xi_i) = e^{\xi_i x_i^*} - 2e^{\xi_i x_i'} + e^{\xi_i x_i^o}$$

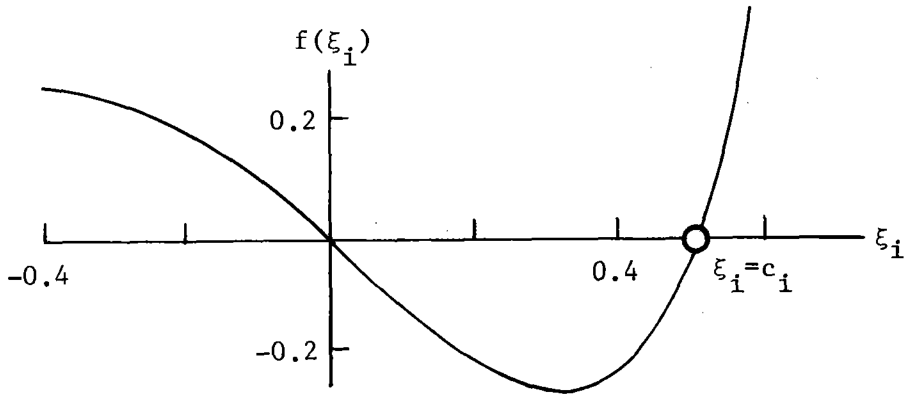


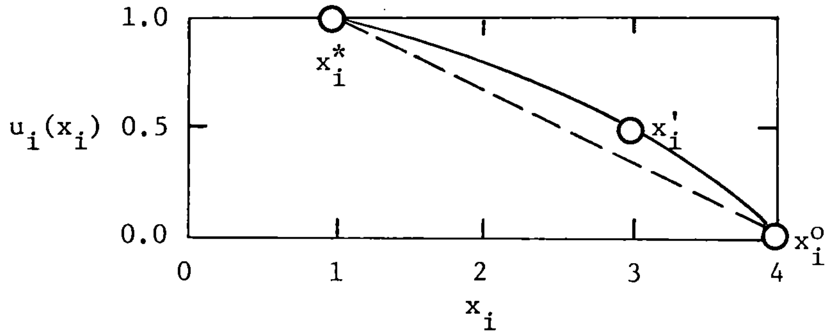
Figure B-2. Utility Function and $f(\xi_i)$ for $u_i(x_i)$ Increasing and Risk Prone.

$x_i^o = 4$ $\qquad$ $a_i = 1.3090$

$x_i' = 3$ $\qquad$ $b_i = -0.1910$

$x_i^* = 1$ $\qquad$ $c_i = 0.4812$

$$u_i(x_i) = a_i + b_i e^{cx}$$



$$f(\xi_i) = e^{\xi_i x_i^*} - 2e^{\xi_i x_i'} + e^{\xi_i x_i^o}$$

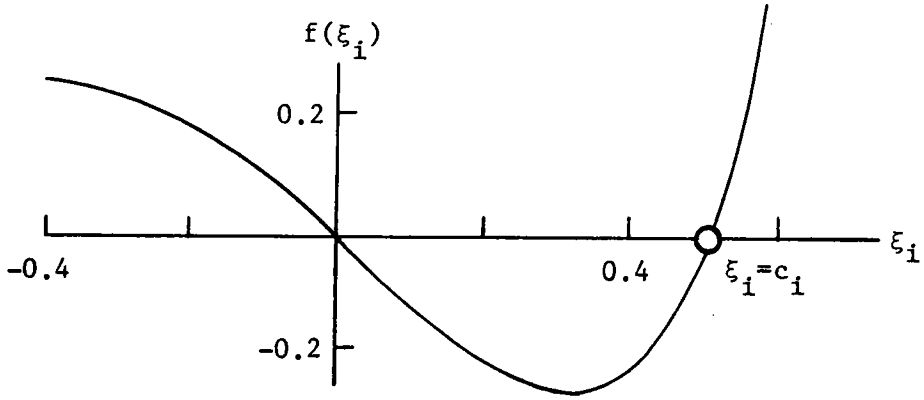


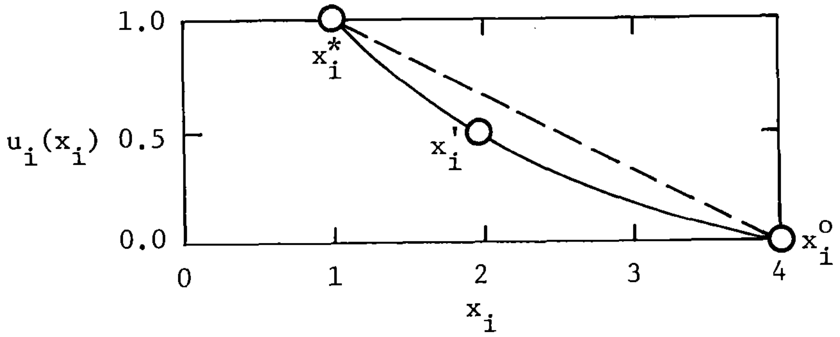
Figure B-3. Utility Function and f(c) for u(x) Decreasing and Risk Averse.

$x_i^o = 4 \qquad a_i = -0.3090$

$x_i' = 2 \qquad b_i = 2.1180$

$x_i^* = 1 \qquad c_i = -0.4812$

$$u_i(x_i) = a_i + b_i e^{c_i x_i}$$



$$f(\xi_i) = e^{\xi_i x_i^*} - 2e^{\xi_i x_i'} + e^{\xi_i x_i^o}$$

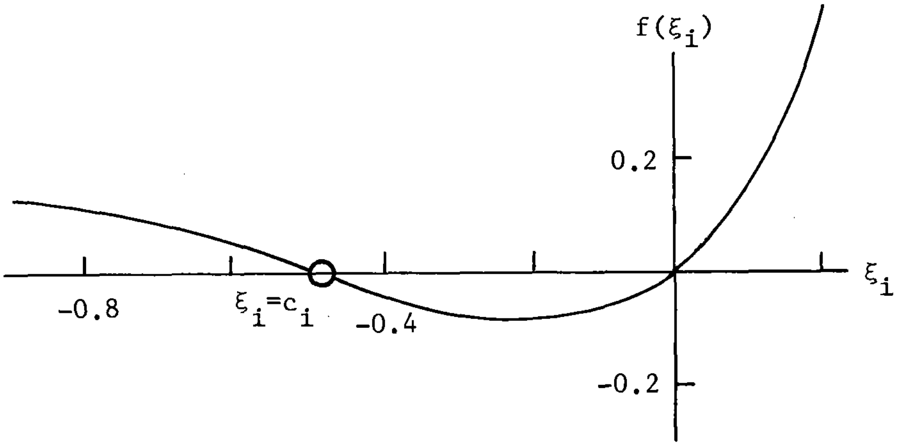


Figure B-4. Utility Function and f(c) for u(x) Decreasing and Risk Prone.

Program Description

Program Title UTICA 97: UTIlity CAlculations of the Form $u(x) = a + be^{cx}$.

Name Ralph F. Miles, Jr. **Date** 12/19/78

Address Jet Propulsion Laboratory, 4800 Oak Grove Drive

City Pasadena **State** California **Zip Code** 91103

Program Description, Equations, Variables, etc. UTICA 97 is a program for Hewlett Packard HP-97 or HP-67 programmable calculators. It calculates utility function values $u(x)$ given only the least-preferred state x^o, the 50/50 certainty equivalent state x', and the most-preferred state x^*. If x^o, x', and x^* are entered, the utility function is assumed to be the exponential form

$$u(x) = a + be^{cx}$$

unless $x' = 1/2(x^o + x^*)$ in which case the utility function is the linear form

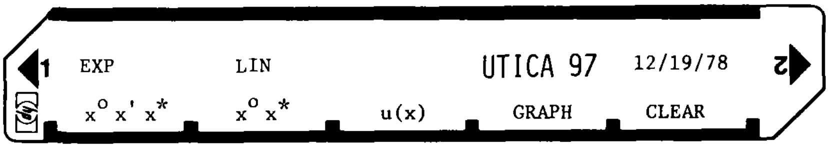
$$u(x) = a + bx \ .$$

If only x^o and x^* are entered, the utility function is assumed to be linear. The constant "c" is obtained by an interval bisection technique to find the root of the equation $f(\xi)$, where $f(c) = 0$:

$$f(\xi) = e^{\xi x^*} - 2e^{\xi x'} + e^{\xi x^o} \ .$$

Operating Limits and Warnings Only monotonic functions can be calculated. The certainty equivalent state x' must lie in the open interval (x^o, x^*).

User Instructions



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	For exponential form go to Step 2.			
	For linear form go to Step 9.			
2	Enter least-preferred state.	x^o	A	x^o
				0.0000
3	Enter 50/50 certainty equivalent.	x'	A	x'
				0.5000
4*	Enter most-preferred state.	x^*	A	x^*
				1.0000
				a
				b
				c
5	Enter x to calculate u(x).	x	C	x
				u(x)
6	Repeat Step 5 to calculate other u(x).			
7	Graph u(x) from x^o to x^*.		D	Graph
8	Clear Program for new utility function.		E	0.0000
9	Enter least-preferred state.	x^o	B	x^o
				0.0000
10*	Enter most-preferred state.	x^*	B	x^*
				1.0000
11	Enter x to calculate u(x).	x	C	x
				u(x)
12	Repeat Step 11 to calculate other u(x).			
13	Graph u(x) from x^o to x^*.		D	Graph
14	Clear Program for new utility function.		E	0.0000
*	Due to program memory limitations, the constants "a" and "b" are not printed if u(x) is linear. They may be displayed by keystrokes RCLA for "a" and RCLB for "b".			

STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	*LBLA
002	STO0	35 00	Exponential form. Store x^o.
003	PRTX	-14	
004	0	00	
005	PRTX	-14	
006	RTN	24	
007	*LBLA	21 11	*LBLA
008	STO1	35 01	Exponential form. Store x'.
009	PRTX	-14	
010	0	00	
011	.	-62	
012	5	05	
013	PRTX	-14	
014	RTN	24	
015	*LBLA	21 11	*LBLA
016	STO2	35 02	Exponential form. Store x^*.
017	PRTX	-14	
018	1	01	
019	PRTX	-14	
020	SPC	16-11	
021	RCL0	36 00	Test for linear form. $c = 0$?
022	RCL2	36 02	
023	+	-55	
024	2	02	
025	÷	-24	
026	RCL1	36 01	
027	X=Y?	16-33	
028	GTO5	22 05	
029	RCL0	36 00	Test for sign of $f'(0)$. $f'(0) > 0 \rightarrow$ Set F1.
030	RCL1	36 01	
031	2	02	
032	x	-35	
033	-	-45	
034	RCL2	36 02	
035	+	-55	
036	X>0?	16-44	
037	SF0	16 21 00	
038	RCL0	36 00	Calculate and store ξ^o.
039	ABS	16 31	
040	RCL2	36 02	
041	ABS	16 31	
042	+	-55	
043	1/X	52	
044	F0?	16 23 00	
045	CHS	-22	
046	STO5	35 05	
047	*LBL1	21 01	$f(\xi^i) \leq 0 \rightarrow \xi^{i+1} = 2\xi^i$. $f(\xi^i) > 0 \rightarrow$ GTO 2.
048	GSBa	23 16 11	
049	X>0?	16-44	
050	GTO2	22 02	
051	RCL5	36 05	
052	2	02	
053	STx5	35-35 05	
054	GTO1	22 01	
055	*LBL2	21 02	Reg. 5 → Reg. 7.
056	RCL5	36 05	

STEP	KEY ENTRY	KEY CODE	COMMENTS
057	STO7	35 07	
058	*LBL3	21 03	Test to end iteration.
059	RCL7	36 07	
060	RCL6	36 06	
061	-	-45	
062	RCL7	36 07	
063	÷	-24	
064	ABS	16 31	
065	EEX	-23	
066	6	06	
067	CHS	-22	
068	X>Y?	16-34	
069	GTO4	22 04	
070	RCL7	36 07	Calculate ξ^{i+1} and store in Reg. 6 or Reg. 7.
071	RCL6	36 06	
072	+	-55	
073	2	02	
074	÷	-24	
075	STO5	35 05	
076	GSBa	23 16 11	
077	CF1	16 22 01	
078	X>0?	16-44	
079	SF1	16 21 01	
080	RCL5	36 05	
081	F1?	16 23 01	
082	STO7	35 07	
083	F1?	16 23 01	
084	GTO3	22 03	
085	STO6	35 06	
086	GTO3	22 03	
087	*LBL4	21 04	Store c.
088	RCL5	36 05	
089	STOC	35 13	
090	RCL2	36 02	Calculate b. $b = \dfrac{1}{e^{cx^*} - e^{cx^o}}$
091	RCLC	36 13	
092	x	-35	
093	e^x	33	
094	RCL0	36 00	
095	RCLC	36 13	
096	x	-35	
097	e^x	33	
098	-	-45	
099	1/X	52	
100	STOB	35 12	
101	RCL0	36 00	Calculate a. $a = -be^{cx^o}$.
102	RCLC	36 13	
103	x	-35	
104	e^x	33	
105	RCLB	36 12	
106	x	-35	
107	CHS	-22	
108	STOA	35 11	
109	PRTX	-14	Print a, b, and c.
110	RCLB	36 12	
111	PRTX	-14	
112	RCLC	36 13	

REGISTERS

0 x^o	1 x'	2 x^*	3 δ	4 $x^o + n\delta$	5 ξ	6 ξ^-	7 ξ^+	8	9
S0	S1	S2	S3	S4	S5	S6	S7	S8	S9
A a	B b	C c	D	E	I Graph Index				

Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	PRTX	-14	
114	SPC	16-11	
115	CLX	-51	
116	RTN	24	
117	*LBLa	21 16 11	Subroutine a.
118	RCL0	36 00	Calculate f(ξ). f(ξ) = $e^{\xi x^*} - 2e^{\xi x'} + e^{\xi x^o}$
119	RCL5	36 05	
120	x	-35	
121	e^x	33	
122	RCL1	36 01	
123	RCL5	36 05	
124	x	-35	
125	e^x	33	
126	2	02	
127	x	-35	
128	-	-45	
129	RCL2	36 02	
130	RCL5	36 05	
131	x	-35	
132	e^x	33	
133	+	-55	
134	RTN	24	
135	*LBLB	21 12	*LBLB
136	STO0	35 00	Linear form Store x^o.
137	PRTX	-14	
138	0	00	
139	PRTX	-14	
140	RTN	24	
141	*LBLB	21 12	*LBLB
142	STO2	35 02	Linear form. Store x^*.
143	PRTX	-14	
144	1	01	
145	PRTX	-14	
146	SPC	16-11	
147	*LBL5	21 05	Calculate b. $b = \frac{1}{x^* - x^o}$
148	RCL2	36 02	
149	RCL0	36 00	
150	-	-45	
151	1/X	52	
152	STOB	35 12	
153	RCL0	36 00	Calculate a. $a = -bx^o$.
154	x	-35	
155	CHS	-22	
156	STOA	35 11	
157	CLX	-51	
158	RTN	24	
159	*LBLC	21 13	*LBLC
160	PRTX	-14	Calculate u(x).
161	RCLC	36 13	
162	X=0?	16-43	
163	GTO6	22 06	
164	GSBb	23 16 12	
165	RTN	24	
166	*LBL6	21 06	
167	R↓	-31	
168	GSBc	23 16 13	

STEP	KEY ENTRY	KEY CODE	COMMENTS
169	RTN	24	
170	*LBLD	21 14	*LBLD
171	RCL2	36 02	Calculate δ.
172	RCL0	36 00	
173	-	-45	
174	2	02	
175	0	00	
176	÷	-24	
177	STO3	35 03	
178	RCL0	36 00	Initialize for graph.
179	STO4	35 04	
180	2	02	
181	1	01	
182	STOI	35 46	
183	RCLC	36 13	Test for linear form.
184	X=0?	16-43	
185	GTO8	22 08	
186	*LBL7	21 07	Graph $u(x) = a + be^{cx}$.
187	RCL4	36 04	
188	PRTX	-14	
189	RCLC	36 13	
190	GSBb	23 16 12	
191	RCL3	36 03	
192	ST+4	35-55 04	
193	DSZI	16 25 46	
194	GTO7	22 07	
195	CLX	-51	
196	RTN	24	
197	*LBL8	21 08	Graph u(x) = a + bx.
198	RCL4	36 04	
199	PRTX	-14	
200	GSBc	23 16 13	
201	RCL3	36 03	
202	ST+4	35-55 04	
203	DSZI	16 25 46	
204	GTO8	22 08	
205	CLX	-51	
206	RTN	24	
207	*LBLb	21 16 12	Subroutine b.
208	x	-35	Calculate $u(x) = a + be^{cx}$.
209	e^x	33	
210	GSBc	23 16 13	
211	RTN	24	
212	*LBLc	21 16 13	Subroutine c.
213	RCLB	36 12	Calculate {a + b(·)}. Print u(x).
214	x	-35	
215	RCLA	36 11	
216	+	-55	
217	PRTX	-14	
218	SPC	16-11	
219	RTN	24	
220	*LBLE	21 15	*LBLE
221	CLX	-51	Clear Program.
222	CLRG	16-53	
223	CF0	16 22 00	
224	RTN	24	

LABELS

A 1, 7, 15	B 135, 141	C 159	D 170	E 220
a 117	b 207	c 212	d	e
0	1 047	2 055	3 058	4 087
5 147	6 166	7 186	8 197	9

FLAGS

0	f'(0) > 0
1	f(ξ) > 0
2	
3	

SET STATUS

FLAGS	ON	OFF
0	☐	☒
1	☐	☒
2	☐	☒
3	☐	☒

TRIG	
DEG	☒
GRAD	☐
RAD	☐

DISP	
FIX	☒
SCI	☐
ENG	☐
n	4

APPENDIX C

FORMULAS AND PROOF FOR CALCULATING ATTRIBUTE SCALING CONSTANTS FROM INDIFFERENCE RELATIONS

APPENDIX C

FORMULAS AND PROOF FOR CALCULATING ATTRIBUTE SCALING CONSTANTS FROM INDIFFERENCE RELATIONS

I. INTRODUCTION

This Appendix presents a derivation of the formulas used for calculating attribute scaling constants determined from indifference relations. The formulas appear in Equations (4-18) and (4-19).

II. FORMULAS

For $k_i < k_1$, $i \neq 1$, where k_1 is the scaling constant for the reference attribute x_1, the formulas is:

$$k_i = k_1 u_1(x_1^i), \tag{C-1}$$

where x_1^i is assessed from the indifference relation:

$$(x_1^i, x_i^o) \sim (x_1^o, x_i^*) \tag{C-2}$$

as shown in Diagram a of Figure C-1.

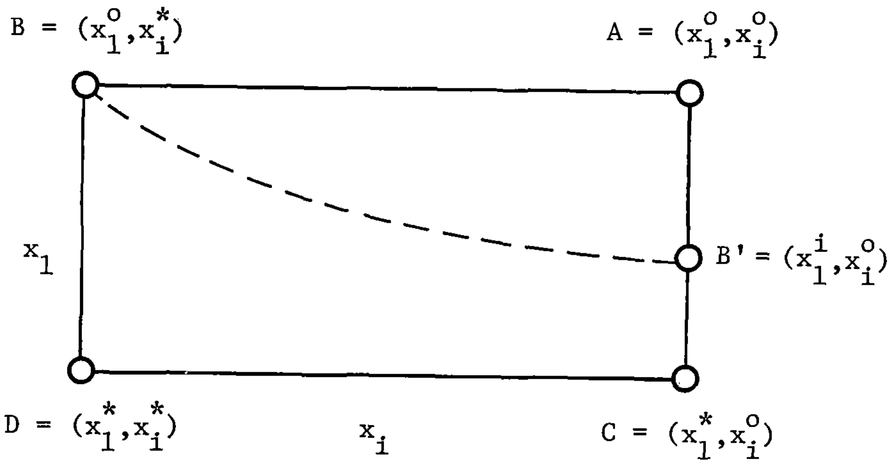
For $k_i > k_1$, $i \neq 1$, where k_1 is the scaling constant for the reference attribute x_1, the formula is:

$$k_i = \frac{k_1}{u_i(x_i^1)}, \tag{C-3}$$

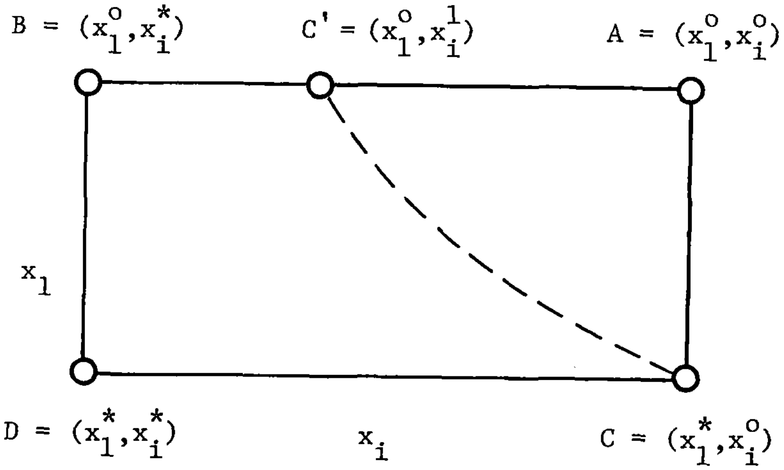
where x_i^1 is assessed from the indifference relation:

$$(x_1^o, x_i^1) \sim (x_1^*, x_i^o) \tag{C-4}$$

as shown in Diagram b of Figure C-1.



a. Trade-off Diagram for $k_i < k_1$. $B' = (x_1^i, x_i^o) \sim B = (x_1^o, x_1^*)$



b. Trade-off Diagram for $k_i > k_1$. $C' = (x_1^o, x_i^1) \sim C = (x_1^*, x_i^o)$

Figure C-1. Trade-off Diagrams for Assessing k_i

III. PROOF

The attribute scaling constant k_i is defined from:

$$u(x_i,\bar{x}_i^o) \triangleq k_i\, u_i(x_i) \tag{C-5}$$

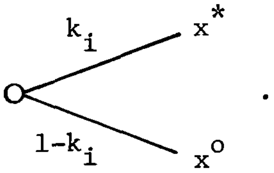
$$u_i(x_i^*) = 1.0 \tag{C-6}$$

Therefore,

$$k_i = u(x_i^*,\bar{x}_i^o), \tag{C-7}$$

and can be assessed from the gamble:

$$(x_i^*,\ \bar{x}_i^o) \sim$$



(C-8)

This follows from the mathematical formulation of the gamble of Equation (C-8) which yields (with $u(x^*) = 1.0$ and $u(x^o) = 0.0$):

$$u(x_i^*,\bar{x}_i^o) = k_i u(x^*) + (1-k_i)\, u(x^o) = k_i\ . \tag{C-9}$$

This gamble will be used to assess k_1. The other scaling constants will be determined from indifference relations.

For $k_i < k_1$, $i \neq 1$, the formula for k_i follows from the assumption that there exists x_1^i such that:

$$(x_1^i,\bar{x}_1^o) \sim (x_i^*,\bar{x}_i^o)\ . \tag{C-10}$$

Assume that preferential independence exists for all two-attribute pairs (x_i, x_j), $i \neq j$. Therefore it is meaningful to assess x_1^i from the indifference relation:

$$(x_1^i, x_i^o) \sim (x_1^o, x_i^*). \qquad \text{(C-11)}$$

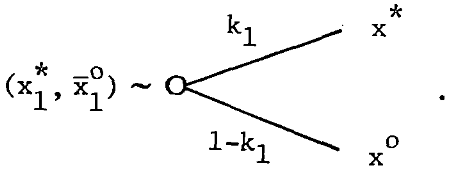
By the indifference relation of Equation (C-10):

$$u(x_1^i, \bar{x}_1^o) = u(x_i^*, \bar{x}_i^o). \qquad \text{(C-12)}$$

By Equations (C-7) and (C-12)

$$k_i = u(x_i^*, \bar{x}_i^o) = u(x_1^i, \bar{x}_1^o). \qquad \text{(C-13)}$$

Now k_1 is assessed by setting $i = 1$ in the gamble of Equation (C-8):



(C-14)

Having assessed x_1^i from Equation (C-11) and k_1 from Equation (C-14), by Equations (C-5) and (C-13) one obtains:

$$k_i = u(x_1^i, \bar{x}_1^o) = k_1\, u_1(x_1^i). \qquad \text{(C-15)}$$

The proof for $k_i > k_1$, $i \neq 1$, parallels that for $k_i < k_1$. For $k_i > k_1$, the formula for k_i follows from the assumptions that there exists x_i^1 such that:

$$(\bar{x}_i^o, x_i^1) \sim (x_1^*, \bar{x}_1^o) . \qquad \text{(C-16)}$$

By preferential independence it is meaningful to assess x_i^1 from the indifference relation:

$$(x_1^o, x_i^1) \sim (x_1^*, x_i^o) \; . \qquad \text{(C-17)}$$

By the indifference relation of Equation (C-16):

$$u(\bar{x}_i^o, x_i^1) = u(x_1^*, \bar{x}_1^o) . \qquad \text{(C-18)}$$

Having assessed x_i^1 from Equation (C-17) and k_1 from Equation (C-14), by Equations (C-5) and (C-7) one obtains:

$$k_i \; u_i(x_i^1) = k_1 . \qquad \text{(C-19)}$$

Therefore, for $k_i > k_1$,

$$k_i = \frac{k_1}{u_i(x_i^1)} \; . \qquad \text{(C-20)}$$

Q.E.D.

APPENDIX D

PROGRAM FOR CALCULATING THE KEENEY MULTIPLICATIVE UTILITY FUNCTION ON AN HP-97 PROGRAMMABLE CALCULATOR

APPENDIX D

PROGRAM FOR CALCULATING THE KEENEY MULTIPLICATIVE UTILITY FUNCTION ON AN HP-97 PROGRAMMABLE CALCULATOR

This Appendix presents a program for calculating multiattribute utility function values from the Keeney Multiplicative Utility Function on a Hewlett Packard HP-97 or HP-67 programmable calculator. The name of the program is MATEUS 97: "An HP-97 Program for the MultiATtribute Evaluation of UtilitieS."

Program Description

Program Title MATEUS 97: MultiATtribute Evaluation of UtilitieS

Name Ralph F. Miles, Jr. **Date** 9/20/77

Address Jet Propulsion Laboratory, 4800 Oak Grove Drive

City Pasadena **State** California **Zip Code** 91103

Program Description, Equations, Variables, etc. MATEUS is a program for Hewlett Packard HP-97 or HP-67 programmable calculators. It calculates the Keeney Multiattribute Utility Function:

$$u(x) = \frac{1}{k}\left\{\prod_{i=1}^{n}\left[1 + kk_iu_i(x_i)\right] - 1\right\}$$

If $\Sigma k_i = 1$, then:

$$u(x) = \sum_{i=1}^{n} k_iu_i(x_i)$$

The initial input data are the k_i's, from which k is calculated from:

$$1 + k = \prod_{i=1}^{n}\left[1 + kk_i\right]$$

The k_i's establish the functional form of the equation. Then $u_i(x_i)$, $i=1,\ldots,n$ are input to calculate u(x).

Reference:

R. L. Keeney and H. Raiffa, Decisions with Multiple Objectives, John Wiley, New York, 1976.

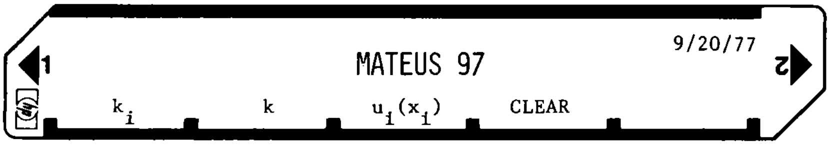
Operating Limits and Warnings

1. $0 \leq k_i < 1.0$.

2. $2 \leq n \leq 20$.

3. $0.0 \leq u_i(x_i) \leq 1.0$.

User Instructions



STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Enter k_i, $i=1,\ldots,n$.	k_i	A	i
				k_i
2	Calculate k .		B	Σk_i
				k
3	Enter $u_i(x_i)$, $i=1,\ldots,n$.	$u_i(x_i)$	C	i
				$u_i(x_i)$
4	After $u_n(x_n)$ entered, MATEUS calculates u(x).			u(x)
5	Repeat Step 3 for calculating other u(x) with			
	same k_i's.			
6	Clear Program for new k_i's.		D	

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	*LBLA	21 11	*LBLA	057	RCLC	36 13	
002	STOi	35 45	Store k_i, i=1,...,n, in R_{i-1}. $2 \leq n \leq 20$. Print i, k_i. Display i. Store n in R_E.	058	÷	-24	
003	ISZI	16 26 46		059	ABS	16 31	
004	RCLI	36 46		060	EEX	-23	
005	PRTX	-14		061	6	06	
006	X⇄Y	-41		062	CHS	-22	
007	PRTX	-14		063	X>Y?	16-34	
008	X⇄Y	-41		064	GTO4	22 04	
009	STOE	35 15		065	RCLA	36 11	
010	RTN	24		066	RCLB	36 12	
011	*LBLB	21 12	*LBLB	067	RCLC	36 13	
012	0	00	Calculate Σk_i and store in R_D. Print Σk_i.	068	÷	-24	
013	STOD	35 14		069	–	-45	
014	STOI	35 46		070	STOA	35 11	
015	*LBL1	21 01		071	GSBb	23 16 12	
016	RCLD	36 14		072	GTO3	22 03	
017	RCLi	36 45		073	*LBL4	21 04	
018	+	-55		074	RCLA	36 11	
019	STOD	35 14		075	PRTX	-14	
020	ISZI	16 26 46		076	RTN	24	
021	RCLI	36 46		077	*LBLb	21 16 12	Subroutine b.
022	RCLE	36 15		078	0	00	Calculate f(q). $f(q) = 1 + q - \pi_i(1 + qk_i)$
023	X≠Y?	16-32		079	STOI	35 46	
024	GTO1	22 01		080	1	01	
025	RCLD	36 14		081	STOB	35 12	
026	PRTX	-14		082	*LBL9	21 09	
027	CF0	16 22 00	Test for linear model. Print k=0 and stop if $\Sigma k_i = 1.0$.	083	RCLA	36 11	
028	1	01		084	RCLi	36 45	
029	X=Y?	16-33		085	x	-35	
030	SF0	16 21 00		086	1	01	
031	0	00		087	+	-55	
032	F0?	16 23 00		088	RCLB	36 12	
033	PRTX	-14		089	x	-35	
034	F0?	16 23 00		090	STOB	35 12	
035	RTN	24		091	ISZI	16 26 46	
036	RCLD	36 14	Calculate k≠0. $1 + k = \pi_i(1 + kk_i)$ Use Newton-Raphson Method to find root of f(q). $q^{j+1} = q^j - \frac{f(q^j)}{f'(q^j)}$ $f(q) = 1 + q - \pi_i(1 + qk_i)$	092	RCLI	36 46	
037	1	01		093	RCLE	36 15	
038	X≤Y?	16-35		094	X≠Y?	16-32	
039	CHS	-22		095	GTO9	22 09	
040	0	00		096	1	01	
041	.	-62		097	RCLA	36 11	
042	1	01		098	+	-55	
043	x	-35		099	RCLB	36 12	
044	STOA	35 11		100	–	-45	
045	*LBL2	21 02		101	STOB	35 12	
046	1	01		102	RTN	24	
047	0	00		103	*LBLc	21 16 13	Subroutine c.
048	RCLA	36 11		104	0	00	Calculate f'(q) $f'(q) = 1 - \left\{\left[\sum \frac{k_i}{1 + qk_i}\right] \times \left[\pi(1 + qk_i)\right]\right\}$
049	x	-35		105	STOI	35 46	
050	STOA	35 11		106	STOC	35 13	
051	GSBb	23 16 12		107	*LBLd	21 16 14	
052	X>0?	16-44		108	RCLi	36 45	
053	GTO2	22 02		109	RCLi	36 45	
054	*LBL3	21 03		110	RCLA	36 11	
055	GSBc	23 16 13		111	x	-35	
056	RCLB	36 12		112	1	01	

REGISTERS

0 k_1	1 k_2	2 k_3	3 k_4	4 k_5	5 k_6	6 k_7	7 k_8	8 k_9	9 k_{10}
S0 k_{11}	S1 k_{12}	S2 k_{13}	S3 k_{14}	S4 k_{15}	S5 k_{16}	S6 k_{17}	S7 k_{18}	S8 k_{19}	S9 k_{20}

A q,k	B f(q), u_i	C f'(q), $\pi(1 + kk_iu_i)$	D Σk_i, Σk_iu_i	E n	I i

Program Listing

STEP	KEY ENTRY	KEY CODE	COMMENTS
113	+	-55	
114	÷	-24	
115	RCLC	36 13	
116	+	-55	
117	STOC	35 13	
118	ISZI	16 26 46	
119	RCLI	36 46	
120	RCLE	36 15	
121	X≠Y?	16-32	
122	GTOd	22 16 14	
123	1	01	
124	RCLA	36 11	
125	+	-55	
126	RCLB	36 12	
127	-	-45	
128	RCLC	36 13	
129	x	-35	
130	CHS	-22	
131	1	01	
132	+	-55	
133	STOC	35 13	
134	RTN	24	
135	*LBLC	21 13	*LBLC
136	STOB	35 12	Initialize for u(x) calculations. Set F1. Set $R_I = R_D = 0$. Set $R_C = 1$.
137	F1?	16 23 01	
138	GTO5	22 05	
139	SF1	16 21 01	
140	SPC	16-11	
141	0	00	
142	STOI	35 46	
143	STOD	35 14	
144	1	01	
145	STOC	35 13	
146	*LBL5	21 05	Test for linear model.
147	F0?	16 23 00	
148	GTO7	22 07	
149	RCLB	36 12	Calculate u(x) for multiplicative model. Print i. Print $u_i(x_i)$. Print u(x).
150	RCLi	36 45	
151	x	-35	
152	RCLA	36 11	
153	x	-35	
154	1	01	
155	+	-55	
156	RCLC	36 13	
157	x	-35	
158	STOC	35 13	
159	ISZI	16 26 46	
160	RCLE	36 15	
161	RCLI	36 46	
162	X=Y?	16-33	
163	GTO6	22 06	
164	PRTX	-14	
165	RCLB	36 12	
166	PRTX	-14	
167	X⇄Y	-41	
168	RTN	24	
169	*LBL6	21 06	
170	CF1	16 22 01	
171	PRTX	-14	
172	RCLB	36 12	
173	PRTX	-14	
174	RCLC	36 13	
175	1	01	
176	-	-45	
177	RCLA	36 11	
178	÷	-24	
179	SPC	16-11	
180	PRTX	-14	
181	SPC	16-11	
182	RTN	24	
183	*LBL7	21 07	Calculate u(x) for linear model. Print i. Print $u_i(x_i)$. Print u(x).
184	RCLB	36 12	
185	RCLi	36 45	
186	x	-35	
187	RCLD	36 14	
188	+	-55	
189	STOD	35 14	
190	ISZI	16 26 46	
191	RCLE	36 15	
192	RCLI	36 46	
193	X=Y?	16-33	
194	GTO8	22 08	
195	PRTX	-14	
196	RCLB	36 12	
197	PRTX	-14	
198	X⇄Y	-41	
199	RTN	24	
200	*LBL8	21 08	
201	CF1	16 22 01	
202	PRTX	-14	
203	RCLB	36 12	
204	PRTX	-14	
205	RCLD	36 14	
206	SPC	16-11	
207	PRTX	-14	
208	SPC	16-11	
209	RTN	24	
210	*LBLD	21 14	*LBLD
211	CLX	-51	Clear program.
212	ENT↑	-21	
213	ENT↑	-21	
214	ENT↑	-21	
215	CLRG	16-53	
216	P⇄S	16-51	
217	CLRG	16-53	
218	CF0	16 22 00	
219	CF1	16 22 01	
220	SPC	16-11	
221	SPC	16-11	
222	SPC	16-11	
223	RTN	24	
224	R/S	51	

LABELS					FLAGS
A 001	B 011	C 135	D 210	E	0 $\Sigma k_i = 1$
a	b 077	c 103	d 107	e	1 Initial u
0	1 015	2 045	3 054	4 073	2
5 146	6 169	7 183	8 200	9 082	3

SET STATUS

FLAGS	ON	OFF	TRIG		DISP	
0	☐	☒	DEG	☒	FIX	☒
1	☐	☒	GRAD	☐	SCI	☐
2	☐	☒	RAD	☐	ENG	☐
3	☐	☒			n	6

Interview: #1

Date: 5/15/79

Position and Organization:

Task Manager: Applications Analysis and Development
Point-Focusing Thermal & Electric Applications Project
Jet Propulsion Laboratory
Pasadena, California

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	75 70/85	0.8		
x_2	Capital Cost $/kWh	1500 - 3000	2000		112	
x_3	Reliability Forced Outage %	0 - 25	8		110	
x_4	Plant Output Capacity Factor %	20 - 35	30		70	
x_5	Safety Man-Days-Lost/Year	0 - 60	35		120	
x_6	R&D Requirements $10^6$$	200 - 600	450		75	
x_7	Environment Subjective	0 - 10	-		85	
x_8	Applications Subjective	0 - 10	-		80	

$\Sigma k_i =$
$k =$

x_1	65	75	85	98				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:

Figure E-1. Worksheet No. 1 for the Interview #1 Data Analysis

Interview: #2

Date: 6/25/79

Position and Organization:
Technical Manager: Point Focusing Thermal & Electric Applic. Project
Jet Propulsion Laboratory
Pasadena, California

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	80 70/100	0.550		
x_2	Capital Cost $/kWh	1500 - 3000	2000		95/90/105	
x_3	Reliability Forced Outage %	0 - 25	17		75	
x_4	Plant Output Capacity Factor %	20 - 35	28		100	
x_5	Safety Man-Days-Lost/Year	0 - 60	30		75	
x_6	R&D Requirements $10^6$$	200 - 600	350		80	
x_7	Environment Subjective	0 - 10	-		110	
x_8	Applications Subjective	0 - 10			70/65/90	

$\Sigma k_i =$

$k =$

x_1	67	80	95	110				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:

Figure E-2. Worksheet No. 1 for the Interview #2 Data Analysis

Interview: #3

Date: 5/23/79

Position and Organization:
Manager, Thermal Power Systems Project
Jet Propulsion Laboratory
Pasadena, California

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	80	0.900		
x_2	Capital Cost \$/kWh	1500 - 3000	2250		100	
x_3	Reliability Forced Outage %	0 - 25			90	
x_4	Plant Output Capacity Factor %	20 - 35			90	
x_5	Safety Man-Days-Lost/Year	0 - 60			110	
x_6	R&D Requirements 10^6\$	200 - 600			115	
x_7	Environment Subjective	0 - 10			110	
x_8	Applications Subjective	0 - 10			90	

$\Sigma k_i =$

$k =$

x_1	65	80	100	115				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:

Figure E-3. Worksheet No. 1 for the Interview #3 Data Analysis

Interview: #4

Date: 5/17/79

Position and Organization:
Manager, Solar Electric Program
Jet Propulsion Laboratory
Pasadena, California

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	65 65/80	0.968		
x_2	Capital Cost \$/kWh	1500 - 3000	2000		70/55/80	
x_3	Reliability Forced Outage %	0 - 25			90	
x_4	Plant Output Capacity Factor %	20 - 35			90	
x_5	Safety Man-Days-Lost/Year	0 - 60			80	
x_6	R&D Requirements 10^6\$	200 - 600			85	
x_7	Environment Subjective	0 - 10			95	
x_8	Applications Subjective	0 - 10			90/75/90	

$\Sigma k_i =$

$k =$

x_1	60	65	75	80				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:

Figure E-4. Worksheet No. 1 for the Interview #4 Data Analysis

Interview: #5

Date: 5/25/79

Position and Organization: Manager of R&D Programs
A Public Utility Company
Arizona

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	80 75/85	0.667		
x_2	Capital Cost $/kWh	1500 - 3000	1750		70/55/80	
x_3	Reliability Forced Outage %	0 - 25	8			1.34/1.9/ 0.8
x_4	Plant Output Capacity Factor %	20 - 35	21		125	
x_5	Safety Man-Days-Lost/Year	0 - 60	30		100/90/ 100	
x_6	R&D Requirements $10^6$$	200 - 600	510		110	
x_7	Environment Subjective	0 - 10	-		70/60/77	
x_8	Applications Subjective	0 - 10	-		110	

$\Sigma k_i =$
$k =$

x_1	65	80	100	115				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:

Figure E-5. Worksheet No. 1 for the Interview #5 Data Analysis

Interview: #6

Date: 5/24/79

Position and Organization:

Power System Engineer
A Municipal Utility Company
Pasadena, California

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	91 70/90	0.50		
x_2	Capital Cost \$/kWh	1500 - 3000	2250		100(1)	
x_3	Reliability Forced Outage %	0 - 25	15			11.55
x_4	Plant Output Capacity Factor %	20 - 35	30		100	
x_5	Safety Man-Days-Lost/Year	0 - 60	30		80	
x_6	R&D Requirements 10^6\$	200 - 600	400		120	
x_7	Environment Subjective	0 - 10	-		100	
x_8	Applications Subjective	0 - 10	-		122	

$\Sigma k_i =$

$k =$

x_1	80	91	110	120				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments: (1) u_2 (3000) $> u_2$ (1500), a reversal of the usual preference.

Figure E-6. Worksheet No. 1 for the Interview #6 Data Analysis

Interview: #7

Date: 5/30/79

Position and Organization:
Manager of Resource Development
A Municipal Utility Company
Los Angeles, California

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	90 90/100	0.500		
x_2	Capital Cost \$/kWh	1500 - 3000	2250		100/90/ 100	
x_3	Reliability Forced Outage %	0 - 25	15		110/105/ 115	
x_4	Plant Output Capacity Factor %	20 - 35	27.5		110/105/ 115	
x_5	Safety Man-Days-Lost/Year	0 - 60	30	0.700(1)		50(1)
x_6	R&D Requirements 10^6\$	200 - 600	450		110/105/ 115	
x_7	Environment Subjective	0 - 10	-		110	
x_8	Applications Subjective	0 - 10	-		100	

Σk_i =
k =

x_1	75	90	110	115				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments: (1) k_1 and u_5 (x_5^1) are inconsistent. k_5 was assessed directly.

Figure E-7. Worksheet No. 1 for the Interview #7 Data Analysis

Interview: #8

Date: 5/24/79

Position and Organization:

Supervising Research Engineer
A Public Utility Company
California

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	90 60/100	0.700		
x_2	Capital Cost \$/kWh	1500 - 3000	2250		60	
x_3	Reliability Forced Outage %	0 - 25	20		110	
x_4	Plant Output Capacity Factor %	20 - 35	25		100	
x_5	Safety Man-Days-Lost/Year	0 - 60	30		80	
x_6	R&D Requirements 10^6\$	200 - 600	400		80	
x_7	Environment Subjective	0 - 10	-		125	
x_8	Applications Subjective	0 - 10	-		120	

$\Sigma k_i =$

$k =$

x_1	70	90	100	100				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:

Figure E-8. Worksheet No. 1 for the Interview #8 Data Analysis

Interview: #9

Date: 6/4/79

Position and Organization:

American Public Power Association
Director of Energy Research
Washington, D.C.

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	75 90/70	0.167		
x_2	Capital Cost $/kWh	1500 - 3000	2000		117.5/ 100/122	
x_3	Reliability Forced Outage %	0 - 25	12.5			15
x_4	Plant Output Capacity Factor %	20 - 35	27.5		105	
x_5	Safety Man-Days-Lost/Year	0 - 60	30			30
x_6	R&D Requirements $10^6$$	200 - 600	400	.267 .550(1)		350 .625
x_7	Environment Subjective	0 - 10	-		105	
x_8	Applications Subjective	0 - 10	-	.278 .650(1)		0.60(1)

$\Sigma k_i =$

$k =$

x_1	65	75	85	90				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments: (1) Scaling constants or utility directly assessed.

Figure E-9. Worksheet No. 1 for the Interview #9 Data Analysis

Interview: #10

Date: 6/5/79

Position and Organization:
Manager of Planning and Assessment
Northeast Solar Energy Center
Cambridge, Massachusetts

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	95 70/100	0.900		
x_2	Capital Cost $/kWh	1500 - 3000	2000		124	
x_3	Reliability Forced Outage %	0 - 25	15		110	
x_4	Plant Output Capacity Factor %	20 - 35	27.5		110	
x_5	Safety Man-Days-Lost/Year	0 - 60	30		110	
x_6	R&D Requirements $10^6$$	200 - 600	400		125	
x_7	Environment Subjective	0 - 10	-		120	
x_8	Applications Subjective	0 - 10	-		90/70/ 110	

$\Sigma k_i =$
$k =$

x_1	70	95	105	110				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:

Figure E-10. Worksheet No. 1 for the Interview #10 Data Analysis

Interview: #11

Date: 6/7/79

Position and Organization: Prof. of Nuclear Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	75 85/70	0.700		
x_2	Capital Cost $/kWh	1500 - 3000	2500		90/75/ 105	
x_3	Reliability Forced Outage %	0 - 25	10		105	
x_4	Plant Output Capacity Factor %	20 - 35	27.5		(1)	
x_5	Safety Man-Days-Lost/Year	0 - 60	20	0.333(2)		20(2)
x_6	R&D Requirements $10^6$$	200 - 600	450		115	
x_7	Environment Subjective	0 - 10	-		110/90/ 110	
x_8	Applications Subjective	0 - 10	-		105/85/ 110	

$\Sigma k_i =$

$k =$

x_1	60	75	95	108				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:
(1) Utility independence grossly violated. See text.
(2) Inconsistent with $k_1 = 0.7$. k_5 assessed directly.

Figure E-11. Worksheet No. 1 for the Interview #11 Data Analysis

Interview: #12

Date: 6/9/79

Position and Organization:

President
TERA: Transportation & Economic Research Associates
Arlington, Virginia

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	87.5 87.5/60	0.250		
x_2	Capital Cost $/kWh	1500 - 3000	2250		95/85/ 100	
x_3	Reliability Forced Outage %	0 - 25	14	0.710 0.333$^{(1)}$	-/80/-	17.5/-/ 20
x_4	Plant Output Capacity Factor %	20 - 35	27.5		87.5	
x_5	Safety Man-Days-Lost/Year	0 - 60	30	0.500 0.333$^{(1)}$		30
x_6	R&D Requirements $10^6$$	200 - 600	400		100	
x_7	Environment Subjective	0 - 10	-		110	
x_8	Applications Subjective	0 - 10	-	0.312 0.250$^{(1)}$		0.80$^{(1)}$

$\Sigma k_i =$

$k =$

x_1	69	87.5	106	115				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments: (1) Scaling constants or utility value assessed directly.

Figure E-12. Worksheet No. 1 for the Interview #12 Data Analysis

Interview: #13

Date: 6/8/79

Position and Organization: Past President Sierra Club

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	80	0.75		
x_2	Capital Cost $/kWh	1500 - 3000	2250	0	125	
x_3	Reliability Forced Outage %	0 - 25	12.5		94	
x_4	Plant Output Capacity Factor %	20 - 35	27.5		125	
x_5	Safety Man-Days-Lost/Year	0 - 60	30		124	
x_6	R&D Requirements $10^6$$	200 - 600	400		110	
x_7	Environment Subjective	0 - 10	-		80	
x_8	Applications Subjective	0 - 10	-		110	

$\Sigma k_i =$

$k =$

x_1	65	80	100	115				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:

Figure E-13 Worksheet No. 1 for the Interview #13 Data Analysis

Interview: #14

Date: 6/19/79

Position and Organization:
Commissioner
Energy Resources Conservation and Development Commission
State of California

x_1	Attribute Unit of Measure	Range	x_i'	k_i	x_1^i $u_1(x_1^i)$	x_i^1 $u_i(x_i^1)$
x_1	First Year Cost mills/kWh	50 - 125	80	0.022		
x_2	Capital Cost $/kWh	1500 - 3000	2250		80	
x_3	Reliability Forced Outage %	0 - 25	5		90	
x_4	Plant Output Capacity Factor %	20 - 35	27.5		110	
x_5	Safety Man-Days-Lost/Year	0 - 60	30		80	
x_6	R&D Requirements $10^6$$	200 - 600	300		90	
x_7	Environment Subjective	0 - 10	-		90	
x_8	Applications Subjective	0 - 10	-	0.056 0.200(2)		0.40(2)

$\Sigma k_i =$

$k =$

x_1	65	80	100	110				
$u_1(x_1)$	0.75	0.5	0.25	0.125				

Additional Comments:
(1) Insufficient time to verify utility & preference independence.
(2) Scaling constant and utility value assessed directly.

Figure E-14. Worksheet No. 1 for the Interview #14 Data Analysis