THEORETICAL PRINCIPLES FOR SOLAR ENERGY COLLECTOR STUDIES

Energy Conversion Technical Report No. 3

by

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1. INTRODUCTION

The operation of all types of solar energy collectors depends upon the setting up of an equilibrium between the incoming radiant energy and the outgoing energy which comprises the various collector losses as well as the useful components. It is customary to group the useful component and the losses together as a function of absorber temperature and thus distinguish them from the incoming radiant energy.

The present author is inclined towards the alternative method of grouping viz. to consider both the incoming and outgoing radiative components together and distinct from the remaining outgoing components. The advantage of this alternative method lies chiefly in the fact that the two radiative mechanisms have much in common physically as well as analytically and this enables one to visualise the processes of optimisation more readily.

This paper will develop, through sections 2, 3 and 4, the proof of a useful theorem which relates the radiative equilibrium temperature of an absorber to geometrical parameters. The consequences of this theorem are explored in sections 5 and 6, and in 7, these consequences are enlarged upon by introducing the non radiative losses and useful output. Finally sections 8 and 9 deal with the overall Carnot efficiency of an absorber and thermodynamic conversion system.

2. ENERGY TRANSFER IN A RAY

Consider two independent elemental "black" areas dA_1 and dA_2 at respective temperatures T_1 and T_2. The line joining the two elements is at angles θ_1 and θ_2 respectively to the normals of the two elements.

Consider the radiation dF, received by dA_2 from dA_1

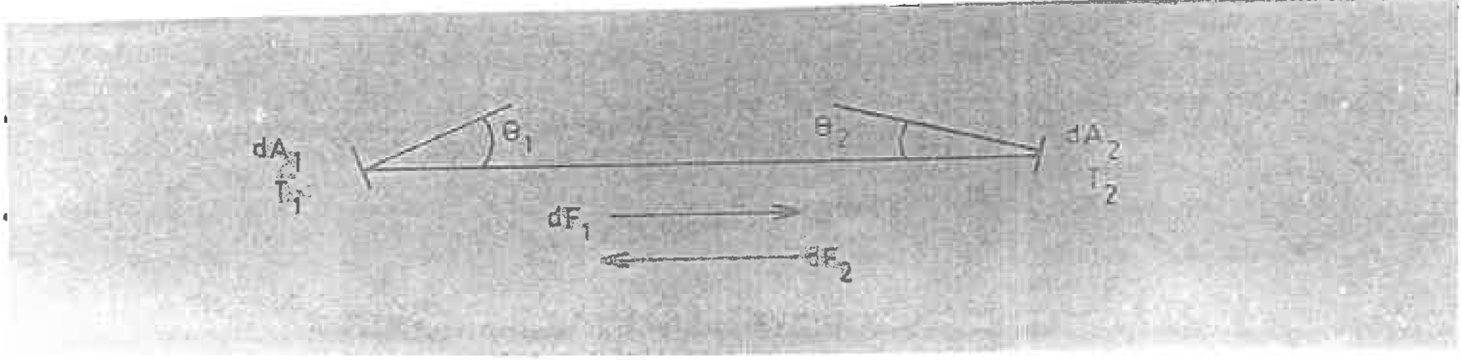

Fig. 2.1

4. EQUILIBRIUM OF A NON-STORING BLACK BODY

Here we consider two black bodies one of which (subscript W) is connected to an energy source or store and the other (subscript E) suffers no losses to the surroundings other than by radiation and is connected neither to source, sink or energy store. We are concerned with the equilibrium temperature of body E given the temperature T_W of body W.

Using equation 3.1 the energy transfer W to E is: $W_{WE} = K\,T_W^4$

Similarly from E to W: $W_{EW} = K\,T_E^4$

However the total radiation from E may be expressed by

$$W_E = F\,T_E^4$$

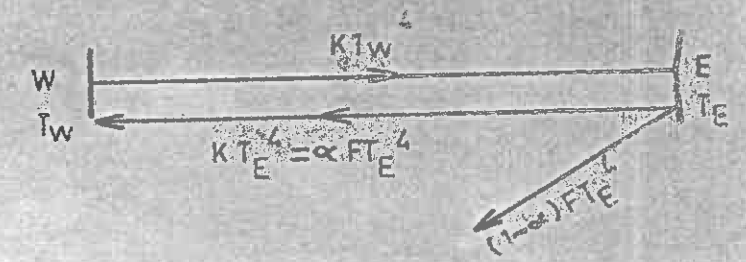

Fig. 4.1

Since E is in equilibrium it follows that

$$F\,T_E^4 = K\,T_W^4$$

ie $$\left(\frac{T_E}{T_W}\right)^4 = \frac{K}{F} = \alpha \qquad 4.1$$

where α is that fraction of the radiation emitted by E that is intercepted by W.

One practical interpretation of this result is:

In a collecting system using direct radiation from the sun, the maximum possible absorber temperature (black body) is simply derived by considering the emission of radiation from the absorber and determining the fraction α of the total that is intercepted by the sun.

Then $T_{absorber} = T_{sun} \cdot \alpha^{1/4}$ 4.2

T_{sun} can be taken as $5900^{o}K$. (Brinkworth, Daniels and Duffie, A.A.C.a). This is true for absorbers with or without optical concentrators.

5. EFFECTS OF ATTENUATING MEDIUM AND SELECTIVE ABSORPTION

Here we consider non-black absorbing surfaces and the effects of non-scattering attenuation in the transmission medium.

Fig. 5.1 illustrates the situation with an attenuating medium in which the power of the emitted radiation from W to E is attenuated by factor H. This definition of H implies that H is a function of the spectral distribution of the radiation entering the attenuating medium. In particular, if the radiation source is a black body at temperature T_W, H is a function of T_W.

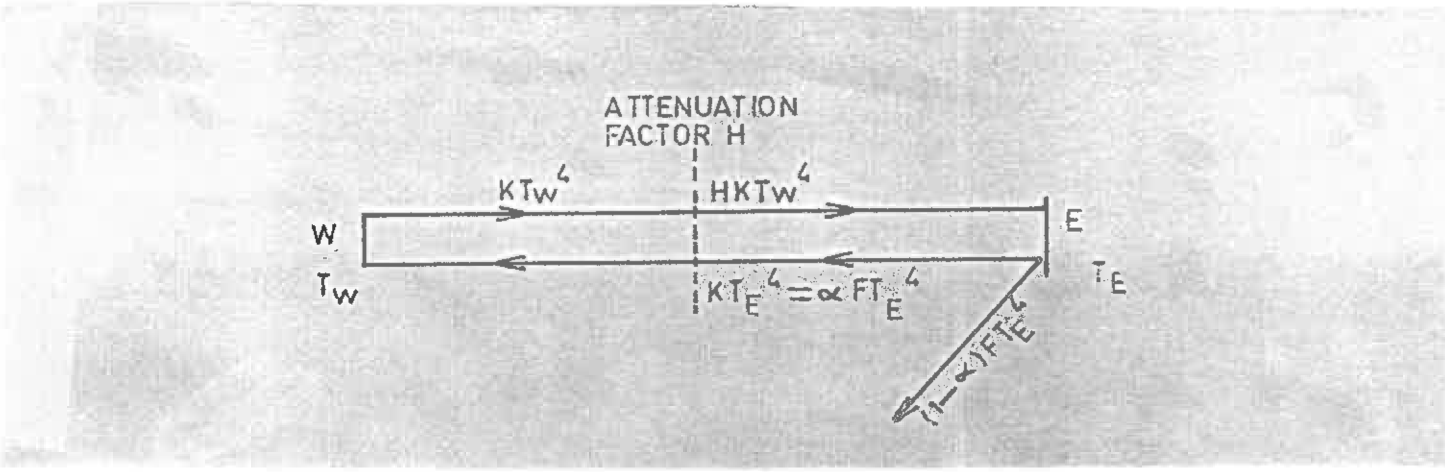

Fig. 5.1

If we assume that none of the energy absorbed in the medium reaches back to E then the essential equilibrium at E occurs between the radiation received and emitted at its surface. It follows that:

$$H K T_W^{\ 4} = F T_E^{\ 4} = \frac{K T_E^{\ 4}}{\alpha}$$

whence $$\left(\frac{T}{T_W}\right)^4 = \alpha H \qquad 5.1$$

If E is non-black, its optical properties may be described in terms of its absorptivity a and emissivity e. These two factors are defined as follows:

Absorptivity, a, is the fraction of incident radiation that is absorbed over the entire spectrum of the radiation. This definition implies that absorptivity is a function of both the spectral distribution of the incident radiation and the spectral behaviour of the absorptivity function for the absorbing surface.

Emissivity, e, is the fraction:

(emitted power)/(power that would be emitted if emitting surface were black and at the same temperature).

This definition implies that emissivity is a function of the temperature of the emitter and the spectral behaviour of the absorptivity function.

The case of a non-black absorber E is illustrated in fig. 5.2.

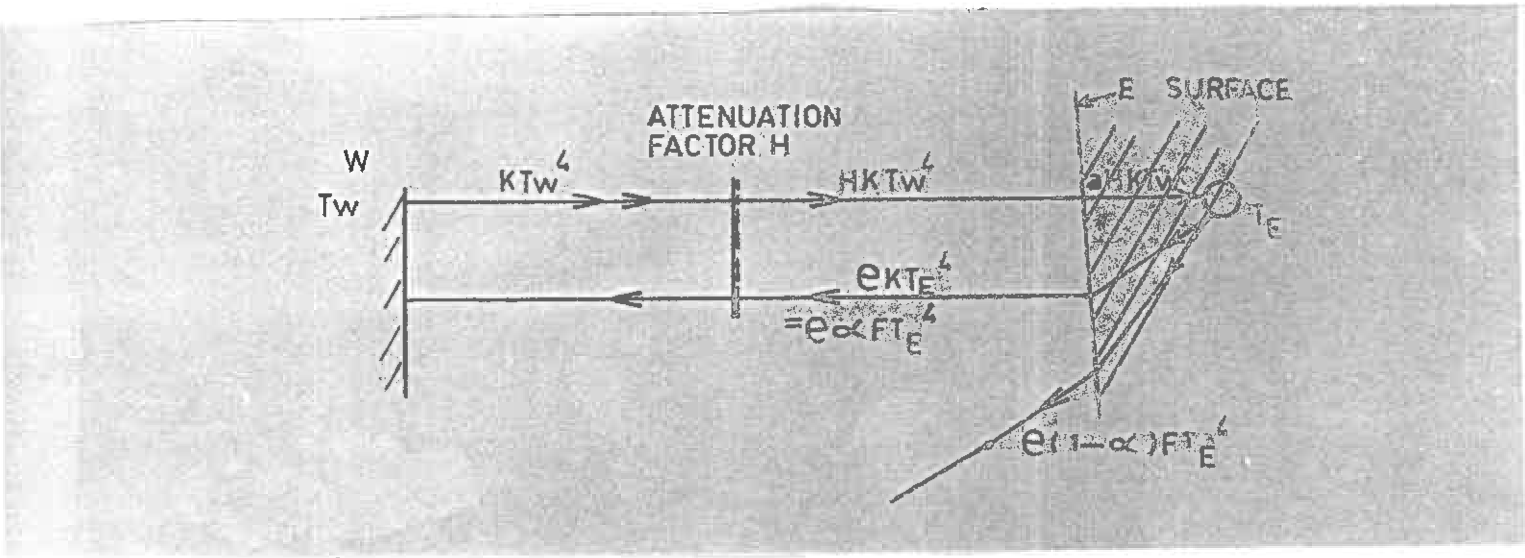

Figure 5.2

Equilibrium requires that:

$$a\,H\,K\,T_W^{\;4} = e\,F\,T_E^{\;4} = \frac{e\,K\,T_E^{\;4}}{\propto}$$

$$\text{whence}\left(\frac{T_E}{T_W}\right)^4 = \propto H\,\frac{a}{e} \qquad 5.2$$

In Appendix I it is shown that the maximum value of (T_E / T_W) with $\propto = 1$ is defined by:

$$\left(\frac{T_E}{T_W}\right) < \left(\frac{T_s}{T_W}\right) < 1 \qquad 5.3$$

where T_s is the spectral temperature of the radiation of power $H K T_W^4$ received by E. T_s is defined in the Appendix.

6. EFFECT OF A DIFFUSE SOURCE TOGETHER WITH SELECTIVE ABSORPTION

A diffuse source is a scattering medium which behaves like a source when irradiated by another source. In the ideal case the scattered radiation obeys Lamberts Law. One might expect to be able to assign an effective temperature T_D to the diffuse source and also an effective emissivity (or attenuation) H should the scattered radiation be non-black.

In Fig. 6.1 W is a source irradiating an element dD of the diffuser which is shown as a surface.

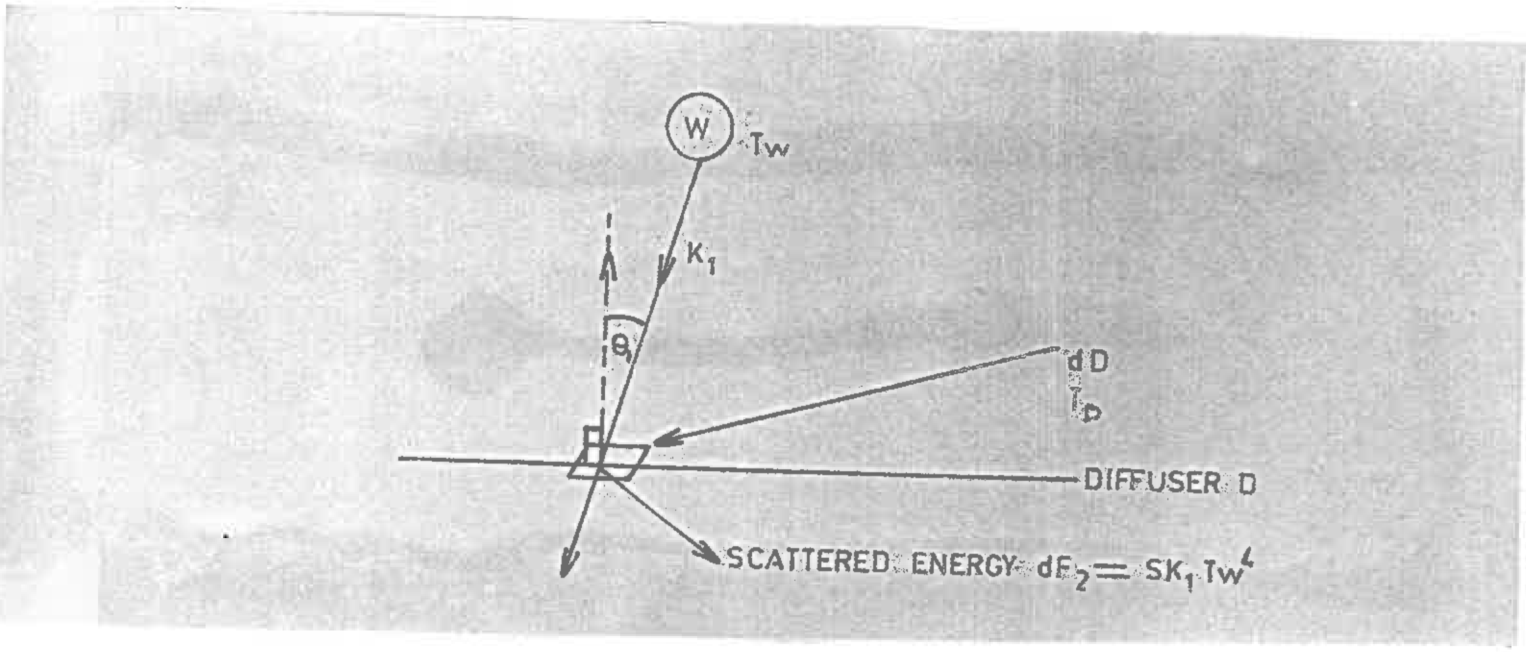

Fig. 6.1

The energy transferred from W to element dD of the diffuser D is

$$d F_{WD} = K_1 T_W^4$$

where $K_1 = \frac{\sigma}{\pi} \cos \Theta_1 \;\; \phi_W \;\; dD \qquad 6.1$

ϕ_W is the solid angle subtended by W at dD

Θ is the angle of incidence shown in fig. 6.1

Suppose fraction S of this energy is scattered forward (with a Lambert angular distribution). The total scattered energy from dD is therefore $dF_2 = S K_1 T_W^4$.

Accepting that the diffuser has, as a source, an effective temperature T_D and emissivity H, the energy radiating from element dD is $H \sigma \, dS \, T_D^4$ which may be equated to $d F_2$.

It follows, with appropriate substitution of 6.1, that:

$$E = S\left(\frac{T_W}{T}\right)^4 \frac{\cos\Theta_1 \phi_W}{\pi} \qquad 6.2$$

For many applications, e.g. where W is the sun and the diffuser is the atmosphere, $\cos\Theta \simeq 1$ in which case both H and T_D are uniform over the whole of the diffuser.

Provided H and T_D <u>are</u> uniform, the diffuser may now be treated as a source in the normal way, e.g. as depicted in fig. 5.2, and equation 5.2 describes the equilibrating temperature of a surface exposed to this source.

Thus, changing T_W in equation 5.2 to T_D and substituting 6.2 for E, we have:

$$\left(\frac{T_E}{T_D}\right)^4 = \frac{\propto aS}{e} \; S\left(\frac{T_W}{T_D}\right)^4 \frac{\phi_W}{\pi}$$

for which equation it has been assumed $\cos\Theta = 1$ throughout the diffuser.

It follows that:

$$\left(\frac{T_E}{T_W}\right)^4 = \frac{a}{e} \propto S \frac{\phi_W}{\pi} \qquad 6.3$$

This equation shows that the idea of an effective source temperature and emissivity is irrelevant to diffuser sources.

The derivation of equation 6.3 is further discussed in Appendix II together with some quantitative consequences.

7. EQUILIBRIUM OF A NON-STORING BLACK BODY INCLUDING NON-RADIATIVE LOSSES AND USEFUL OUTPUT

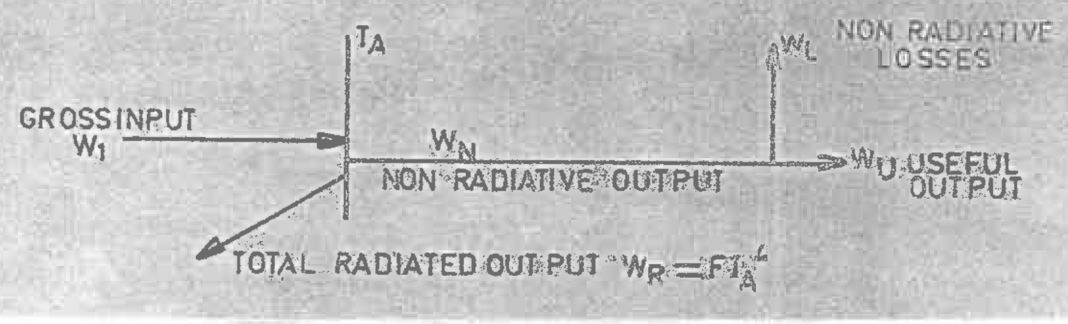

Fig. 7.1

Once derived, the equilibrating temperature T_E is one of two important parameters which together provide the basis for the further calculation of the relationship between absorber temperature T_A, radiative losses and useful energy output.

The second of the two important parameters is the gross input radiative energy W_1. This is determined by the general formula: $W_1 = \beta A w$ 7.1

where

A is the projected area of the collector on a designated plane (plane X in fig. 7.2),

w is the incident energy flux on unit area of the designated plane,

β is an efficiency of collection.

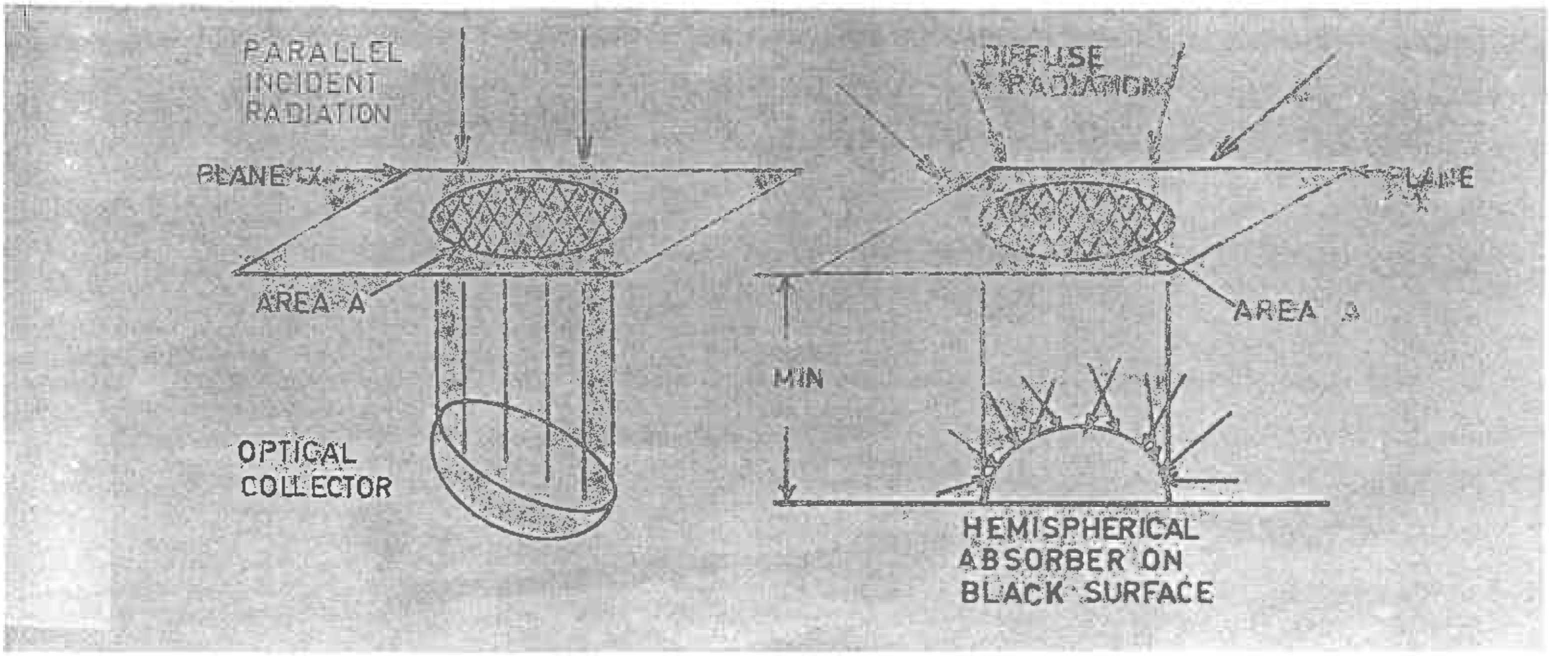

Fig. 7.2

The position and orientation of the designated plane X is chosen as a matter of convenience in performing the calculations and obviously W_1 is independent of this choice. It is usually most convenient to orientate plane X to achieve maximum w as is indicated in the examples of fig. 7.2.

The efficiency of collection, β is simply the fraction of energy falling on area A that is primarily absorbed. It is possible, in the case of diffuse radiation, for β to be greater than one but only for actual surface areas greater than A.

The gross input flux W_1 is considered as being divided into the total re-radiated energy W_R denoted, as before (with $T_E \rightarrow T_A$) by $W_R = F\,T_A^{\,4}$ and the non radiated output W_N

i.e. $W_N = W_1 - F\,T_A^{\,4}$

However it is known from the definition of T_E that $W_1 = F\,T_E^{\,4}$

Hence $$\frac{W_N}{W_1} = 1 - \left(\frac{T_A}{T_E}\right)^4 \qquad 7.2$$

Often the non radiative losses are proportional to $(T_A - T_K)$ i.e. the sink temperature is equal to ambient temperature. In this case W_L/W_1 can be cast in a more convenient form viz $\gamma\left(\frac{T_A}{T_E} - \frac{T_K}{T_E}\right)$ where use is made of the fact that both W_1 and T_E are constant as T_A is varied. Equation 8.1 may now be written more conveniently in terms of a variable $x = T_A/T_E$ and a constant $c = T_K/T_E$ thus:

$$\frac{W_o}{W_1} = \eta_w = (1 - c/x) \cdot \left(1 - x^4 - \gamma(x - c)\right) \quad 8.2$$

or

$$\eta_w = C(U-L) \quad 8.3$$

where C $(= 1 - c/x)$ is the Carnot factor;

U $(= (1-x^4))$ represents the maximum useful energy;

and L $(= \gamma(x-c))$ represents the thermal loss.

The factors C, U and L are illustrated in fig. 8.2 together with η_w as functions of x.

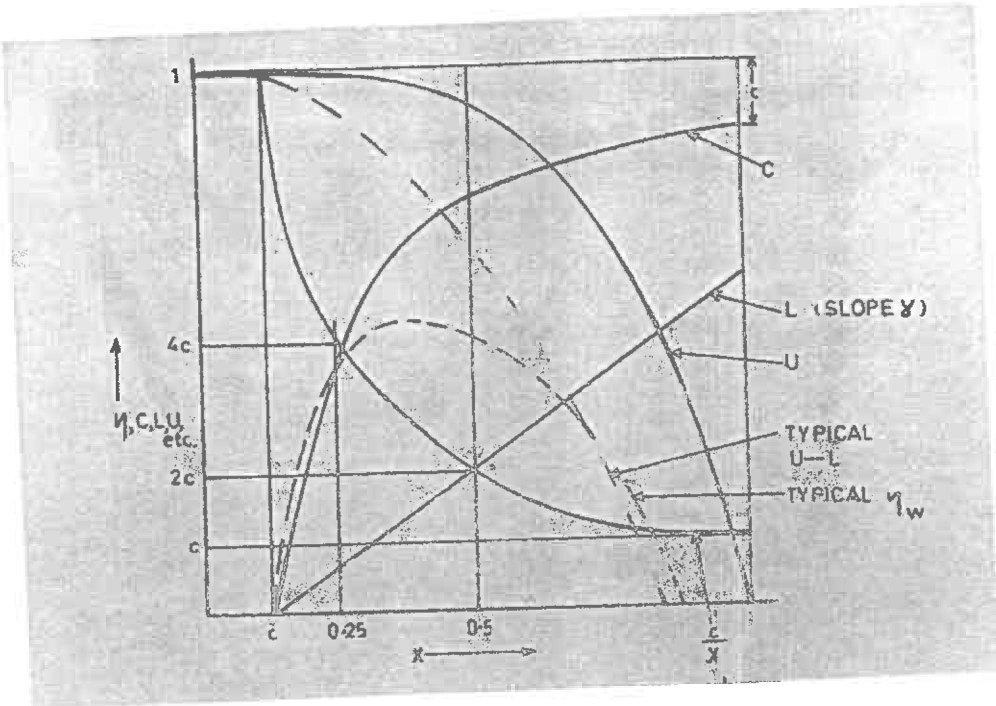

Fig. 8.2

Clearly the peak value of η_w is a function of c and γ.

It is easily identified that:

$$\gamma = k / F\,T_E^{\;3} = k\,T_E\,/W_1 \qquad 8.4$$

where $W_L = k(T_A - T_S)$

9. OVERALL CARNOT EFFICIENCY FOR A SELECTIVE ABSORBER

If the discussion of section 7 is followed through again for a selective absorber with absorptivity a and emissivity e, equation 7.2 becomes:

$$\frac{W_N}{W_1} = a_A \left(1 - \left(\frac{e_A\, a_E}{e_E\, a_A}\right)\cdot\left(\frac{T_A}{T_E}\right)^4\right) \qquad 9.1$$

where the subscripts refer to the two absorber temperatures T_A and T_E.

The effect of this modification on equation 8.3 is:

$$\eta_{ws} = C\,(a_A\, U' - L) \qquad 9.2$$

where $U' = 1 - f\,x^4$

and $f = \left(\frac{e_A}{a_A}\right) \Big/ \left(\frac{e_E}{a_E}\right)$

Now clearly f is a function of x previously defined, and when $x \rightarrow 1$, $f \rightarrow 1$.

Fig. 9.1 illustrates how f would vary with a typical e/a function. There are two scales, one for e/a as a function of T/T_W and another for f and U' as functions of T_A/T_E (i.e. of x).

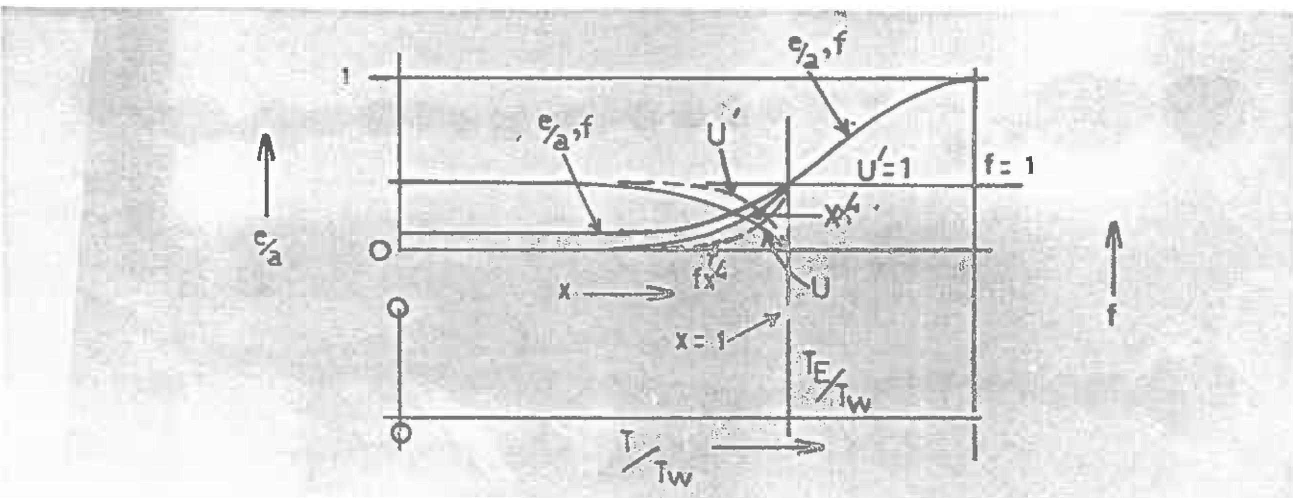

Fig. 9.1

It is clear that the amount U' differs from U is related to how rapidly f is varying in the vicinity of $x = 1$.

APPENDIX I

The question is here posed: what is the maximum possible value of (T_E / T_W) (eqn. 5.2) that can be achieved through manipulation of a and e while taking into account the effect of H.

To answer this question it is useful to define the Spectral Temperature of radiation.

Spectral Temperature. Radiation from direction $\overline{D}$ falling on normal area dA through angular aperture $d\phi$ may be characterised by a temperature T_ω for each spectral frequency . This follows from the well known fact that the radiation intensity per unit bandwidth $dW/d\phi.dA.d\omega$

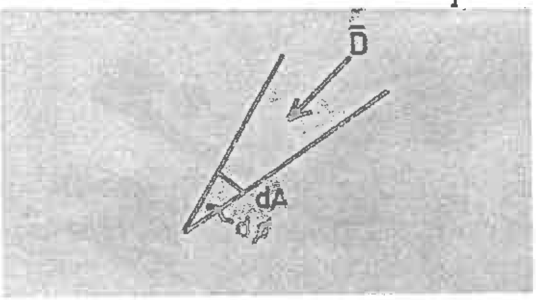

Fig. I.1

from a black body is a unique function of ω for each black-body temperature (figure I.2). The spectral temperature T_s of radiation from a particular direction is defined as the maximum value of T_ω.

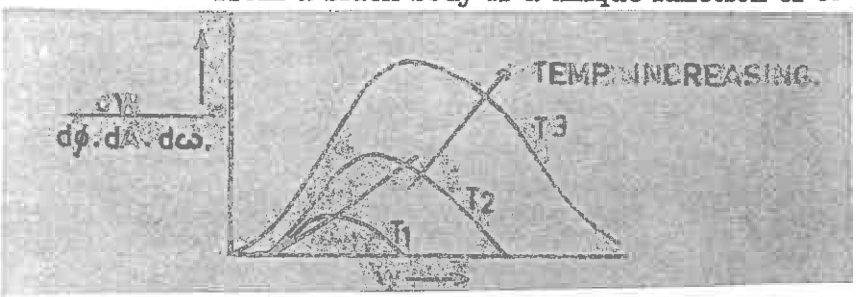

Fig. I.2

A corrolary to this definition is that, for radiation H K T_W^4 in fig. 5.2, $T_S < T_W$.

Using this definition it follows that (uniform) radiation W from a source (e.g. radiation H K T_W^4 fig. 5.2) may be represented by:

$$W = T_S(\omega) . H(\omega)$$

where

$X(\omega) . Y(\omega)$ represents the operation $\int_{\Delta X}\int_{\Delta\phi}\left(\int_0^\infty X.Y.d\omega\right)dX.d\phi$

ΔA and $\Delta\phi$ are respectively, using the notation of fig. 5.2, the area of E and the solid angle subtended by W at E;

$T_s(\omega)$ represents the function $dW/d\phi . dA. d\omega$ for the radiation from a black body at temperature T_s and of the same size and location as the source; and $H(\omega)$ is a dimensionless function of ω which is 1 at one value of ω and ≤ 1 elsewhere.

The selective absorptivity of the non-black surface can also be represented by a function similar to $H(\omega)$ - a function which we will call $A(\omega)$.

Radiation $T_s(\omega) . H(\omega)$ falling on E will result in power $A(\omega). T_s(\omega). H(\omega)$ being absorbed. In turn, if the surface temperature is T_E, the power radiated back to the source will be $T_E(\omega) . A(\omega)$.

For equilibrium we therefore have

$$\alpha\left[\underline{A(\omega) . T_s(\omega)} . H(\omega)\right] = \left[\underline{T_E(\omega) . A(\omega)}\right]$$

Now the quantities underlined are in fact the same function of the variable T. The following conclusions may therefore be drawn:

1) If $\alpha < 1$, $T_E < T_s$;

If $\alpha = 1$, then either:

2) $H(\omega) = 1$ for all ω where $A(\omega) \neq 0$, and then $T_s = T_E$;

or

3) $H(\omega) < 1$ for some ω (and always $H(\omega) \not> 1$), and then $T_E < T_s$.

The answer to our original question is therefore that $\frac{T_E}{T_w} < \frac{T_s}{T_w} < 1$.

APPENDIX II

For the special case of the diffuser being the atmosphere and the primary source the sun ($\alpha = 1$), equation 6.3 may also be derived directly from the energy balance of the small area $\triangle$ E where the incoming flux is $S\,K\,T_W^{\,4}$ and the outgoing energy is $\sigma \triangle E\,T_E^{\,4}$.

Equation 6.3 may also be checked by considering the system illustrated in fig. II.1.

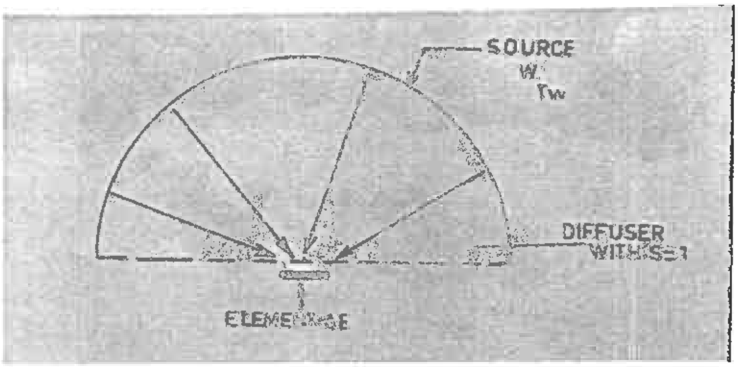

Fig. II.1

If element $\triangle$ E is close to the diffuser, $\alpha = 1$. Further, assume a = e = 1 and S = 1.

It is evident that the radiation crossing an imaginary surface just below the diffuser in fig. II.1 will be the same whether the diffuser is present or absent and in the latter case we know from equation 4.1 that $T_E = T_W$.

The same result follows from 6.3 putting $\phi_W = 2\pi$ and multiplying by $\frac{1}{2}$ to take into account the fact that cos Θ_1 averages to this value for every position of the element Δ_E.

In the special case of the atmospheric diffuser, the result expressed by equation 6.3, viz.

$$\left(\frac{T_E}{T_W}\right)^4 = \frac{a}{e}\,\alpha\;S\,\frac{\phi_W}{\pi}$$

leads to some interesting conclusions. In this case α = 1 and we may take S = .1 as a representative real case. Practical selective surface treatments e.g. as developed by the CSIRO, have $a/e \approx 10$ The solid angle subtended by the sun is 68μ sterads

We may expect therefore for these figures:

$$\frac{T_E}{T_W} \approx 6.8 \times 10^{-2}$$

Hence, taking $T_W = 6000^{\circ}K$, $T_E \approx 409^{\circ}K$ or $136^{\circ}C$. It is possible therefore, with absorbers using this surface treatment, to boil water using diffuse radiation only. However, the result of section 7 viz:

$$\text{max. possible collection efficiency} = \frac{W_N}{W_1} = 1 - \left(\frac{T_A}{T_E}\right)^4$$

indicates that it is not possible to have a thermal output greater than 31% of the diffuse radiation intensity (i. e. $\approx$ 31 watts per m^2). When allowance is made for thermal conduction and convection losses there is little or no margin for a useful net output.

On the other hand if a T_E of twice the boiling point of water could be achieved, i. e. $T_E \approx 746^{\circ}K$, the maximum collection efficiency for diffuse radiation would be close to 1 (i. e. 15/16). This in turn could be achieved by an a/e ratio of 110 (and, of course, we require $a \approx 1$).

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