REPRINTED FROM:

IECEC-89
August 6-11, 1989
Washington, D.C.

PROCEEDINGS OF THE 24th INTERSOCIETY ENERGY CONVERSION ENGINEERING CONFERENCE

Participating Societies

INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS

United Engineering Center • 345 East 47th Street • New York, NY 10017

Reprinted from PROCEEDINGS OF THE 24th INTERSOCIETY ENERGY CONVERSION ENGINEERING CONFERENCE, Washington, DC, August 6-11, 1989

899053

Charge Pressure Effects in Kinematic Stirling Engines

J. R. Senft

Department of Mathematics & Computer Science
University of Wisconsin
River Falls, Wisconsin 54022

ABSTRACT

Recent work has shown in a general way how the mechanical efficiency of reciprocating kinematic heat engines depends upon the relation between the engine workspace pressure and the external or bufferspace pressure. These results go beyond the intuitive and empirical level of understanding of the past and permit a precise evaluation of various schemes to improve engine performance by controlling charge pressure levels in the workspace or bufferspace. This paper applies these results to determine the effects on the performance of Stirling engines when charging the workspace with the buffer pressure fixed. A new general theorem is introduced, proven, and then used to show that the description of charge pressure effects given in the paper is complete. With this we now have a more thorough understanding of pressurization effects in Stirling engines.

INTRODUCTION

Pressurizing Stirling engines to increase specific power is a long established practice. Charging the engine with more working fluid ideally increases the indicated cyclic work and so potentially increases shaft output. In the case of double-acting engines, for example the 1827 engine of the Stirling brothers [1], the mechanical efficiency is virtually unaffected by the charge level of the workspace and so the result of workspace charging is directly beneficial. In single acting engines, the situation is not so straightforward. As has been shown in recent papers by this author [2,3,5], the mechanical efficiency of single acting engines depends upon the relationship between the workspace pressure cycle and the external or buffer pressure which acts on the non-workspace side of the pistons.

For single acting engines equipped with a pressure-worthy crankcase or buffer chamber, the buffer pressure can be changed in principle independently of the workspace to maintain or improve mechanical efficiency. The effects of charging only the buffer space with the workspace charge fixed and the effects of charging both spaces uniformly are now fully understood for Stirling engines [4,5] and beyond [6]. The effects of the third possibility, namely charging only the workspace with the buffer pressure fixed, has been up to now only partially investigated [4]. This paper is intended to more fully investigate this remaining case and thus complete our theoretical understanding of charge effects in single acting reciprocating kinematic Stirling engines.

The case of charging only the workspace with a fixed buffer pressure is quite important from a practical point of view. For engines not equipped with a pressure-worthy crankcase or bufferspace the only hope of increasing specific power through pressurization is by workspace-only charging. There are other practical advantages as well [4], and these also apply to entirely enclosed engines.

GENERAL THEOREM ON WORKSPACE CHARGING

In this section a result will be formulated and proven which generally applies to workspace charging of single-acting reciprocating kinematic engines with a fixed buffer space charge. The result will then be applied in the following sections to the specific analysis of Stirling engines.

The terminology and notation to be used follows that of reference [6] to which the reader should refer for complete details as necessary. Let the piecewise smooth function

$$\Gamma = (\alpha(t), m\beta(t), \gamma(t)) \qquad t \in J = [a,b]$$

represent the workspace volume V, workspace pressure p, and buffer pressure p_b respectively in terms of the parameter t. The mass of fluid in the workspace is represented by m and the form of p is characteristic of engines called <u>monomorphic</u> [6]. Monomorphic engines respond to charging in a uniform idealized way inasmuch as their indicated cyclic work is directly proportional to the workspace charge mass m.

Given a workspace monomorphic engine there is a minimum value of m, denoted by m' for which $m'\beta(t) \geq \gamma(t)$ for all $t \in J$. For $m \geq m'$, p is always at least equal to p_b and the engine is said to be <u>buffered from below.</u> The effects of workspace charging when buffering is from below is completely described in Theorem V of reference [6]. The theorem shows that in the range $[m',\infty)$, cyclic shaft work W_s is a linear function of positive or negative slope according to whether the square of <u>mechanism effectiveness</u>, e^2, is greater or less than the ratio W_c/W_e of <u>absolute compression</u> to <u>absolute expansion work.</u> Thus in the case when $e^2 > W_c/W_e$, there is no absolute maximum for W_s, that is, shaft output can be indefinitely increased by more workspace charging. We complete the picture by proving here the following result.

Theorem. In a workspace monomorphic engine, if $e^2 < W_c/W_e$, then W_s has an absolute maximum with respect to workspace charge mass m for a fixed buffer pressure trace.

CH2781-3/89/0000-2187 $1.00 © 1989 IEEE

In order to establish this theorem, we use the Mechanical Efficiency Theorem from [6] in the following form:

$$W_s = eW - \left[\frac{1}{e} - e\right] W_- - e\,W_b \qquad (1)$$

where W_- is the forced cyclic work and W_b is the cyclic buffer space energy dissipation. The last term of (1) is constant by the hypothesis that the buffer pressure trace is fixed and by the general assumption throughout that the mechanism effectiveness is constant. The first term of (1) is linear in m. The factor W_- in the middle term in the present case is by definition

$$W_- := \int_a^b [(m\,\beta(t) - \gamma(t))\,\alpha'(t)]^-\,dt$$

Since Γ is assumed piecewise smooth, $\alpha'(t)$ is piecewise continuous on $J = [a,b]$. This means J can be decomposed into a union of finitely many endpoint-overlapping closed subintervals

$$J = J_1 \cup J_2 \cup \ldots \cup J_K$$

on the interior of each of which α' is continuous and is continuously extendable to the endpoints. Therefore on each rectangle $[0,m'] \times J_i$ we may regard the function

$$(m\,\beta(t) - \gamma(t))\,\alpha'(t)$$

as continuous in m and t. Therefore its negative part is likewise continuous and it follows [7] that each of

$$\int_{J_i} [(m\,\beta(t) - \gamma(t))\,\alpha'(t)]^-\,dt$$

is a continuous function of m on $[0,m']$. W_- is the sum of these terms as i ranges from 1 to k and so W_- is continuous in m on $[0,m']$.

This shows that every term of (1) is continuous in m and therefore so is W_s. It follows that W_s has an absolute maximum on $[0,m']$. By the hypothesis that $e^2 < W_c/W_e$, W_s is decreasing on $[m',\infty)$ and this completes the proof of the theorem.

WORKSPACE CHARGING IDEAL STIRLING ENGINES

With the general results above, the effects of charging the workspace of Stirling engines can now be easily deduced. A best case analysis will result by considering ideal Stirling engines. An ideal Stirling engine in this context is a kinematic single-acting reciprocating engine with a constant mechanism effectiveness, a constant buffer pressure, and an ideal gas working fluid which undergoes an ideal Stirling cycle. Not only does this idealized engine concept yield a best case analysis, but it is the most natural simple representative of practical Stirling engines [4].

If buffer pressure is a constant, p_o, then $W_b = 0$ so that equation (1) becomes

$$W_s = eW - \left[\frac{1}{e} - e\right] W_- \qquad (2)$$

If an engine is given, then p_o, e, the volume extremes V_m and V_M, the temperature extremes T_C and T_H, and the gas constant R are fixed. We have

$$W = m\,R(T_H - T_C)\ln\left[\frac{V_M}{V_m}\right] \qquad (3)$$

Let m_o denote the working fluid mass for which the engine is buffered above, i.e.

$$m_o = \frac{p_o V_m}{R T_H}$$

We use m_o to define the dimensionless mass variable M by

$$M = \frac{m}{m_o} \qquad (4)$$

It is also convenient to use the specific work obtained by dividing the actual work expressions by $p_o V_m$. Equation (3) then becomes

$$\bar{W} = M(1-\tau)\ln r \qquad (5)$$

where $\tau = T_C/T_H$ is the temperature ratio and $r = V_M/V_m$ is the volume compression ratio.

The forced cyclic work function W_- is more difficult to express. There are two types of engine cycle to consider and five cases for each of these.

Efficacious Engines

If $\tau r \le 1$ then the Stirling cycle has the property that the minimum expansion process pressure is at least equal to the maximum compression process pressure. Such an engine is called efficacious because for any constant buffer pressure within these limits $W_- = 0$ and the mechanical efficiency is equal to the effectiveness of the mechanism which is the best efficiency value possible in view of the Mechanical Efficiency Theorem (1).

To calculate the forced work of a Stirling, five subintervals must be considered. The intervals arise from the four working fluid mass values which put one of the corners of the Stirling cycle on the buffer pressure line. For efficacious Stirlings these points expressed in terms of the mass variable M are:

$$1 < r \le \frac{1}{\tau} < \frac{r}{\tau} = M'$$

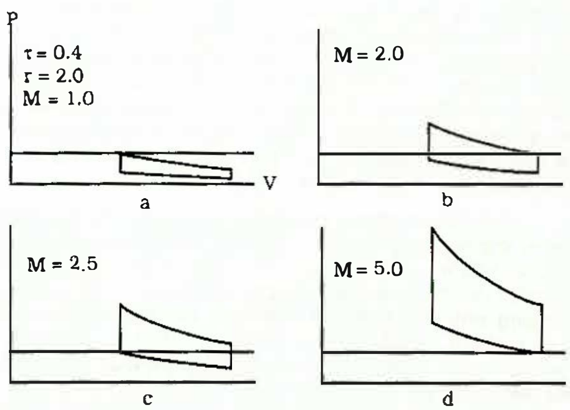

Fig.1 Pressure/Volume diagram of an efficacious Stirling cycle with fixed buffer pressure shown at the workspace charge levels M = 1, r, 1/τ and r/τ.

Figure 1 shows an efficacious cycle with charge mass corresponding to each of these four values. A straightforward calculation yields the following expression for the <u>specific forced work</u>, $\bar{W}_- = W_-/(p_0 V_m)$:

$$\bar{W}_-(M) = \begin{cases} r-1-M\ln r & 0 \le M \le 1 \\ r-M\,(1-\ln M + \ln r) & 1 \le M \le r \\ 0 & r \le M \le 1/\tau \\ M\tau\,(\ln M + \ln \tau - 1) + 1 & 1/\tau \le M \le r/\tau \\ M\tau \ln r - r + 1 & r/\tau \le M \end{cases} \qquad (6)$$

Combining equations (5) and (6) as in (2) yields the following <u>specific shaft work</u> function

$$\bar{W}_S(M) = e\,\bar{W}(M) - \left[\frac{1}{e} - e\right]\bar{W}_-(M) \qquad (7)$$

Figure 2 shows graphs of $\bar{W}_S \ge 0$ of an engine with $r = 1.25$ and $\tau = 0.33$. This temperature ratio is typical of high performance Stirlings with operating temperatures in the neighborhood of T_H = 700C and T_C = 50C. Curves are shown in the figure for mechanism effectiveness values ranging from e = 1.0 (no friction loss) down to e = 0.1. Values of e less than about .60 really need not occur in well designed, well built, and properly maintained engines, but these values are included in the graphs to show the effects of poor mechanism performance.

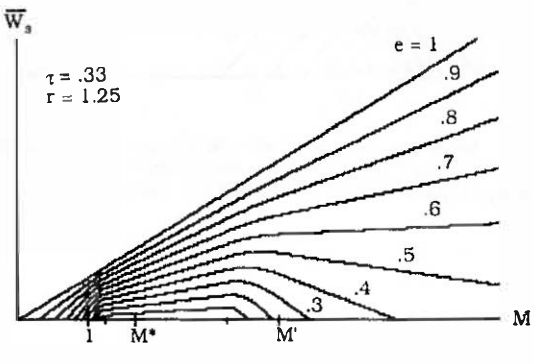

Fig. 2 Graphs of cyclic shaft work as a function of workspace charge for various mechanism effectiveness values of an efficacious Stirling engine with temperature ratio 0.33 and compression ratio 1.25.

Figures 3 and 4 show graphs again for $\tau = 0.33$ with compression ratios of 1.5 and 1.75 for comparison. These compression ratios are also typical of high performance Stirlings having ample internal heat exchangers. Figure 5 shows the graphs of $\bar{W}_S$ for an engine with compression ratio $r = 1.5$ running at a larger temperature ratio $\tau = 0.60$ which would correspond for example to T_H = 300C and T_C = 70C. The horizontal and vertical scales are the same in all the graphs.

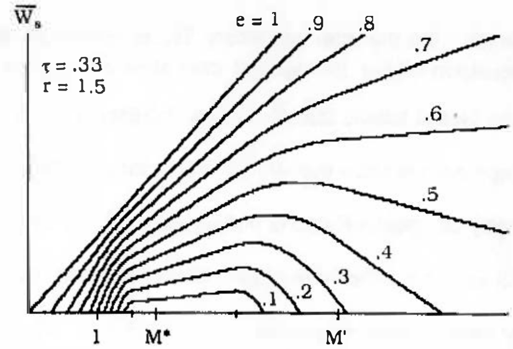

Fig. 3 Same as Fig. 2 but with higher compression ratio 1.5.

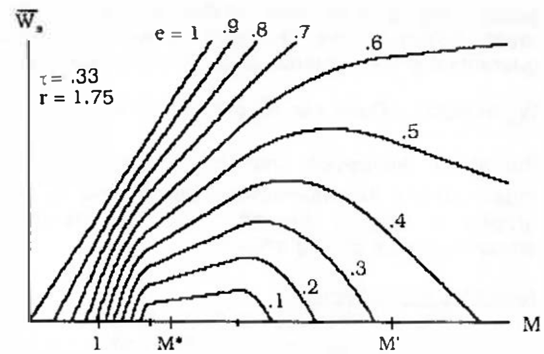

Fig. 4 Same as Fig. 3 with higher compression ratio 1.75.

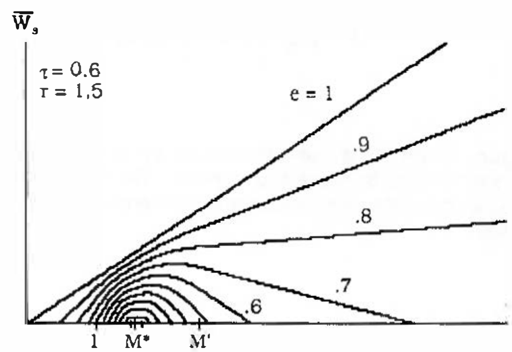

Fig. 5 Same as Fig. 3 for an engine with lower temperature ratio 0.6 and compression ratio 1.5.

The characteristics of these sets of graphs are similar, and indeed this is the case in general. Examination of (5) and (6) will show that $\bar{W}_S$ is linear in M in the initial, middle, and final intervals. In the last interval, namely $[M', \infty)$, W_S increases or decreases according to whether e^2 is greater or less than τ (and is constant when $e^2 = \tau$) [5]. It is readily verified that the function $\bar{W}_S(M)$ is continuously differentiable on $[0,\infty)$ (we are interested only in the portion where $W_S \ge 0$ of course). On the intervals where $\bar{W}_S$ is not linear, a simple calculation shows the second derivative is negative. From these facts it follows that $\bar{W}_S'$ is non-increasing on $[0,\infty)$ and it is then easy to show that $\bar{W}_S$ is never concave upward, that is, for any two points P and Q on the graph of $\bar{W}_S$, the segment [PQ] lies on or below the graph. This implies $\bar{W}_S$ cannot have any interior local minimums. Now if $e^2 > \tau$, $\bar{W}_S$ increases without bound on $[M',\infty)$ and so cannot have a local maximum on $[0,M')$ which means $\bar{W}_S$ is monotone increasing over the entire charging range. If $e^2 < \tau$ then because $\tau = W_c/W_e$ for ideal Stirlings, the theorem proven above applies guaranteeing the existence of an absolute maximum value for $\bar{W}_S$ in $[0,M')$. There can be only this one local maximum by the above discussion and so $\bar{W}_S$ must increase to this maximum and then decrease to zero. Thus as claimed the graphs of Figs. 2 through 5 are representative of all efficacious ideal Stirling engines.

Non-Efficacious Engines

The non-efficacious Stirling cycles are those for which $\tau r > 1$. The interval decomposition required in this case is slightly different because the minimum expansion pressure is below the maximum compression pressure. The division points in terms of M are

$$1 < \frac{1}{\tau} < r < \frac{r}{\tau} = M'$$

Figure 6 shows a non-efficacious cycle with mass charge corresponding to these four values. Comparison with Fig. 1 will point out the differences and similarities of the two engine types.

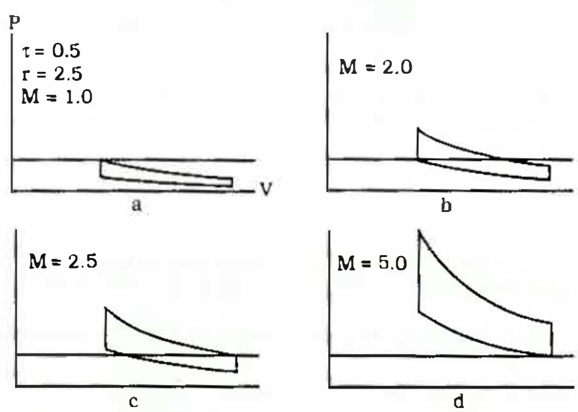

Fig. 6 Pressure/Volume diagram of a non-efficacious Stirling cycle with fixed buffer pressure shown at the workspace charge levels M = 1, $1/\tau$, r and r/τ.

The specific forced work function for the non-efficacious engine is found to be as follows:

$$\bar{W}_-(M) = \begin{cases} r-1-M\ln r & 0 \le M \le 1 \\ r-M(1-\ln M+\ln r) & 1 \le M \le 1/\tau \\ r+M[(\tau+1)\ln M+\tau\ln\tau-\ln r-\tau-1]+1 & 1/\tau \le M \le r \\ M\tau(\ln M+\ln\tau-1)+1 & r \le M \le r/\tau \\ M\tau\ln r-r+1 & r/\tau \le M \end{cases}$$

Note that the formulas for $\bar{W}_-$ differ only in the middle interval and also that the adjacent intervals have different endpoints.

Figures 7 and 8 show graphs of $\bar{W}_S$ for two non-efficacious engines. The scales are the same as in the efficacious case for convenience in making comparisons. By an argument similar to the one in the previous section it can be shown that the graphs of Figs. 7 and 8 are representative of all non-efficacious ideal Stirling engines.

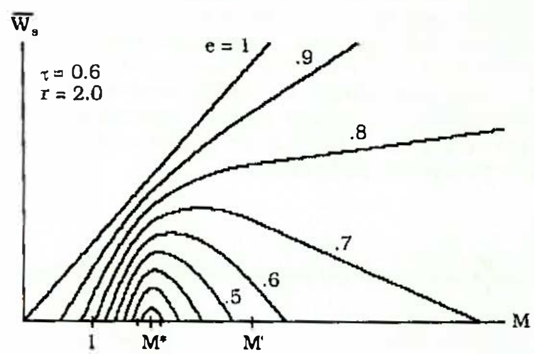

Fig. 7 Graphs of cyclic shaft work as a function of workspace charge for various mechanism effectiveness values of a non-efficacious Stirling engine with temperature ratio 0.6 and compression ratio 2.0

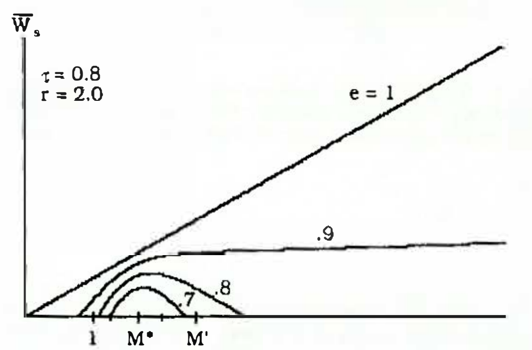

Fig. 8 Same as Fig. 7 but with higher temperature ratio 0.8.

Charging Effects on Shaft Output

Comparing the discussions in the two previous sections, the influence of workspace charge mass on cyclic shaft work is similar for both efficacious and non-efficacious Stirling engines. In both cases, as workspace charge is increased, shaft output either increases without bound, or increases to a maximum and then remains constant, or increases to a maximum and then decreases to zero. These behaviors are characterized by the relative magnitudes of the temperature ratio and the mechanism effectiveness.

If $e^2 > \tau$, then W_S increases without bound as m increases. This increase is linear beyond the point where the engine is buffered from below, namely for $M \geq M'$. The closer e^2 is to τ, the slower is the rate of increase beyond M'.

When $e^2 = \tau$, W_S increases until $M = M'$ and is then constant for $M \geq M'$. This case represents the transition between ever increasing and diminishing output.

If $e^2 < \tau$ then W_S increases to a maximum value and then decreases to zero. The decrease is linear after the engine is buffered from below. The maximum shaft output occurs at a charge level greater than that where the engine has the maximum mechanical efficiency and less than the point where the engine is buffered at minimum cycle pressure. Again, the closer e^2 is to τ the slower is the rate of decrease in this case.

Practical Implications

In practice special devices are required to maintain a workspace charge level with a mean pressure much different than that of the buffer pressure. Leakage past piston or displacer rod seals tends to equalize mean cycle pressure and external pressure. It has been shown in [3,4] that mean cycle pressure and the pressure

$$p^* = \frac{mRT_H}{V_m}\,\frac{1+\tau}{1+r} \tag{8}$$

are essentially the same for the compression ratios typical of real Stirling engines. (This is a happy circumstance because a buffer pressure equal to p^* gives the maximal mechanical efficiency.) Thus without special devices to prevent it, we may assume the workspace mass charge self-adjusts to make $p_O = p^*$. Denoting this mass by m^*, equation (8) shows

$$M^* := \frac{m^*}{m_O} = \frac{1+r}{1+\tau}$$

Other workspace charge levels can be obtained and maintained by fitting pumps or regulator valves. But this adds complexity to the engine and usually introduces mechanical or thermal losses. However, there is a charge level other than M^* which can be obtained by a very simple device. A simple check valve into the workspace will yield a workspace charge very near M', the point where the external pressure is equal to the minimum cycle pressure. This was standard practice on the Rider and Ericsson pumping engines. The benefits of this simple charging scheme can be considerable as for example the graphs in Fig. 3 indicate. Even with a rather poor mechanism such as $e = 0.6$, W_S increases by more than a factor of two when charge is increased from M^* to M' by fitting an ideal check valve. The increase is better for higher e values and appreciable benefit even occurs for e values below the critical $\sqrt{\tau}$ such as $e = 0.5$. Worthwhile benefits also occur in the engine of Fig. 7, but there only for the higher e values. When $e = 0.7$, there is virtually no gain and for lesser e values there is a *decrease* in shaft output. Indeed, if $e = 0.5$, the engine will not even run with a check valve fitted. This kind of decrease is more dramatic in Fig. 8.

Another fairly simple scheme for increasing specific power is workspace charging beyond $M' = r/\tau$. When $e > \sqrt{\tau}$, W_S increases without bound as M is increased beyond M'. However, in practice, this is advisable only when e is *considerably above* $\sqrt{\tau}$. As the graphs clearly show, the closer e is to $\sqrt{\tau}$, the lower the rate of increase and so very high working pressures may be needed to achieve a desired power level. As was pointed out in [5], mechanical efficiency decreases in this situation, so it is doubly beneficial to make e as large as possible and τ as small as possible when considering charging above M'.

The graphs will yield many more insights of this kind and the equations given in the paper allow computation of any other desired case. Reference [4] contains additional numerical examples.

CONCLUSION

With the results of this paper, we now have a complete general understanding of the effects of pressurization on the performance of kinematic single-acting Stirling engines. In the present paper the response curves of shaft output to workspace charge have been described and proven to be completely representative of ideal Stirling engines. Although obtained for idealized engines, the results are useful for indicating trends followed by real Stirling engines. Valuable practical guidelines are easily drawn from these results and examples have been given in the paper.

It is worth pointing out that the limitations on Stirling engine performance described here also impose limitations on all possible reciprocating engines. This is because of the following theorem proven in [3] but restated here in a slightly different form more relevant to the results of this paper.

Stirling Comparison Theorem. Of all single-acting reciprocating kinematic heat engines having the same volume and temperature extremes, the same constant buffer pressure and mechanism effectiveness, and the same mass of the same ideal working gas, the ideal Stirling has the maximum cyclic shaft work.

Therefore, the analysis of ideal Stirlings is inherently a general best case analysis. Understanding of the behavior of the ideal Stirling engine provides a general understanding of the intrinsic performance limits of all comparable reciprocating engines.

ACKNOWLEDGEMENT

This material is based upon work supported by the National Science Foundation under Grant No. CBT-8705576.

REFERENCES

1. T. Finkelstein, "Air engines", The Engineer, April 3, 1959, p. 522-527.

2. J. R. Senft, "Limits on the mechanical efficiency of heat engines", Proc. 22nd Intersociety Energy Conversion Engineering Conference, Paper #879071, Philadelphia, August, 1987.

3. J. R. Senft, "Mechanical efficiency of kinematic heat engines", Jour. of the Franklin Institute, Vol. 324, No. 2, p 273-290, 1987.

4. J. R. Senft, "Buffer pressure effects in Stirling engines", Proc. 4th International Conference on Stirling Engines, Paper #011, Japan Society of Mechanical Engineers, Tokyo, November, 1988.

5. J. R. Senft, "Mechanical efficiency of Stirling engines-general mathematical considerations", Proc 20th Intersociety Energy Conversion Engineering Conference, Paper #859436, Society of Automotive Engineers, Miami, August, 1985.

6. J. R. Senft, "General results on mechanical efficiency of heat engines", Proc 23rd Intersociety Energy Conversion Engineering Conference, Paper #889340, Amer. Society of Mechanical Engineers, Denver, July/August, 1988.

7. R. C. Buck, Advanced Calculus, Second Edition, McGraw-Hill, 1965.

NOMENCLATURE

V	engine workspace volume
V_m	minimum cycle volume
V_M	maximum cycle volume
p	workspace pressure
p_b	variable buffer pressure
p_o	constant buffer pressure
T_C	minimum cycle temperature
T_H	maximum cycle temperature
R	gas constant
m	mass of fluid in workspace
m_o	$(p_o V_m) / (R T_H)$
τ	temperature ratio = T_C / T_H
r	compression ratio = V_M / V_m
e	mechanism effectiveness
M	m / m_o
M^*	$(1+r)/(1+\tau)$
M'	r/τ
W_b	cyclic buffer loss
W_e	absolute expansion work
W_c	absolute compression work
W	indicated cyclic work = W_e-W_c
W_-	forced cyclic work
W_s	cyclic shaft work
$\overline{W}$	$W / (p_o V_m)$
$\overline{W}_-$	$W_- / (p_o V_m)$
$\overline{W}_s$	$W_s / (p_o V_m)$